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Modelling Social Learning in an Agent-Based New Keynesian Macroeconomic Model

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Modélisation de l'Apprentissage Social dans un Modèle Macroéconomique Nouveau Keynésien à Base d'Agents

Résumé

Nous proposons un modèle à base d'agents (ABM) inspiré par le modèle canonique d'équilibre général des Nouveaux Keynésiens (NMK, Woodford 2003). Nous analysons les dynamiques des variables agrégées générées par un processus d'apprentissage social des agents (ménages et firmes). L'offre de travail des ménages et leur demande de biens de consommation, ainsi que la demande de travail des firmes et les salaires qu'elles offrent, évoluent selon une logique d'imitation et d'expérimentations aléatoires suivies par ces agents. Nous étudions dans ce cadre les propriétés agrégées de l'économie et la capacité de ces mêmes agents qui sont engagés dans un processus d'apprentissage à se coordonner sur l'équilibre statique (optimal) du NMK. Notre approche est clairement différente de celle retenue par la littérature existante sur l'apprentissage dans le NMK (à la Evans et Honkapohja), car l'apprentissage est directement intégré dans le comportement des agents individuels. Cette approche originale ouvre de nouvelles perspectives concernant le NMK, et permet d'analyser de nouvelles questions sur des problèmes de coordination qui peuvent être engendrés par l'apprentissage social. Premièrement, notre analyse computationnelle (simulations de type Monte Carlo) montre que l'apprentissage social ne permet pas aux agents d'apprécier correctement les interdépendances entre les marchés à cause de l'émergence de problèmes de coordination qui se traduisent par une offre de travail insuffisante et par des dynamiques dépressives. Deuxièmement, nous mettons en évidence que ces propriétés générales de l'apprentissage social se produisent dans un contexte de (dés)équilibre général, et nous montrons que leur neutralisation, au moins sur un côté des marchés, est susceptible d'améliorer significativement les performances de l'économie. Nos résultats soulignent l'importance de modéliser dans des ABM macroéconomiques les mécanismes d'apprentissage avec précaution.

Mots-clés : Modèles économiques de simulation informatique, Modèles à Base d'Agents, Apprentissage Social, Nouveau modèle Keynésien, Equilibre Général, Problèmes de Coordination

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Keywords: Computational Economics, Agent-Based Modelling, Social Learning, New Keynesian Model, General Equilibrium, Coordination Problems

JEL: C63, D11, D21, D51, D83, E21, E32

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Modelling Social Learning in an Agent-Based New Keynesian Macroeconomic Model

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Abstract

We propose an agent-based macroeconomic model (ABM) inspired by the New Keynesian general equilibrium model (NKM, Woodford 2003). We analyse the aggregate economic dynamics resulting from social learning of agents (households and firms). Households' labour supply and consumption demand, as well as firms' labour demand and wage offers evolve through imitation and random experimenting by the agents. We study, in this setting, the aggregate properties of the economy and the ability of those learning agents to coordinate on the intra-temporal equilibrium of the original model. Our approach is quite different from the existing learning literature in the NKM (à la Evans & Honkapohja, that mainly focuses on learning for testing local stability of equilibria), since learning is directly embedded in the behaviour of the individual agents. This original approach opens new perspectives about the NKM, and allows us to ask new questions about the coordination problems that can result from social learning. First, our computational analysis (Monte Carlo simulations) shows that social learning does not allow the agents to correctly learn about the interdependence between markets, because of the emergence of coordination problems that result in insufficient labour supply and depressive dynamics. Second, we shed light on the general properties of social learning that are behind these results in a general (dis)equilibrium setting, and prove that their neutralisation, at least on the one side of the markets, can significantly improve the performance of the economy. Our results point to the importance of carefully modelling learning mechanisms within macroeconomic ABMs.

1 Introduction

The question of agents' learning process, and of their ability to coordinate their activities is central to our understanding of the properties of macroeconomic dynamics, and of their global performance in terms of production, unemployment and inflation. Unfortunately that coordination process is not easy to model. Consequently, the canonical macroeconomic approach has mainly evolved under the equilibrium assumption, and the analysis has generally been restricted to the equilibrium states of the economy. Concerning the modelling of agents' behaviour and their expectations, a natural complement to the equilibrium analysis has been proposed by the Rational expectations revolution, giving rise to the rational expectations equilibrium (REE) concept as the main tool of analysis of macroeconomic dynamics. By definition of an REE, agents use their full knowledge about the economy to determine their optimal behaviour in an REE, and that optimal behaviour is compatible across all markets. As a consequence, the REE is computed as a fixed point of a system composed of optimal behaviour of the agents given their expectations, and their rational expectations, given that behaviour (and the equilibrium conditions on the markets, since these conditions guide their expectations).

The REE concept has been quite fruitful in allowing the economic analysis of many important issues under these assumptions. But, the fundamental question of convergence towards this kind of situation has recurrently surfaced in economic discussions. The rational expectations theory has disqualified the traditional simple adaptive expectations approach. But, by construction, that theory is not able to tackle the question of the global stability of REE: are agents able to converge onto a REE (by coordinating their expectations and behaviour), if the economy starts far from it. The construction of the REE at the microeconomic level, developed following the micro foundations of macroeconomic dynamics paradigm, makes clear that this coordination problem necessarily possesses two interrelated components: how do the agents form their expectations, and how do they determine their behaviour, when the economy is far from the equilibrium?

Most of the tentative answers to that coordination problem have chosen to address the first component, by formulating an expectation formation process at the aggregate level. They in general adopt an adaptive representation of the model of economy, as perceived by the agents, and this representation takes the form of a set of aggregate behavioural equations. Agents adapt this model following their experience in the economy, and that adaptation corresponds to their learning on the economy. This deliberate deviation from the rational expectations approach aims to take into account agents' bounded rationality. Sargent (1993) presents the early modelling attempts of the 80s.[1] One of the approaches already discussed by Sargent (1993) has more recently attracted considerable attention. Evans & Honkapohja (2001) propose an exhaustive overview of the mathematical, as well as conceptual dimensions of this modelling method that has recently become a standard in the analysis of the stability of REE under adaptive learning.[2] In this approach, learning of agents is represented just along the lines of the behaviour of an econometrician who updates the sample of her observations so as to refine the estimation of the perceived model of the economy. More specifically, the learning process is usually modelled through a sophisticated version of the recursive least-squares (**RLS**) algorithm. The latter is applied to a system of aggregate equations (the perceived model) whose specification is tied to the linearisation of the equilibrium conditions corresponding to a REE in the model. Using such a locally defined model, this approach can therefore study the possibility, for this continuously updated RLS estimations, to converge to the REE of the model. This approach consequently extends the REE analysis, by testing its local stability in respect with this learning process (*E-stability* in the terminology of Evans and Honkapohja). Especially, when the REE is not unique, E-stability can become an interesting criteria for discriminating between the different REE.

This new method definitely contributes to our understanding of aggregate economic dynamics in the neighbourhood of a REE. Unfortunately, it gives rise to new problems when it is deployed in a macroeconomic model with micro foundations, like the canonical New Keynesian DSGE model (NKM henceforth, see Woodford 2003). In a micro-founded model, this approach represents the evolution of expectations only at the level of the aggregate reduced model, defined in the neighbourhood of a REE. But this REE necessarily depends, on its turn, on the optimality conditions of agents' decisions. Consequently, in such a microeconomic model, the behaviour, as well as the expectations must be defined at the individual level, since agents' expectations must guide their decisions, and the optimality of the latter depends on these expectations (see Preston 2005b,a). Otherwise,

[1]See also Sargent (1999).

[2]For a more recent survey on that literature and its developments, see Evans & Honkapohja (2009).

nothing really ensures the consistency of individual behaviours, on which the REE is founded, with the aggregate representation of adaptive expectations, defined in the vicinity of this REE. Evans, Honkapohja and other authors propose some schemes to eliminate this inconsistency. Unfortunately they are not very convincing, because they are mainly based on the substitution of some form of imperfect knowledge for the genuinely bounded rationality of the agents (*à la* Simon 1955, 1976). In Branch *et al.* (2012), for example, households are able to compute the Euler equation, but only with a finite horizon, instead of an infinite one. Consequently, more than their computational ability (they are able to optimise), their perception of the problem to solve is *bounded* in this case.[3] These schemes try in fact to give some bounded rationality to individual decisions (mainly limited to the modification of the constraints of their optimisation program), while keeping their consistency with the adaptive expectations formulated at the aggregate level. As a consequence, this analysis is constrained from the start by the aim to give some micro foundations to the RLS approach formulated at the aggregate level, and do not fully take into account the bounded rationality of the agents: an optimisation based decision process is finally proposed as micro-foundation of adaptive expectations, formulated at the aggregate level.

We think that an alternative way of tackling this inconsistency problem in the NKM would indeed consist in fully introducing bounded rationality and learning at the individual level, in the formulation of agents' behaviour. One approach has already been tested in other macroeconomic models, and it constitutes a natural candidate in our eyes: social learning of individual agents (see Arifovic 2000). Agents observe other agents' past behaviour (for example, consumption and labour supply levels) and performance (for example, utility), and they imitate them as a function of that performance. They also make random experiments from time to time, by randomly choosing some other completely new decision. In this case, agents learn directly on their behaviour, and we do not need to formulate separate processes for their decisions and expectations. Expectations are directly embedded in agents' behaviour, and they learn both on different alternatives available, and on their potential performance (in terms of utility or profit). Consequently, the approach we propose suppresses the separation between learning on expectations and learning on behaviour, and necessarily eliminates the potential inconsistency between those processes.[4] We can also easily take into account, with such an approach, the emergence of agents' heterogeneity as a consequence of their learning process.

When we include those more complex processes in our models, we must accept to reduce their analytical tractability, and adopt a computational approach for analysing their properties. The issues that can be studied using such models based on bounded rationality, adaptive individual learning, and heterogeneity of the agents, are really fundamental to our understanding of economic dynamics. Consequently, many new computational agent based models (ABM) have been recently developed for tackling economic issues at different levels and scope (going from organisational dynamics to macroeconomic dynamics, for a review, see Tesfatsion & Judd 2006; Tesfatsion 2006). Moreover, adopting such a modelling approach has a specific advantage for the micro foundations of macroeconomics paradigm: it allows to fully micro-found macroeconomic models, and allow their analysis without any *a priori* reference to an equilibrium. Equilibria can emerge only if they belong to the set of attractors of the learning dynamics.

We propose, in this article, to analyse the properties of macroeconomic dynamics in an ABM with social learning. We introduce bounded rationality and learning at the level of individual decisions

[3]See Evans *et al.* (2009) for another approach based on imperfect information, in terms of structural knowledge this time.

[4]But see also Arifovic *et al.* (2012) for an approach that is based on social learning, but that is afflicted by the inconsistency problem.

in the canonical NKM, in order to be able to refer to the properties and the REE of this model. We hence construct a NK-ABM where the ABM is kept as close as possible to the NKM in terms of general structure, and functional forms, while allowing for learning at the individual level, and rationing on the markets. This deliberate homeomorphism should allow us to qualify the specific role played by social learning. Consequently, our ABM may be called New Keynesian, at least for three reasons: first, because of its structural proximity with the NKM; second, because the learning process we specify for the firms might bring about to price stickiness from the part of the firms; finally, because Keynesian features are part of the construction of the model, such as the absence of the market clearing assumption, as well as of intertemporal concerns from the part of the agents. This model is used to address the following questions. Can social learning allow the resolution of the coordination problems in the economy, and the convergence towards a REE? More generally, what are the properties of macroeconomic dynamics resulting from this learning process? Can we determine a specific and general role played by social learning? What can we learn, more generally, about the potential role of this approach in micro-founded macroeconomic models with multiple markets?

We proceed as follows to answer these questions. Section 2 presents the agent-based macroeconomic model that we build and we use for our analysis. As we depart in that respect from the standard assumptions made in the canonical NKM, we put particular attention to the specification and modelling of the behaviour, learning processes and interactions of the agents. In section 3, we describe the protocols we use for the generation of our results using computer simulations, and for the statistical analysis of these results. Particular attention is provided to the kind of sensitivity analysis we implement here, given the computational dimensions of the model. Sections 4 presents our results. We observe that social learning cannot ensure satisfactory dynamics, and aggregate performances, when this kind of learning takes place in a context of complex, interdependent functioning of markets. Section 5 shows that these negative results concerning social learning are based on a general mechanism: imitation is necessarily associated, in this setting, with negative externalities and coordination problems. The latter lead the economy into depressive dynamics. Those externalities are reinforced by the presence of rationing mechanisms on the markets. We consider in section 6 a variant of our baseline model where those externalities are neutralised on the one side of the markets (on the households' side). We show that firms' learning is able, in this case, to steer the economy towards a path with increasing economic performance and welfare. Section 7 briefly concludes.

2 The model

We build a framework whose basic structural features are as close as possible to the ones that underlie the NKM. In particular, as in the canonical setting, we model a perishable single-good economy where labor is the only input and, as a consequence, we disregard capital (cf. Woodford 2003, chapter 2 to 4 or Galí, 2008, chapter 2 and 3). However, and while the NKM assumes intertemporal optimisation from the part of households and firms in a setting with rational expectations and market clearing, the agent based model we build displays radically distinct rules of functioning. We model the economy as a complex adaptive system (Holland, 1996): households and firms operate under bounded rationality, choosing their strategies on the basis of a social learning process. This process has two components: imitation between agents, and random experiments (mutations). Furthermore, the matching between agents' supplies and demands on the labor and goods markets does not necessarily lead to market clearing, and agents can be rationed. Our assumptions on the rationing mechanisms on the markets are kept as close as possible to the functioning of the markets in the NKM (competitive labour market, and imperfect competition on the good market). Thus, our model adopts a structure very close to the NKM, but where disequilibrium, as well as equilibrium configurations can emerge on the markets.

In what follows, we present the ABM used throughout this study. First, we specify the behaviours of the n households and m firms that populate the economy that we consider in the model (see subsections 2.1 and 2.2). Aggregate interactions and dynamics that ensue from such behaviours as well as learning mechanisms are then presented in subsections 2.3 and 2.4.

It appears beforehand necessary to detail the sequence of the interactions between the households and the firms that we assume to take place within each of the periods that are covered by the ABM. In the equilibrium approach that underlies the NKM, such a step is not required: the decision processes of the agents are envisioned simultaneously as their mutual consistency obtains each period through market clearing. Because we depart from such a process in our setting, we need to explicitly account for the sequence of operations that we consider in the ABM, as this sequence does presumably frame the decisions that the agents take on the different markets on which they successively interact. Thus, at any given period, five different steps prevail:

1. First, the firms determine their labor demand and their related wage offers while the households decide upon their labor supply and desired consumption (see subsections 2.1 and 2.2). Except for the initial period, those decision variables directly stem from learning of agents (see step 5).

2. Households and firms meet on the labor market. When all feasible transactions have taken place, the quantity of labor hired by each firm, as well as the amount of labor actually supplied (worked) by each household and the associated wage bill are determined.

3. Each firm uses the quantity of labor she has hired to produce the consumption good. Summing all the wage bills she received, each household is able to determine her labor income. With the addition of its financial component (dividends and saving proceeds), she computes her total income that bounds her consumption decisions (budget constraint).

4. Firms and households meet on the good market. At the end of this process, the product of the sales made by each firm allows her to compute her profits. The actual consumption level of each household, together with her labour supply, allows her to compute her satisfaction level (utility). Moreover, she saves the amount of her income that has not been spent in consumption (if she has been rationed on the good market).

5. The strategies of the firms (labor demand, wage offers) and of the households (labor supply, desired consumption) are updated through learning, that shapes their behaviour during the next time period.

2.1 Households

At any given period, each household has to decide upon the level of two individual variables: the quantity of labor that she wants to supply on the labor market (noted as $h^s_{i,t}$); the desired level of consumption (i.e. the desired level of real consumption in the single good) noted as $c^d_{i,t}$, that she would like to buy on the good market.

In our setup, those two decision variables are not treated as the outcome of an optimising behaviour from the part of the household, that would typically lead to a tradeoff between leisure and labor in the first case and an intertemporal smoothing of consumption in the second case. We rather assume that the households proceed in this respect along the lines of bounded rationality and modify their choices overtime through learning. Eventually, such an adaptive behaviour may deliver consumption (and labor supply) paths that are close to the ones associated with the approach based on the maximisation of utility. But it may also exhibit rather different trajectories featuring strong deviations from this benchmark.

We assume that, in the first period, the values of the two decision variables composing the strategy of the consumer, $(h^s_{i,0},\ c^d_{i,0})$, are drawn from (independent) uniform distributions. At each further period, the strategies are updated according to the learning mechanism we specify in subsection 2.4.

Labor supply As labor suppliers, households are implicitly considered as price-takers on the labor market. Although the household's labor supply does not depend at each period on the level of the wage, the introduction of learning and the ensuing dynamics may lead to a relationship between the two variables over time.

Consumption Regarding consumption, we assume that households are subject to a period-by-period budget constraint that prevents them from consuming more than their current income. This constraint in nominal terms may be expressed as follows:

$$\widetilde{c}_{it} \quad \leq \quad \widetilde{y}_{it} \tag{1}$$

where $\widetilde{c}_{it}$ stands for the level of nominal consumption by household i and $\widetilde{y}_{it}$ refers to the level of (nominal) income of this household.[5]

In a bounded rationality context, households are not supposed to be able to forecast their future income streams and the level of interest rates, so as to implement an optimal smoothing of their consumption paths. Consequently, only the flow budget constraint is relevant for the households' decisions. This does not prevent, however, that intertemporal concerns may emerge from the functioning of the economy due to the wealth dynamics (driven by savings) and learning. Furthermore, we also take the view that credit constraints (not explicitly modelled) may prevail on the financial markets so that households are not allowed to borrow for financing their consumption expenditures over time.

Income As we have mentioned, household income is made up of two components: labor income on the one side and financial income on the other side. Financial income has itself two origins: the first relates to the profits of the firms of previous period that are distributed to the households.[6] The second comes from the proceeds of saving that the households may have built up at the end of the previous period. Accordingly, the (period t) whole (nominal) income $\widetilde{y}_{it}$ that household i receives is computed as:

$$\widetilde{y}_{it} = w_{i,t} \cdot h_{i,t} + d_{it} + b_{i,t-1} \cdot (1 + i_{t-1}) \tag{2}$$

Labor income corresponds to the product $(w_{i,t} \cdot h_{i,t})$. h_{it} stands for the whole quantity of labor that is actually supplied by household i and hired by possibly different firms at period t. Accordingly $w_{i,t}$

[5]Note that this budget constraint refers to nominal consumption, while desired consumption is in real terms. The relationship between the two depends on how the matching between supply and demand obtains on the good market (see also subsection 2.3).

[6]The New Keynesian framework allows for simultaneous determination of all endogenous variables; this is not possible in agent-based models, because computer simulations follow scrupulously the train of events. This feature of computational economics explains why period $t-1$ profits cannot be paid to households in $t-1$, but have to "wait" till period t.

stands for the composite wage that household is paid for her labor services.[7] Regarding financial income and as in the NKM framework, we assume that firms convert their profits into dividends which are paid to the households. We assume that only households with positive wealth can hold shares of the firms, and that each household receives a dividend (d_{it}) whose amount is tied to the relative wealth of the household at the end of period $t-1$ (relative wealth is computed as the share of household's income in the sum of all the (recorded) positive incomes for a given period). Thus, we have: $d_{it} = \varpi_{i,t-1} \cdot \left(\sum_{j=1}^{j=m} \pi_{j,t-1} \right)$ where $\varpi_{i,t-1} \geq 0$ refers to the relative positive wealth of household i at the end of period $t-1$ and $\pi_{j,t-1}$ to firm j's profit at the end of period $t-1$. More precisely

$$\varpi_{i,t-1} = \begin{cases} \dfrac{\widetilde{y}_{it-1} - \widetilde{c}_{it-1}}{\sum_{j \in \Delta} (\widetilde{y}_{jt-1} - \widetilde{c}_{jt-1})} & if \ \widetilde{y}_{it-1} - \widetilde{c}_{it-1} > 0 \\ 0 & otherwise \end{cases} , with \ \Delta = \{ i \,|\, \widetilde{y}_{it-1} - \widetilde{c}_{it-1} > 0 \}$$

Finally, $b_{i,t-1}$ is the amount of savings that household i has built up at the end of period $t-1$ and invested on the financial market. Saving proceeds are thus given by $b_{i,t-1} \cdot (1 + i_{t-1})$ where i_{t-1} stands for the interest rate prevailing between $t-1$ and t on the financial market.

2.2 Firms

Each firm j has also a two-components strategy $(h^d_{j,t},\, w_{j,t})$. $h^d_{j,t}$ is the quantity of labor that j wants to hire (labor demand) in period t while $w_{j,t}$ is her wage offer.

Labor demand and wage offers As in the case of households, we suppose that the firms set their labor demand and wage offers under bounded rationality. The initial values of the two decision (individual) variables $(h^d_{j,0}, w_{j,0})$ are drawn from (independent) uniform distributions, while at each further period, firms' strategies are updated according to the learning mechanisms we specify in subsection 2.4.

Good production and price setting The good production of each firm j results from the quantity of labour $h_{j,t}$ that j has hired, and by the production function. For the latter, we retain the standard Cobb–Douglas specification (as in the New Keynesian framework, see Galí 2008, p. 18)

$$y^s_{j,t} = A h^{1-\alpha}_{j,t} \tag{3}$$

$A > 0$ represents the level of technology; assuming decreasing returns, we set $\alpha \in [0, 1[$. All firms are identical with regard to their production technology, i.e. A and α are the same for all firms and we moreover normalise $A = 1$ (as in Woodford 2003, p. 225).

Following the New Keynesian framework, we assume imperfect competition on the good market. As a consequence, each firm j sets its price according to a markup scheme over marginal cost, with a markup factor $\mu \geq 1$

$$p_{j,t} = \mu \left[\frac{w_{j,t} \cdot \left(y^s_{j,t} \right)^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \right], \tag{4}$$

where μ reflects the market power of the firms.

[7] $w_{i,t}$ is a composite wage whenever household i has supplied labor to different firms that offer different wages $w_{j,t}$.

2.3 Functioning of markets

The individual behaviour of firms and households that we assume in this model departs from the standard approach that is based on the utility or profit optimisation. In a related way, there is no implicit market clearing process ensuring that transactions proceed at equilibrium so as to fulfil these optimal choices. We need therefore to explicitly model the interactions that prevail on the markets, and how individual supplies and demands are matched.

Labor market In order to stay close to the structure of the New Keynesian framework, we assume an efficient rationing mechanism that maximises the households' surplus on the labor market. Firms are ranked by decreasing wage offers and meet households which are ranked by decreasing labor supply. The first ranked firm starts by hiring labor supplied by the first-ranked household. Whenever a firm has succeeded to hire labour so as to fulfil her labour demand, she exits from the matching process (the same is true for an household who has fulfilled her labour supply). When all transactions on the labor market have proceeded, the quantities of labor that are hired by each firm ($h_{j,t}$) on the one hand, and actually supplied by each household ($h_{i,t}$) on the other hand are determined[8] (as well as the different wage levels that apply to these quantities). Accordingly, when the labor market closes, firm j's labor cost $h_{j,t} \cdot w_{j,t}$ as well as household i's labor income $h_{i,t} \cdot w_{i,t}$ can be computed. Rationing may occur from either side of the market: firm j is rationed when $h^d_{j,t} > h_{j,t}$ while for household i, this happens when $h^s_{i,t} > h_{i,t}$.

Good market We also assume an efficient rationing mechanism on this market. Firms are ranked by increasing prices and meet households which are ranked by decreasing good demand. The same process operates as for the labour market: firms offering the lowest prices do interact first with households holding the highest level of real desired consumption. When all the transactions have proceeded, the quantity of the good that is sold by each firm ($y_{j,t}$) on the one side and that is bought (i.e. consumed) by each household ($c_{i,t}$) on the other side is determined[9] (as well as the price that values each of the underlying transactions). Whenever she transacts with a firm, each household does check whether her related (desired) nominal expenditure is in line with her budget constraint.

When the good market closes, firm j's profit is computed as follows:

$$\pi_{j,t} = p_{j,t} \cdot y_{j,t} - w_{j,t} \cdot h_{j,t}, \tag{5}$$

while the savings amount made by household i is defined as

$$b_{i,t} = \widetilde{y}_{i,t} - \widetilde{c}_{i,t}, \tag{6}$$

with $\widetilde{c}_{i,t} \equiv p_{i,t} \cdot c_{i,t}$ and $p_{i,t}$ is the composite price paid by the household i on her consumption bundle.[10]

2.4 Model dynamics and learning

Two kinds of dynamics prevail in our model. The first one stems from savings that provide an intertemporal transfer of income, thereby potentially affecting consumption choices over time. The

[8] We have $\sum_{j=1}^{j=m} h_{j,t} = \sum_{i=1}^{i=n} h_{i,t}$.

[9] We have $\sum_{j=1}^{j=m} y_{j,t} = \sum_{i=1}^{i=n} c_{i,t}$.

[10] $p_{i,t}$ is a composite price index if the household has bought different quantities of the good from firms that set different prices $p_{j,t}$.

second and main dynamics, on which we focus in the paper, is due to learning. Learning is a natural component of the bounded rationality of households and firms in this economy, and allows those agents to adapt and update their strategies over time.

We assume that this adaptation occurs through a social learning mechanism (imitation) coupled with a random exploration of the space of strategies by each of the agents under concern (random experimenting). That twofold learning process is widely used in the literature[11] and well-suited to represent learning in a heterogeneous population of agents, who aim to adapt their behaviour to the evolution of their environment.

Households' learning Household's learning is driven by their performance measured through a standard utility function. Following Galí (2008), the period t utility of each consumer i is given by the CES function:

$$u_{i,t} = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \frac{h_{i,t}^{1+\phi}}{1+\phi}, \tag{7}$$

where utility is increasing in consumption and decreasing in labor.

Imitation With a given probability (of imitation), Pr_{imit}, each household i can have a possibility of imitating another household's strategy. In case of imitation, the probability of being imitated ($Pr_{copy,k,t}$) for an household k is proportional to her relative performance:

$$Pr_{copy,k,t} = \frac{exp(u_{k,t})}{\sum_{i=1}^{n} exp(u_{i,t})}, \tag{8}$$

If household k is designated to be imitated by i, i's strategy is updated as follows:

$$(h_{i,t+1}^{s}, c_{i,t+1}^{d}) = (h_{k,t}, c_{k,t}). \tag{9}$$

This imitation process favours the diffusion, in the households population, of strategies corresponding to the highest performances. We furthermore assume that what is imitated are the actual levels of consumption and labour supply of the household k, at the beginning of period $t-1$, and not the desired levels of those variables. Only the former correspond indeed to the actual relative performance that underlies the imitation process. Moreover, the desired levels are private information, and are normally unobservable by other agents, especially when k is potentially subject to rationing.

Random experimenting (mutation) With a given probability of mutation, Pr_{mut}, household i's strategy can be modified by random experiments. In this case, household i draws a new labor supply out of the normal distribution[12] $\mathcal{N}\left(\frac{\sum_{i=1}^{n} h_{i,t}}{n}, \sigma_{mu}^2 \times \frac{m}{n}\right)$ and a new level of desired consumption out of $\mathcal{N}\left(\frac{\sum_{i=1}^{n} c_{i,t}}{n}, \sigma_{mu}^2 \times \frac{m}{n}\right)$.

In case household i neither imitates nor performs a mutation of her strategy set, the latter set remains unchanged for the next period.

[11] *See* notably Holland *et al.* (1989), Sargent (1993) and Brenner (1999) for general statements. Applications to economic issues include for example Arifovic (1995) or Vallée & Yildizoglu (2009).

[12] Note that the means of these normal distributions are equal to the average quantity of labor actually supplied by the households and to their average actual consumption. The variance σ_{mu}^2 is calibrated using the factor m/n, in order a) to allow the consumers to discover strategies potentially compatible with the number of firms in the economy, b) to neutralise any bias we could introduce by changing the number of agents in our experiments. Negative draws out of these normal distributions are converted into 0.01 in order to avoid negative labor supplies and negative good demand.

Firms' learning Firms learn much in the same way as households. With probability Pr_{imit}, each firm j is allowed to imitate another firm's strategy. High profit firms should be more likely to be imitated than low profit firms. Consequently, the probability of being imitated ($Pr_{copy,l,t}$) for a firm l is proportional to her relative profit:

$$Pr_{copy,l,t} = \frac{exp(\pi_{l,t})}{\sum_{j=1}^{m} exp(\pi_{j,t})}. \tag{10}$$

If firm l is designated to be imitated by j, j takes up l's *observable* strategy:

$$(h^d_{j,t+1}, w_{j,t+1}) = (h_{l,t}, w_{l,t}). \tag{11}$$

With probability Pr_{mut}, firm j is subject to mutation: j draws a new labor demand strategy out of the normal distribution[13] $\mathcal{N}\left(\frac{\sum_{j=1}^{m} h_{j,t}}{m}, \sigma^2_{mu}\right)$ and a new wage strategy out of $\mathcal{N}\left(\frac{\sum_{j=1}^{m} w_{j,t}}{m}, \sigma^2_{mu}\right)$.

In the absence of mutation or imitation, the strategy set of the firm i remains the same. Let us note that the specification of such a learning process for the firms does allow for some degree of price stickiness in the economy. Indeed, the firms whose wage and labor demand strategies remain unchanged between two periods, set the same price in period t as in $t-1$ (unless rationing conditions on the labor market change for these firms).[14]

2.5 Computation of aggregate variables

In what follows, the functioning of the economy will be analysed through aggregate indicators.[15] The latter are obtained by summing the related individual variables. For the labor market, we look at aggregate labour demand $\left(H^d_t \equiv \sum_{j=1}^{m} h^d_{j,t}\right)$, aggregate labour supply$\left(H^s_t \equiv \sum_{i=1}^{n} h^s_{i,t}\right)$ and unemployment $\left(U_t \equiv \sum_{i=1}^{n} \left(h^s_{i,t} - h_{i,t}\right)\right)$. On the good market, aggregate good demand (equivalent to aggregate real, desired consumption) is computed as: $Y^d_t \equiv \sum_{i=1}^{n} y^d_{i,,t} \left(= \sum_{i=1}^{n} c^d_{i,,t}\right)$. aggregate supply $\left(Y^s_t \equiv \sum_{j=1}^{m} y^s_{j,t}\right)$ and aggregate output $\left(Y_t \equiv \sum_{j=1}^{m} y_{j,t}\right)$ indicators complete the picture, as well as the price level that is computed as a composite price index $P_t \equiv \frac{\sum_{j=1}^{m} p_{j,t} y_{j,t}}{\sum_{j=1}^{m} y_{j,t}}$. Finally, the social welfare indicator is computed as: $W_t \equiv \sum_{i=1}^{i=n} u_{it}$.

3 Simulation protocol

The main objective of this paper is to analyse what key mechanisms underlie the functioning and evolution of the economies we have considered in the ABM and, in particular, to look at the role of social learning in that regard. To this aim, we perform extensive sensitivity analysis of the model whereby the performances of different economic configurations are compared. Each economy is characterised by a set of values for the parameters that frame its functioning. These structural

[13]Negative draws out of these normal distributions are converted into 0.01 in order to avoid negative labor demands and negative wages.

[14]Unlike the New Keynesian framework, there is no need here to introduce partial price stickiness by some Calvo (1983) process, i.e. an exogenous Poisson process that forces each firm with some probability to keep the price used in the previous period.

[15]Small letters stand for individual variables and capital letters stand for aggregate ones.

parameters refer, respectively, to the marginal disutility of labor (ϕ), the marginal utility of consumption (σ), the markup over the marginal cost (μ), the returns to scale of the production function (α), the initial wealth of households (b_0), the number of households (n), the number of firms (m), as well as the learning parameters, the probability of imitation (P_{imit}), the probability of mutation (P_{mut}) and the standard deviation for mutation draws $\left(\sigma^2_{mut}\right)$.[16] We run Monte Carlo simulations over the multidimensional space generated by the set of parameters and observe the distributions of the main aggregate indicators (that we have defined *supra*) resulting from them. We will mainly use the first moments of these distributions (for each period) in our analysis.

Parameters values in Monte Carlo simulations				
α $[0, 0.5[$	μ $[1, 2.5]$	σ $\{0.16, 0.9\}$	ϕ $\{0.1, 0.9\}$	b_0 $\{1, 5, 10, 50\}$
P_{imit} $[0.01, 0.3]$	P_{mut} $[0.01, 0.1]$	σ^2_{mut} $[0.5, 2]$	n $\{100, 150, 200, 500\}$	m $\{10, 20, 30, 50\}$
Initialisation				
$w_{j,0}$ $]0, 1]$	$h^d_{j,0}$ $[0, \frac{2n}{m}[$	$c^d_{i,0}$ $]0, 2]$	$h^s_{i,0}$ $[0, \frac{2n}{m}[$	$(i_t)_{t=0,1,2,...}$ 0

Table 1: Summary of the Monte Carlo experiments

4 Results

We first look at the performance of the different economies that we simulate, and try to identify the main underlying mechanisms of the dynamics we observe.

Figure 1 shows that social learning is not able to support a steady increase and stabilisation of the social welfare. After an initial phase of utility increase, the welfare of the consumers decreases in a continuous way, even if the profits increase. The paradox is only illusory, however, as we show in the next paragraph.

The emergence of a depressive dynamics in time drives the inferior performance of social learning in this economy. Figure 2 clearly shows that both markets suffer from a continuous crunch of both demand and supply sides of the market.

Compression of labour supply is more clearly shown in Figure 3 where we represent full distributions of this variable at different time periods. We observe that this supply is important in the early periods, but it progressively collapses in time, and its mass is concentrated on levels close to zero in the last periods. Without any labour supply, no production is possible and consequently, the whole economy collapses. The welfare can only decrease under such an evolution. Why does such a collapse arise in this economy?

The collapse results from the inability of the agents' learning to diffuse levels of labour supply favourable to social welfare. Table 2 shows that the culprit is the imitation process since its negative effect on labour supply is significant. Imitation favours the diffusion of increasingly lower labour supplies in the economy, while larger exploration ($deviationMutateRate$) is associated with higher labour supply. What are the mechanisms behind these depressive role of imitation in this economy?

[16] Table 1 gives the corresponding ranges of values considered for each of the parameters, as well as for the initial values for the individual variables. See also Appendix A.2 for the description of other notations and experiments.

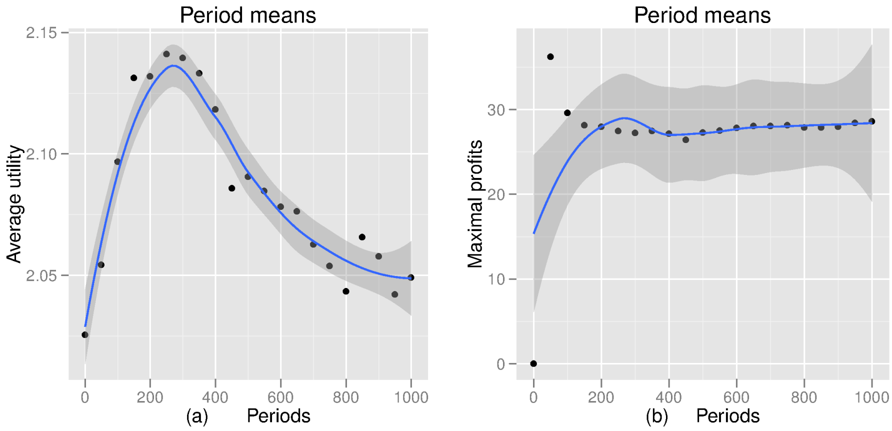

Figure 1: Evolution of the global performance of the economy: average utility and maximal profits in the full model

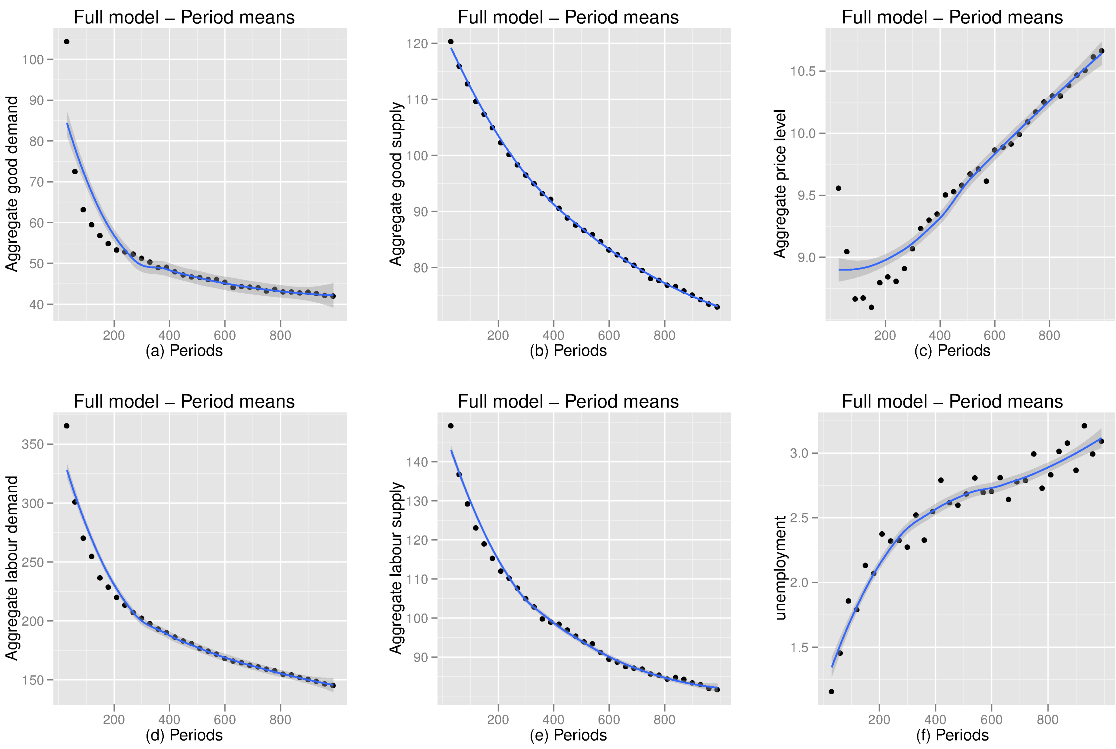

Figure 2: Compression of economic activity in time in the full model

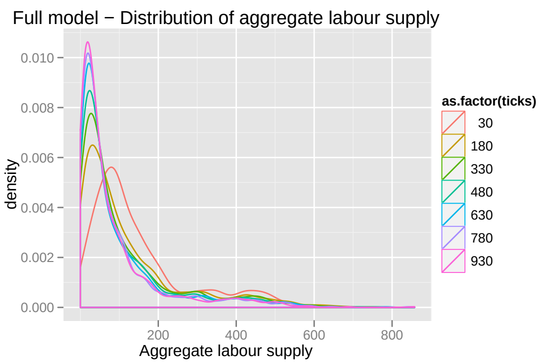

Figure 3: Compression of labour supply in time: shift of the distribution towards the left over time

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.2767	0.0087	31.78	0.0000
nbconsumers	0.0001	0.0000	13.99	0.0000
nbfirms	0.0029	0.0001	30.00	0.0000
initialCWealth	-0.0014	0.0001	-19.11	0.0000
probImitate	**-1.0312**	0.0157	-65.69	0.0000
deviationMutateRate	0.0740	0.0025	29.07	0.0000
sigma	0.5382	0.0041	130.07	0.0000
mu	-0.0063	0.0029	-2.21	0.0271
alpha	-0.1847	0.0070	-26.37	0.0000
phi	0.1067	0.0035	30.17	0.0000
ticks	-0.0002	0.0000	-33.02	0.0000

Residual standard error: 0.2443 on 29989 degrees of freedom
Multiple R-squared: 0.4726, Adjusted R-squared: 0.4724
F-statistic: 2687 on 10 and 29989 DF, p-value: $< 2.2e\text{-}16$

Table 2: Determinants of the average labour supply in the full model

5 Discussion: the depressive role of imitation externalities

The negative results of the preceding section indicate that a learning process mainly based on imitation can be unfavourable to the emergence and persistence of high social welfare in this economy. This result is not due to the specificity of our numerical setup or simulations. We now show how negative externalities can arise, resulting from the interaction of the social dimension of learning and the adjustment process on labour and good markets (associated with rationing). In a nutshell, imitation creates externalities between the dynamics of agents' strategies on both markets, and drives the adoption of strategies with less and less labour supply, hence yielding lower production and consumption levels in the economy. That dynamics is sustained by the rationing process that favours the diffusion of strategies associated with low levels of labour (supplied and hired). As a consequence, the externalities tend to pull the economy towards the lower equilibrium with $0-$consumption. After having discussed these mechanisms, we introduce, in the next section, a variant of our model that neutralises them (on the side of the households) and allows for the continuous increase of social welfare.

We first describe the mechanisms that can arise on the households side. For a given income level, a household with a consumption level $c \geq 0$, and a zero labour supply ($h = 0$) will obtain a higher utility than another one with the same consumption, but a strictly positive labour supply ($h > 0$). Consequently, during the imitation process, the behaviour of the former will have a higher probability of being imitated, and will more easily diffuse in the population. It ensues that, at the next period, the zero labour supply strategy is more likely to be adopted by other households than the one associated with a positive labour supply (and the same level of consumption). This mechanism can more generally be observed between any pair of strategies with a similar consumption level, but one having a higher labour supply than the other. It has to be noted that rationing mechanisms would intensify the diffusion of strategies with low labour content, since the utility levels are based on the actual behaviour resulting from market interactions.

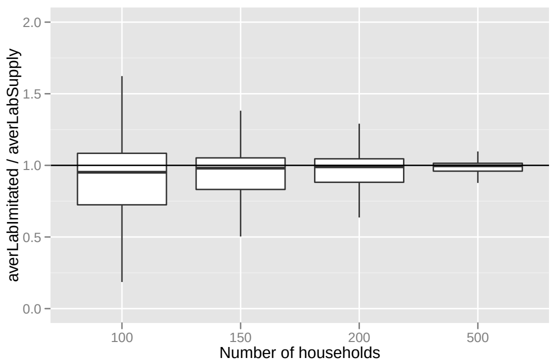

Figure 4: Diffusion of low labor supply strategies

In Figure 4, we illustrate the extent to which the aforementioned externalities arise within the economies we have considered. We plot in this graphic, the distribution of the average level of the actual imitated quantities of labour, normalised using the average of the imitable quantities of labour in each period. We distinguish experiments with different numbers of households. In each case, we observe that this indicator has a significant proportion of its values below 1 (globally, in more than 55% of the cases the imitated quantities are on average lower that the actual labour supply), indicating that strategies with labour supplies lower that the average worked labour diffuse indeed in the population with a significant frequency, even more when the number of agents is lower. Given that we do not control here for the imitated consumption levels, this ratio would be even higher if we compared only the imitations with a consumption level lower than the average. Indeed, some of the strategies are imitated even if they contain higher labour levels, because they also contain higher consumption levels that compensate the disutility of the labour.

The rationing process on the good market does also favour the diffusion of low labour content strategies on the side of the firms. The lower the amount of labour a firm is asking for, the lower the level at which that firm sets her price for the quantity of the good she desires to sell, *ceteris paribus*. That firm is accordingly less likely to be rationed on the good market (as the ranking of the firms follows increasing prices). She is, in turn, more likely to generate a higher profit than another firm that hires a higher amount of labor but is rationed on the good market. As the imitation of the firms proceeds according to the relative level of their profits, the diffusion of the firms' strategies that are associated with a low level of labour demand is also favoured.

6 An alternative model without learning externalities

We now introduce a version of our model where the learning externalities are neutralised on the households' side. We consider whether the economy can exhibit a better performance for this case. To check this result, we assume that the fixed labour supply of the consumer is equal to the optimal labour supply that she would propose in the static optimal equilibrium of the model given in the Appendix A.1 (this optimal value depends on the parameters corresponding to each simulation's setup). Moreover, we also assume that the consumers desire to spend their whole income on consumption in each period. We are consequently able to check if the social learning on the firms' side is able to discover, in this very favourable setup, a time path with increasing economic performance. If this is impossible, that would indicate an even more profound problem with social learning in a general (dis)equilibrium setup.

Consequently, we call Model 0 a variant of our initial model where we assume for the labour supply (see equation (17), Appendix A.1)

$$h_{i,t} = h_i^* \tag{12}$$

and for the desired level of nominal consumption

$$\widetilde{c}_{it}^d = max\left\{0, \widetilde{y}_{it}\right\} \tag{13}$$

where $\widetilde{c}_{i,t}^d \equiv p_{i,t} \cdot c_{i,t}^d$, where again $p_{i,t}$ is the composite price payed by the household.

Under these assumptions, the externality in the social learning of the consumers is completely neutralised, and the firms must learn to distribute the optimal amount of income in order to converge to the equilibrium outcome.

Figure 5 clearly exhibits much better performance when we neutralise the negative learning externalities on the side of the households. Now we observe a continuous increase of social welfare, even if the labour supply remains constant. Table 3 shows that learning of firms becomes favourable to the utilisation of the the economy's potential. When we neutralise the coordination problems that result from the harmful effects of imitation on the households' side, the firms are able to learn to use the supplied labour by increasing their labour demand. The increase in time of labour demand can be observed in panel (d) of Figure 6, and the resulting decrease in unemployment (panel f), increase in the good supply (panel b).

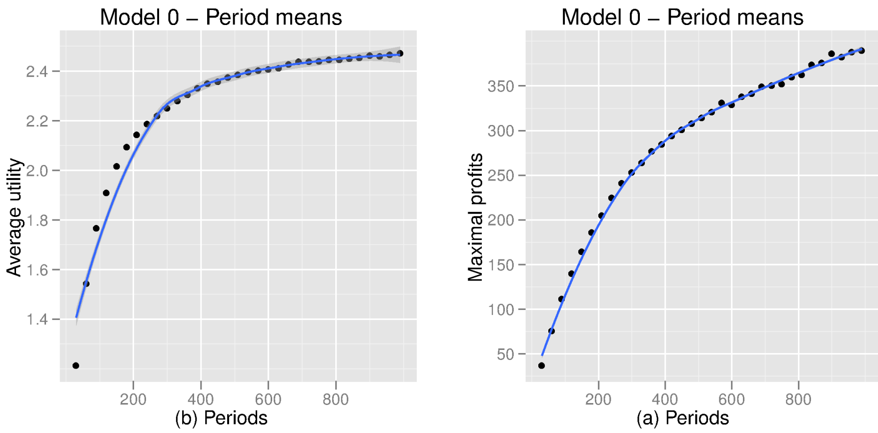

Figure 5: Evolution of the global performance of the economy: average utility and maximal profits in Model 0

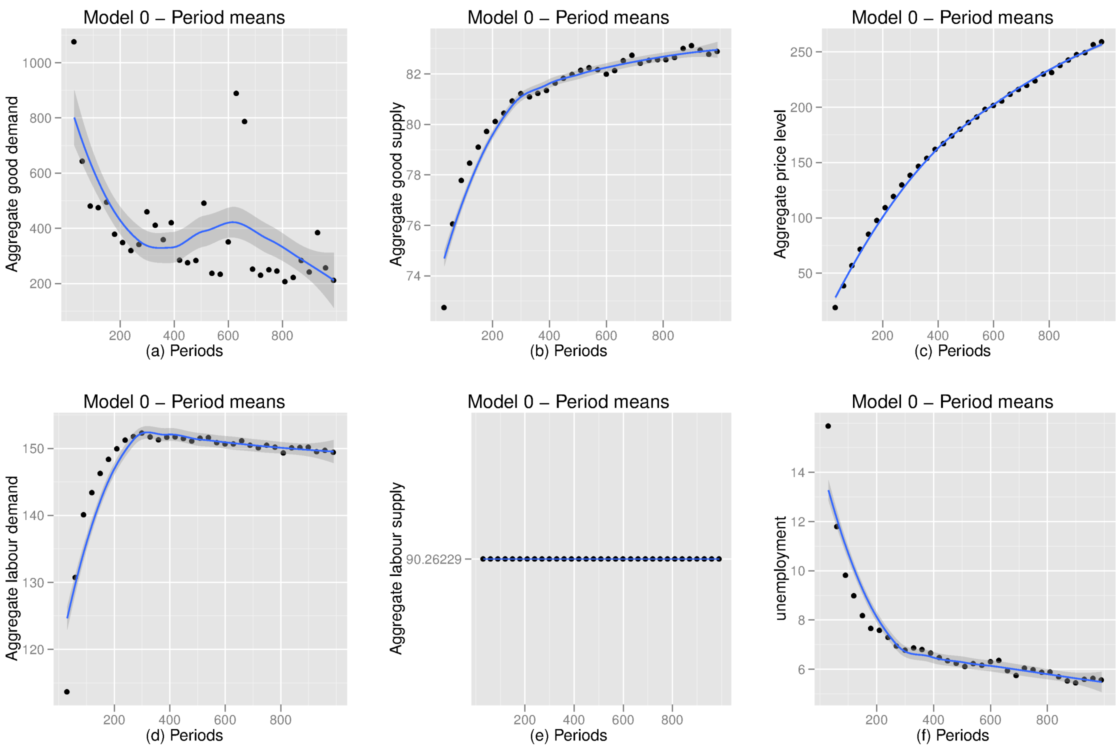

Figure 6: Much better utilisation of the potential of the economy in the Model 0

	Full model	Model 0
$meanLabourSupply$	$-$	constant
$meanLabourDemand$	$-$	$+$
$unemployment$	$-$	$-$

Table 3: Influence of the imitation probability of the labour market

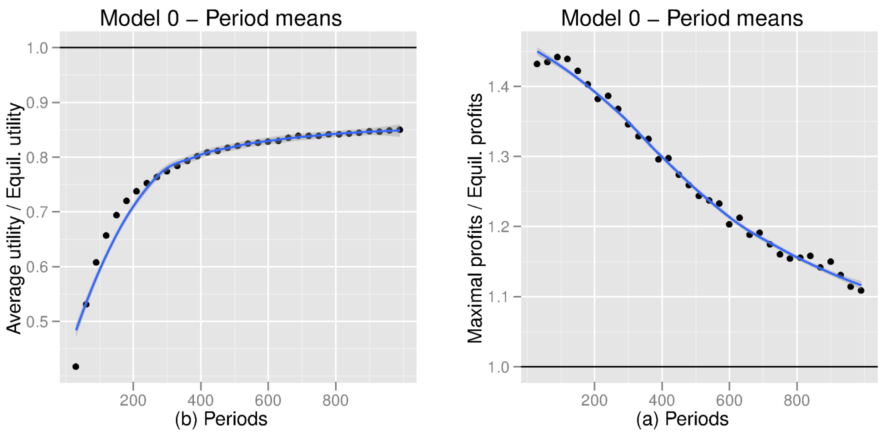

Figure 7: Convergence towards optimal performances in Model 0

More precisely, we can also check the convergence of the behaviour and of the economic performance towards the steady–state equilibrium values. Figure 7 gives the relative values for the utility and the maximal profits, the equilibrium values corresponding to the horizontal lines for $y = 1$ in each plot. We observe that some convergence takes place, even if the speed of convergence seems to decrease with time in the case of social welfare. The last steps necessary to come close to the steady–state equilibrium appear to be quite difficult in this case and the distance remains higher than 10% (the ratio converges to 0.85 at the end of the periods here).

The latter figures plot all the observations from all the experiments for each period, they do not distinguish configurations corresponding to different sets of parameters (experiments). We can use a regression analysis to analyse the role of different parameters of the economy in the convergence towards the equilibrium values. Table 4 shows that the relative distance to equilibrium welfare is influenced in a differentiated way by these parameters. We can first check that this distance decreases with the imitation probability. Consequently, the social learning's harmful externality has effectively been neutralised in this case, and imitation helps performance and welfare. All variables are significant, except the mutation size (σ_{mut}), and all factors, but the initial wealth of consumers, the number of firms and α in the production function, have a positive effect on convergence towards the equilibrium welfare. Concerning the role of α, we quite systematically observe in this model (also in other variants of the baseline model) that the non-linearity resulting from decreasing returns significantly hinders the learning of firms. This non-linearity increases the sensitivity of the global performance to the distribution of firms' market shares, and the convergence of this performance towards the equilibrium one. When the returns are close to constant, only the total output is determinant in the global performance, and this flexibility facilitates the coordination of firms on better global performance. A similar mechanism explains the role played by the number of firms, since the coordination process is more complex when this number increases.

Neutralising the negative externalities yields a better performance. Comparing relative distance to the optimal utility between the full model and Model 0 through a Wilcoxon-Mann-Whithney test implies that we can reject the null hypotheses of the equality of the distances between these two models, on the benefit of the alternative assumption that the distance is higher in the full model. Figure 8 shows that the average utility is closer to the the equilibrium value for medium values of the imitation probability (corresponding to the middle line) and high values of the mutation probability (right column): distribution corresponding to these values put more mass around the relative value

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.8480	0.0090	94.41	0.0000
nbconsumers	-0.0001	0.0000	-15.83	0.0000
nbfirms	0.0012	0.0001	12.23	0.0000
initialCWealth	0.0021	0.0001	29.27	0.0000
probImitate	-0.2418	0.0161	-15.00	0.0000
probMutate	-0.3184	0.0491	-6.48	0.0000
deviationMutateRate	0.0050	0.0026	1.92	0.0548
sigma	-0.4283	0.0041	-103.26	0.0000
mu	-0.1028	0.0028	-36.90	0.0000
alpha	0.2762	0.0070	39.54	0.0000
phi	-0.2330	0.0035	-65.72	0.0000
ticks	-0.0001	0.0000	-14.72	0.0000

Residual standard error: 0.2444 on 29988 degrees of freedom
Multiple R-squared: 0.3967, Adjusted R-squared: 0.3965
F-statistic: 1793 on 11 and 29988 DF, p-value: $< 2.2e\text{-}16$

Table 4: Determinants of the relative distance to the optimal utility $((\overline{u} - u^*)/u^*)$

of $u/u^* = 1$. Consequently, imitation process is necessary for approaching the equilibrium welfare, but we can have too much of it, and introducing enough novelty, through mutations, remains a definitely important dimension of learning.

7 Conclusion

This article explores the properties of macroeconomic dynamics that can be observed when we introduce bounded rationality and social learning in the New Keynesian DSGE model (NKM). To this end, we reformulate the NKM as an Agent based computational model (ABM), where agents' individual decisions are guided through a social learning process based on imitation between agents and random experimenting by them. While keeping the main structure of the NKM concerning the structure of the markets, their interactions, and functional forms (utility and production functions), we extend the analysis to disequilibrium dynamics. Equilibrium steady state can only be observed if it endogenously emerges as a result of the agents' learning, and hence, if it is globally stable. We consequently analyse this possibility, as well as general properties of dynamics emerging in this economy, under agents' social learning and heterogeneity.

Our main results show that simple social learning does not allow coordination of agent's behaviour on equilibrium dynamics. We show that such a learning process indeed diffuses free riding labour supply strategies (households preferring to offer low labour supply, while expecting to consume the production realised with the labour provided by other households), and gives rise to depressive dynamics. This mechanism becomes a vicious circle when it is combined with the rationing on the markets, that depresses even more the actual labour supplies imitated by the households. Consequently, social learning of the agents does not allow the economy to converge to the general equilibrium steady state. We show that the interaction between the markets is the main culprit in this deficiency, because in a version of the baseline model where we neutralise the free riding effect on the side of the households, the learning of the firms is able to pull the economy towards much more favourable dynamics.

Our results consequently underline the sensitivity of the NKM to the assumptions of substantive

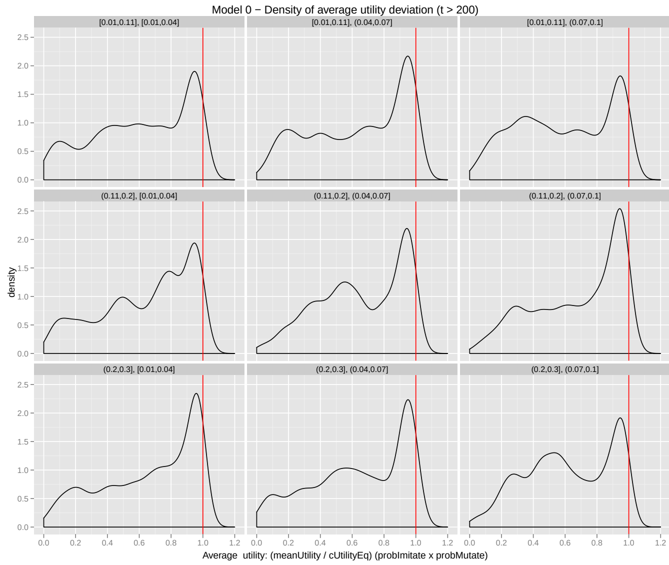

Figure 8: Role of $probImitate$ (as lines) and $probMutate$ (as columns) in the distribution of deviations from equilibrium of the average utility in Model 0

rationality and rational expectations, but they also call our attention on the care that must be taken while we relax those assumptions. Our preliminary explorations using a version of this model that we are developing using individual evolutionary learning seem to indicate that, even without the learning externality observed here, purely adaptive learning is not able to give rise to coordination between agents. Consequently, the embedding of bounded rationality in the individual behaviour of the agents (and not in the reduced version of the NKM, like in Evans & Honkapohja, 2001) sheds light on the fragility of that coordination, and hence on the equilibrium assumption we use in our models.

One potential extension that must be considered is the possibility of an adaptive learning process with forward looking and expectations. Yildizoglu *et al.* (Forthcoming) propose a framework for including such a process, while keeping the expectations purely adaptive: without any explicit knowledge on the structure of the real model, agents develop an individual representation of the economy (a mental model), on the basis of their past experience in this economy. We represent this mental model, and its dynamics, using an artificial neural network. In the very simple context of the canonical intertemporal consumption decision problem, we show that only the inclusion of such representation is able to give rise to a consumption behaviour that is close to the buffer-stock rule of Allen & Carroll (2001). We should now test this approach in the context of market interactions to assess its results in a complete macroeconomic framework.

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A Appendixes

A.1 Optimal static symmetric equilibrium values of the baseline model

We compute the equilibrium values of the variables under concern in the model assuming that households and firms are optimising agents (and do not proceed under bounded rationality). We focus on the intra temporal (static) equilibrium, assuming that the households are mainly concerned with the level of the periodic utility flow they receive (no smoothing objective is thus envisioned). Taking intertemporal concerns into account in our framework (and notably from a bounded rationality perspective) is left for further research.

General equilibrium values Under perfect competition on the labour market, all firms pay the same wage. This implies $w_{j,t} = W_t \ \forall j$. Thus, all the firms face an identical profit maximisation problem: they pay the same wage, they use the same production technology (3) and the same mark-up scheme (4) while they have to cope with the same aggregate good demand. Consequently, each firm hires the same labour quantity $h_{j,t} = \tilde{h}_t,\ \forall j$ and produces the same good quantity $y_{j,t} = \tilde{y}_t,\ \forall j$. This implies identical labour costs for all the firms and thus identical prices: $p_{j,t} = P_t \ \forall j$.

Household i's budget constraint is given by

$$b_{i,t} = w_{i,t}h_{i,t} + \frac{\sum_{j=1}^{m} \pi_{j,t-1}}{n} + b_{i,t-1}(1 + i_{t-1}) - p_{i,t}c_{i,t} \tag{14}$$

Maximisation of utility (7) under (14) gives the first order condition $\frac{p_{i,t}}{w_{i,t}} = h_{i,t}^{-\phi} c_{i,t}^{-\sigma}$. The uniqueness of wages and prices implies identical first order conditions for all households:

$$\frac{P_t}{W_t} = h_{i,t}^{-\phi} c_{i,t}^{-\sigma} \tag{15}$$

So each household provides the same quantity of labour $h_{i,t} = \check{h}_t,\ \forall i$ and consumes the same quantity of goods $c_{i,t} = \check{c}_t,\ \forall i$.

Now we replace in (4) $w_{j,t}$ by W_t, $p_{i,t}$ by P_t and $y_{j,t}$ by $h_{j,t}^{1-\alpha}$ (cf. (3)):

$$\frac{P_t}{W_t} = \frac{\varepsilon}{\varepsilon - 1} \times \frac{W_t h_{j,t}^{\alpha}}{1 - \alpha}. \tag{16}$$

Market clearing on the labour and on the good markets implies $n \times h_{i,t} = m \times h_{j,t}$ and $n \times c_{i,t} = m \times y_{j,t}$. Consequently, we can substitute $h_{i,t}$ in (15) by $\frac{m}{n} \times h_{j,t}$ and $c_{i,t}$ in (15) by $\frac{m}{n} \times y_{j,t}$. Equalising the right hand sides of (15) and (16), we find:

$$h_{j,t}^{*} = \left[\frac{\varepsilon - 1}{\varepsilon} \left(\frac{m}{n} \right)^{-\sigma - \phi} (1 - \alpha) \right]^{\frac{1}{\sigma(1-\alpha)+\phi+\alpha}} \tag{17}$$

Firm j's optimal good supply is therefore $y_{j,t}^{*} = (h_{j,t}^{*})^{1-\alpha}$. Now, we can express all optimal quantities as functions of $h_{j,t}^{*}$ and $y_{j,t}^{*}$: $h_{i,t}^{*} = \frac{m}{n} \times h_{j,t}^{*}$, $c_{i,t}^{*} = \frac{m}{n} \times y_{j,t}^{*}$, $u_{i,t}^{*} = \frac{(c_{i,t}^{*})^{1-\sigma}}{1-\sigma} - \frac{(h_{i,t}^{*})^{1+\phi}}{1+\phi}$, $H_t^{*} = m \times h_{j,t}^{*}$, $Y_t^{*} = m \times y_{j,t}^{*}$, $\Omega_t \equiv \frac{W_t}{P_t} = \frac{\varepsilon-1}{\varepsilon}(1-\alpha)(h_{j,t}^{*})^{-\alpha}$.

Subtracting optimal labour costs $W_t h_{j,t}^{*}$ from optimal turnover $P_t y_{j,t}^{*}$ and dividing the result by P_t, we find the optimal real profit of each firm j:

$$\left(\frac{\pi_{j,t}}{P_t} \right)^{*} = y_{j,t}^{*} - \Omega_t h_{j,t}^{*} \tag{18}$$

The market clearing condition $Y_t = C_t$ implies that aggregated real savings $\frac{S_t}{P_t} \equiv Y_t - C_t$ are always equal to zero. Given $P_t > 0$, aggregated nominal savings S_t and aggregated bond holdings must be zero as well: $B_t = 0$. Taking into account that all households are characterised by identical income and expenditure streams, each household i must hold the same quantity of bonds: $b_{i,t}^{*} = \frac{B_t}{n} = 0$.

A.2 Parameters and Variables

A.2.1 Parameters and Individual Variables

Symbol	R-code	Variation	Description
α	alpha	$[0, 0.5[$	$(1-\alpha)$: labour–elasticity of supply
$b_{i,0}$	initialCWealth	$\{1, 5, 10, 50\}$	household i's initial bond holdings
A		1	technology level
$c^d_{i,0}$		$[0, 2[$	initial desired consumption of household i
c_i			consumption of household i
c^*_i			household i's equilibrium consumption
μ	mu	$[1, 2.5]$	firms' margin rate
$h^s_{i,0}$		$[0, \frac{2n}{m}[$	household i's initial labour supply
h_i			quantity of labour provided by household i
h^*_i	cLabourEq		household i's equilibrium labour supply
$h^d_{j,0}$		$[0, 2n/m]$	firm j's initial labour demand
h_j			quantity of labour hired by firm j
h^*_j			firm j's equilibrium labour demand
i		0	nominal interest rate (coupon rate)
n	nbConsumers	$\{100, 150, 200, 500\}$	number of households
m		$\{10, 20, 30, 50\}$	number of firms
ϕ	phi	$\{0.1, 0.9\}$	$(1+\phi)$: labour–elasticity of utility
p_i			composite good price paid by household i
p_j			firm j's good price
$Pr_{copy,k}$			firm j's probability of being imitated
Pr_{imit}	probImitate	$[0.01, 0.3]$	probability of imitation
Pr_{mut}	probMutate	$[0.01, 0.1]$	probability of mutation
π_j			firm j's profit
σ	sigma	$\{0.16, 0.9\}$	$(1-\sigma)$: consumption–elasticity of utility
σ^2_{mut}	deviationMutate	$[0.5, 2]$	standard deviation of mutation draws
u_i			household i's utility
$w_{j,0}$		$[0, 1[$	firm j's initial nominal wage
w_i			composite wage rate received by the household i
w_j			wage rate paid by the firm j
y^d_j			firm j's good production
y_j			quantity of goods sold by firm j
y^*_j			firm j's equilibrium production

A.2.2 Aggregate Variables

Symbol	Description
B	aggregate bond holdings / public debt
H	aggregate labour quantity (provided by consumers / hired by firms)
H^d	aggregate labour demand
H^s	aggregate labour supply
Y	aggregate quantity of goods sold by firms
Y^s	aggregate good production
P	price level
W	wage level
Ω	real wage level

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