GREThA
Groupe de Recherche en
Économie Théorique et Appliquée

(In)Efficient Interbank Networks

Fabio CASTIGLIONESI

EBC, CentER, Department of Finance

Tilburg University

fabio.castiglionesi@uvt.nl

&

Noemí NAVARRO

GREThA, CNRS, UMR 5113

Université de Bordeaux

noemi.navarro@u-bordeaux.fr

Cahiers du GREThA
n° 2016-13
May

GRETHA UMR CNRS 5113
Université de Bordeaux
Avenue Léon Duguit - 33608 PESSAC - FRANCE
Tel : +33 (0)5.56.84.25.75 - Fax : +33 (0)5.56.84.86.47 - www.gretha.fr

Réseaux interbancaires (in)efficaces

Résumé

Nous étudions l'optimalité et la formation (décentralisée) d'un réseau interbancaire. Les banques peuvent offrir des rendements espérés plus élevés aux déposants en établissant des connexions dans le réseau. Cependant, les banques ont intérêt à jouer avec l'argent des déposants s'ils en sont pas suffisamment capitalisés. La faillite d'une banque affecte négativement les banques qui lui sont connectés (risque de contrepartie). Nous montrons que tant le réseau efficace et comme le décentralisé sont caractérisés par une structure noyau-périphérie. Néanmoins, lorsque les transferts de fonds propres des banques ne sont pas possibles, les deux structures de réseau coïncident que si le risque de contrepartie est suffisamment faible. Dans le cas contraire, le réseau décentralisé est sous connecté par rapport à la solution optimale. Enfin, nous montrons que les transferts de fonds propres des banques n'évitent pas la formation de réseaux inefficaces.

Mots-clés : Réseau interbancaire, noyau-périphérie, coassurance de liquidité, risque de contrepartie

(In)Efficient Interbank Networks

Abstract

We study the optimality and the (decentralized) formation of an interbank network. Banks can offer higher expected returns to depositors by establishing connections in the network. However, banks have an incentive to gamble with depositors' money if not sufficiently capitalized. The bankruptcy of a bank negatively affects the banks connected to it (counterparty risk). We show that both the efficient network and the decentralized one are characterized by a core-periphery structure. Nevertheless, when bank capital transfers are not possible, the two network structures coincide only if counterparty risk is sufficiently low. Otherwise, the decentralized network is under-connected as compared to the optimal one. Finally, we show that bank capital transfers do not avoid the formation of inefficient networks.

Keywords: Interbank Network, Core-periphery, Liquidity Coinsurance, Counterparty Risk

JEL: D85, G21

Reference to this paper: CASTIGLIONESI Fabio, NAVARRO Noemí (2016) (In)Efficient Interbank Networks, *Cahiers du GREThA*, n°2016-13

http://ideas.repec.org/p/grt/wpegrt/2016-13.html.

GRETHA UMR CNRS 5113
Université de Bordeaux
Avenue Léon Duguit - 33608 PESSAC - FRANCE
Tel : +33 (0)5.56.84.25.75 - Fax : +33 (0)5.56.84.86.47 - www.gretha.fr

1 Introduction

It is theoretically established that banks have incentives to form bilateral lending relationships. The most intuitive reason is to coinsure future and uncertain idiosyncratic liquidity shocks (Allen and Gale [6] and Freixas, Parigi and Rochet [25]). There is robust evidence that support such theories by documenting how relationship lending among banks plays a crucial role in providing liquidity insurance both in normal times (Furfine [26], King [31] and Cocco *et al.* [20]) and in times of crisis (Furfine [27]). The overall amount of these bilateral links that banks establish form what is generally known as the interbank network.

Following the 2007/2008 financial crisis, which involved heavily the interbank market, there has been a surge of empirical evidence about the actual shape of the interbank network. Based on transaction data from the Fedwire system, Soromäki et al. [35] and Beck and Atalay [9] find that the interbank networks formed by US commercial banks is quite sparse. It consists of a core of highly connected banks, while the remaining peripheral banks connect to the core banks. An almost identical feature is found in interbank networks in other countries like the UK, Canada, Japan, Austria and Germany (see, respectively, Bank of England [8], Embree and Roberts [23], Inaoka et al. [29], Boss et al. [12] and Craig and von Peter [21]). The aim of this paper is to rationalize this consistent evidence. In particular, we establish under which conditions a core-periphery network could be optimal. Then we show under what circumstances banks have incentives to form an (in)efficient decentralized interbank network.

We model an interbank network composed of several banks that anticipate how the structure of the network affects their payoff. Each bank is financed by depositors and shareholders. While the former need banks to take advantage of the investment opportunities in the economy, the latter provide capital and decide the type of investment the bank chooses.

We thank Martin Brown, Sandro Brusco, Fabio Feriozzi, Jiro E. Kondo, Emanuela Sciubba, Wolf Wagner for helpful comments. We are also grateful to seminar participants at Bank of England, Universidad Carlos III, Bank of Italy, 1st Swiss Conference on Banking and Financial Intermediation (Champery), Society for Economic Dynamics (Boston), European Finance Association (Athens) and Association for Public Economic Theory (Galway) annual meetings where an earlier version of this paper was presented. The usual disclaimer applies. Castiglionesi acknowledges financial support from the Marie Curie Intra European Fellowship and Navarro from the Spanish Ministry of Economics, grant ECO2014-53767P.

Participating in the interbank network is beneficial since it allows banks to increase the expected payoff paid to their depositors. We posit that such expected payoff is increasing in the number of links that a bank establishes in the network. The rationale behind this assumption is the ability of the interbank network to allow banks to coinsure future idiosyncratic liquidity needs. The larger the connections in the network are, the higher the probability to find coinsurance is, the less resources have to be invested in liquid low-return assets, and more resources can be invested in illiquid high-return investment projects. This benefit however has to take into account the potential cost of participating in the interbank network, which is captured by a bank's moral hazard problem.

Shareholders in each bank have two types of investment projects in which they can invest the bank's resources. One project is risk-free, i.e. it guarantees a certain return. The other project is risky since it has the same payoff of the safe project if it succeeds and it delivers nothing if it fails. The risky project however gives private benefits to the bank's shareholders, therefore it represents a gambling project from the depositors' point of view.[1]

Since shareholders are protected by limited liability, they find it convenient to invest in the gambling project when the bank is poorly capitalized. We assume that a bank, by establishing a link with banks that invest in the risky project, reduces the ex-ante probability of serving its own depositors. In particular, we assume that the higher the ratio between the number of neighboring banks that invest in the risky project over the total neighboring banks is, the lower the probability to serve the depositors is. This represents the risk of making connections in the interbank network and we refer to it as counterparty risk.

Overall the banks' decision of belonging to the interbank network, and eventually how many connections to establish, entails a trade-off. The benefits represented by the higher expected payoff have to be weighted against the counterparty risk, i.e. the possible bankruptcy of a neighboring bank that invests in the risky project.

First, we characterize the optimal interbank network as the solution of the planner's problem. The planner can avoid the moral hazard problem in all banks only if a sufficient amount of bank capital is available in the economy. In this case, the (unconstrained)

[1]Through the paper we use the expressions risk-free and safe bank (or project) as synonymous. Similarly for risky and gambling bank (or project).

efficient network is characterized by a fully connected structure. Otherwise, if bank capital in the economy is scarce, the planner has to allow some banks to gamble and a constrained first-best (CFB) network is obtained.

The presence of banks investing in the risky project implies that the CFB network does not necessarily coincide with the fully connected one. Only if the counterparty risk is sufficiently low the two structures coincide. Otherwise, the higher the counterparty risk is, the less connected the periphery banks are. That is, the CFB network is characterized by a core-periphery structure. The core includes all the banks that invest in the safe project and form a complete network structure among themselves. The periphery includes all the gambling banks that can be connected among themselves and/or with the core banks according to the parameters' value. With the additional assumption that the benefit of participating in the network (i.e., depositors' expected payoff) is increasing at decreasing rate in the number of connections, we are able to fully characterize the conditions under which risky banks should or should not be connected among themselves and with the core (safe) banks.

Second, we adopt the equilibrium notion of pairwise stability to analyze the decentralized interbank network formation in which banks decide whether to establish links among themselves. Also in this case a core-periphery structure emerges as an equilibrium outcome. Nevertheless, the connectivity in the decentralized network does not necessarily coincide with the CFB network. We show that the structure of the decentralized interbank network is the same as the CFB one if the counterparty risk is sufficiently low. Even if the decentralized and the CFB networks have the same structure, the core of the CFB network can be larger than the core of the decentralized network.

On the other hand, the decentralized network does not coincide with the CFB network when the counterparty risk is not low enough. The reason is that the planner finds it optimal to link a safe bank with a gambling bank when the expected losses of the former (because of counterparty risk) are lower than the expected gains of the latter (represented by the higher expected payoff due to an higher liquidity coinsurance). However, these expected gains are not internalized by the safe banks that severe the link with the gambling banks even when this is not efficient. The decentralized network can be characterized by an inefficiently low degree of connectivity compared to the CFB network when counterparty risk is sufficiently high.

Finally, we allow for decentralized bank capital transfers once the network has been formed. Banks investing in the safe project could find it convenient to transfer part of their bank capital to a neighboring gambling banks in 'exchange' for financial stability (and to achieve a higher expected payoff). We show that if the probability of success of the bank's risky project is sufficiently high, then banks do not have incentive to make any bank capital transfer. More generally we argue that the decentralized network formation does not induce banks to form the CFB network when ex-post bank capital transfers are allowed.

The type of inefficiency that our model highlights likely arose during the 2007/2008 financial crisis, where banks feared losses in their counterparts. Our model predicts that when the risk associated with the lending of funds is too high, connections become too costly relative to their benefits and safe banks inefficiently sever their interbank links. Nevertheless, our paper also stresses the fact that safer banks still have incentive to maintain their links among each other. These predictions are supported by Afonso et al. [3] who find that interbank lending in the US decreased substantially during the 2007/2008 crisis but it did not freeze completely. They find that riskier banks were cut off from the interbank market while safer banks would be still active.[2] Moreover, consistently with our model, they show that the interbank market stress was likely coupled with inefficient provision of liquidity to the risky banks.

Our paper is inspired by the strand of literature that models contagion as the outcome of links established by banks. In particular, banks are connected through interbank deposits that are desirable ex-ante, but the failure of one institution can have negative effects on the institutions to which it is linked (see Allen and Gale [6], Freixas, Parigi and Rochet [25], Brusco and Castiglionesi [14]). The common feature of all these models is to assume an exogenous and very stylized interbank network. The present model captures the features of the banking models (such as the benefits stemming from liquidity coinsurance as in Allen and Gale [6], and the gambling behavior of low capitalized banks as in Brusco and Castiglionesi [14] or Morrison and White [34]), but it directly addresses the issues of the optimal design of the network and its decentralized formation.

Even if the theory of network formation has been successfully applied to several eco-

[2]Notice that theoretical explanations of the liquidity dry-up based on adverse selection arguments predict that only the risky banks (i.e., the 'lemons') remain in the market (see Heider, Hoerova and Holthausen [28] and Malherbe [33]).

nomics fields, only recently there have been attempts to use such theory to understand the working of financial systems (see Allen and Babus [4] for a survey). Among the first attempts, Leitner's [32] and Babus [7] consider models of network formation, where banks form links in order to reduce the risk of contagion. These models rationalize the interbank network as an insurance mechanism. The idea is that banks can be surprised by unexpected liquidity shocks that can make bankrupt at least one bank in the system (like in Allen and Gale [6]). The possibility that this original failure can spread to the entire system gives the rationale of belonging to the interbank network. We provide an alternative rationale for the existence of interbank networks that is based on banks that fully anticipate the trade-off between the benefits and the costs of participating in the network. Our main contribution is to rationalize the optimality of the core-periphery structure and to compare it with the characteristics of the decentralized interbank network. More recently, Farboodi [24] provides a rationale for the existence of a core-periphery banking network. Her model predicts that the core banks are those who invest in risky projects that allow them to offer high expected returns. Banks without such risky investment belong to the periphery. Our model has the opposite prediction.

A related fast-growing literature on systemic risk studies the propagation of negative shocks in financial networks. Contrary to our approach, large part of this literature takes the structure of the network as given (see Acemoglu et al. [1], Caballero and Simsek [15], Castiglionesi and Eboli [17] and Elliott et al. [22]). There are papers in this area that also emphasize the strategic link formation among financial institutions. Acemoglu et al. [2] and Zawadowsky [36], for example, consider banks located on a network shaped as a ring. While the former paper predicts that the equilibrium network can exhibit both under and over connection, the latter provides a rationale for under-insurance. Allen et al. [5] analyze the interaction between financial connections due to overlapping portfolio exposure and systemic risk. Cabrales et al. [16] investigate the optimal properties of a network of firms that trade off risk-sharing and risk of contagion. They find that when big shocks have low probability to occur, the complete network with uniform exposure among firms is the optimal one. Only if the likelihood of big shocks increases the optimality requires to severe links and to form disjoint components. Core-periphery structures are not optimal is their setting. The main difference with our model is that they consider firms either with homogenous risk-return characteristics (i.e., same return and risk) or heterogenous characteristics but only with respect to risk. We instead consider explicitly moral hazard problems in which banks have heterogenous returns (i.e., banks that invest in the gambling project have lower expected return). This turns out to be the main reason for the core-periphery structure to emerge both as the optimal and decentralized network.

The paper is organized as follows. Section 2 sets up the model. Section 3 posits the benefit and cost of the interbank network and how they affect agents' expected payoff. Section 4 analyzes the planner's problem, characterizing the constrained first-best solution. Section 5 studies the decentralized network formation and its efficiency properties. Section 6 concludes. The Appendix collects the proofs.

2 The Model

There are three dates $t = 0, 1, 2$ and one divisible good called 'dollars' ($). The economy is divided into n regional banks, and let $N = \{1, 2, ..., n\}$ be the set of such banks. Each regional bank has the same (large) amount of depositors that consume at $t = 2$. Depositors have to deposit at $t = 0$ in their regional bank to access the investment opportunities of the economy. The endowment of deposits received at $t = 0$ is normalized to 1$ in each regional bank. Beside deposits, banks are funded also through capital. Each regional bank i randomly receives an endowment $e_i \in [0, \overline{e}]$ of dollars, which represents the bank capital and it is owned by the bank's shareholders. The vector $e = (e_1, e_2, ..., e_n)$ represents the realization of the bank capital endowments. The pair (N, e) is called an *economy*.

Let $K_i \subseteq N$ be the set of banks to whom bank i is directly linked, then the number of banks connected to bank i is $k_i \in \{0, 1, ..., n-1\}$. The vector $K = (K_1, K_2, ..., K_n)$ captures the interdependence among the banks, and it represents the interbank network. We restrict ourselves to undirected networks, i.e., bank i is related to bank j if and only if bank j is related to bank i. Let $\mathcal{K}$ denote the set of all possible financial networks for a given economy (N, e).

We also allow for transfers of bank capital across neighboring banks. Let $x_i = e_i + t_i$ be the bank capital for bank $i \in N$ after transfers have been made (i.e., t_i is the transfer and can be positive or negative). A vector of bank capitals $x = (x_1, x_2, ..., x_n)$ is called feasible for a given economy (N, e) if (i) $x_i \geq 0$ for all i, and (ii) $\sum_{i \in N} x_i = \sum_{i \in N} e_i$. Let $\mathcal{X}$ denote the set of all feasible vectors of bank capital for a given economy (N, e).

After the transfers are made, each bank i has $1 + x_i$ dollars to invest. Banks can invest this money in two types of projects (either in one or the other type of project) that mature in $t = 2$:

1. The risk-free, or safe, project rf with an expected return of $\overline{R} > 1$ dollars per dollar invested.

2. The risky, or 'gambling', project r that yields an expected return of $\overline{R} > 1$ dollars with probability ξ, and 0 dollars with probability $(1 - \xi)$ per dollar invested. This type of project yields also a private benefit $B > 0$ to bank's shareholders. Private benefits are realized at the moment of the investment (so they do not have dollar value, consider them as perks or investment in family business).

We refer to $s_i \in \{rf, r\}$ as the project's choice of bank i at $t = 1$. The vector s denotes the investment strategy profile, that is, $s = \{s_i\}_{i \in N}$. Let $\mathcal{S}$ denote the set of all possible investment profiles for a given economy (N, e). The sequence of events is reported in Table 1.

Table 1. Sequence of events

Time	Events
$t = 0$	Bank's capital is realized and interbank network is chosen.
$t = 1$	Bank's capital transfers are made and projects are chosen.
$t = 2$	Projects cash flows are realized and depositors are paid.

The timing represents the interbank network formation game. In $t = 0$, once bank capitals are realized, the banks choose the network structure. In $t = 1$ bank capital transfers are made and the investors choose the type of project. Accordingly, the expected payoff in $t = 2$ will depend on the structure of the financial network chosen in $t = 0$ and on the type of project chosen by the banks in $t = 1$. The timing captures the fact that bank decisions about which interbank link to establish are long term, while the type of the investment is a decision with shorter time horizon.

3 Interbank Network and Expected Payoffs

3.1 The Benefit: liquidity coinsurance

Since establishing interbank lending relationships alleviate the occurrence of uncertain idiosyncratic liquidity shocks that can hit an investment project before it matures, we assume that the benefit of the interbank network is to increase the expected payoff that a bank can offer to its depositors. A possible mechanism behind such assumption is provided by Castiglionesi et al. [18]. They show that an higher ex-ante probability of finding coinsurance in the interbank market reduces the optimal liquidity holding by banks, allowing them to invest an higher amount of resources in illiquid and more profitable projects. This shift in banks' investment decision increases the expected utility of depositors. In this paper we abstract from analyzing banks' liquidity holding, and we take as given the positive relationship between the ex-ante liquidity insurance provided by the interbank network and depositors' expected payoff.

Let us indicate with D_i the expected payoff for the depositors in bank i, then we posit that

$$D_i = f(k_i)R \tag{1}$$

with $f(k_i)$ increasing in k_i ($f'(k_i) > 0$). The assumption is that, all else equal, the higher the number of neighboring banks k_i is and the higher bank i's ex-ante probability to find a suitable counterparty is. This reduces the liquidity risk that bank i is facing, allowing it to offer a higher expected return to its depositors.[3]

To fix boundaries we assume that $f(0) = 1$ and $f(n-1) = \overline{R}/R \equiv \rho$. That is, if bank i is in autarky ($k_i = 0$) then its depositors expect to get the lowest return $R > 1$. The value R can be interpreted as the 'regional' return that bank i promises if it is a stand-alone bank with no relationship with any regional bank. Bank i in this case is fully bearing its idiosyncratic liquidity risk. On the other extreme, if bank i is linked to all the other regional banks ($k_i = n-1$) then its depositors expect the highest return $\overline{R}$. This value can

[3] In more abstract settings, Bloch, Genicot and Ray [10] and Bramoullé and Kranton [13] offer strategic analyses of informal co-insurance in networks. They characterize bilateral agreements that provide co-insurance against income risk as a result of strategic interaction. We can interpret these works as a sort of micro-foundation of the $f(k)$ function that in our setting captures, in a stylized fashion, the benefit of establishing connections in the interbank network.

be regarded as the 'global' return that bank i can offer since its idiosyncratic liquidity risk is now fully coinsured with all other regional banks. For simplicity, we assume a boundary on such global return given by $2R \geq \overline{R} > R$. Then we have $f(k_i) \in [1, \rho]$, with $\rho \leq 2$.

3.2 The Cost: exposure to counterparty risk

The benefit of belonging to an interbank network has to be traded off against its potential cost. Given the natural justification of establishing links in the interbank network as a way to coinsure idiosyncratic liquidity shocks, the main negative consequence is that a bank could end up lending money to a neighboring bank that is actually investing in the risky project. Only if the risky project succeeds, the risky bank is able to pay back the borrowed money. The risk of being linked to one (or more) bank that invests in the risky project is what we call exposure to counterparty risk.

We capture counterparty risk by assuming that a bank that is linked to risky neighboring banks has a reduced probability to serve its depositors. More precisely we assume that the higher the number of risky banks among the neighboring banks k_i is (i.e., the higher bank i's probability ending up lending to one or more risky bank is), the lower the probability bank i will serve its depositors is. While before we abstract from banks liquidity holding, we abstract here from the actual liquidity exchange in the interbank network.

Formally, let $p_i(K, s)$ be the probability that bank i is able to serve its depositors the expected payoff D_i. This probability needs to take into account both the type of project chosen by bank i and the type of investment of the banks connected to bank i. Let g_i denote the number of gambling neighbors of bank i, by definition $g_i \in [0, k_i]$. Then the probability $p_i(K, s)$ is defined as

$$p_i(K, s) = \begin{cases} \eta^{\frac{g_i}{k_i}} & \text{if } s_i = rf, \\ \xi \eta^{\frac{g_i}{k_i}} & \text{otherwise.} \end{cases} \tag{2}$$

When bank i and all its neighbors are investing in the risk-free project (i.e., $s_i = rf$ and $g_i = 0$), then bank i serves its depositors with $p_i(K, s) = 1$. In this case, no matter which bank (or banks) will lend money to, bank i will surely be paid back and then serve its depositors. If instead $g_i > 0$, bank i will serve its depositors with probability less than one, namely $p_i(K, s) = \eta^{\frac{g_i}{k_i}}$. There is now a possibility that bank i will lend to one (or more) of the g_i banks that can default after borrowing money. This would spill over to bank i, reducing its ex-ante probability of being able to serve its depositors. The worse case is when all bank i's neighboring banks invest in the risky project (i.e., $g_i = k_i$). In this case counterparty risk is maximal, since bank i surely will lend to one (or more) risky bank. In this case the probability of bank i to serve its depositors is minimal and equal to $p_i(K, s) = \eta$. The parameter η then captures, all else equal, the exposure to counterparty risk, and the higher η is and the lower the counterparty risk is.[4]

3.3 The Choice of the Investment Project

Given the possible presence in the interbank network of banks that invest in the gambling project, depositors in bank i have the following expected payoff

$$M_i(K, s) = p_i(K, s)D_i = p_i(K, s)f(k_i)R$$

for their dollar deposited in the bank. We assume that $\xi R \geq 1$, so that depositors will participate in a gambling bank in autarky. Notice that among the strategies of the banks there is no possibility of avoiding the investment. Since the banks can choose to be connected or not to the interbank network, it is always possible for them to be disconnected and invest in autarky.

Shareholders in bank i are residual claimant and are protected by limited liability. Accordingly, they expect the following payoff

$$m_i(K, x_i, s) = \begin{cases} p_i(K, s)[(1 + x_i)f(k_i)R - D_i] & \text{if} \quad s_i = rf, \\ \\ p_i(K, s)[(1 + x_i)f(k_i)R - D_i] + B & \text{otherwise.} \end{cases}$$

Considering the expressions of the depositors' expected payoff D_i and the probability $p_i(K, s)$, we have

$$m_i(K, x_i, s) = \begin{cases} \eta^{\frac{g_i}{k_i}} f(k_i)Rx_i & \text{if } s_i = rf, \\ \xi\eta^{\frac{g_i}{k_i}} f(k_i)Rx_i + B & \text{otherwise.} \end{cases} \quad (3)$$

Therefore, for given $f(k_i)$ and s_{-i}, shareholders in bank i will invest in the safe project if and only if

$$\eta^{\frac{g_i}{k_i}} f(k_i)Rx_i \geq \xi\eta^{\frac{g_i}{k_i}} f(k_i)Rx_i + B,$$

[4] Notice that since counterparty risk represents a conditional probability (i.e., the bank that invest in the gambling project has to fail *and* the bank linked to it has to be a lender of the failing bank), it is natural to assume $\eta > \xi$.

which implies

$$x_i \geq \frac{B}{(1-\xi)\eta^{\frac{g_i}{k_i}} f(k_i) R} \equiv I^*(k_i, g_i, \xi, \eta).$$

Accordingly, banks with sufficiently high level of bank capital have incentive to invest in the risk-free project while relatively low capitalized banks find it convenient to invest in the risky project.

4 Constrained First-Best Interbank Network

Let us define an allocation as a vector (K, x, s), where: $x \in \mathcal{X}$, $K \in \mathcal{K}$, and $s \in \mathcal{S}$. An allocation (K, x, s) is an *Investment Nash Equilibrium* (INE) for a given economy (N, e), with $x = (x_i)_{i \in N}$, if

$$m_i(K, x_i, s) \geq m_i(K, x_i, (s_{-i}, \tilde{s}_i)) \qquad \text{for all } i \in N,$$

with $\tilde{s}_i \in \{rf, r\}$. That is, an allocation is an INE for a given economy if taking the financial network and capital as given there are no unilateral profitable deviations in the choice of the investment project. Note that an allocation (K, x, s) is an INE for a given economy if and only if for all $i \in N$

$$s_i = \begin{cases} rf & \text{if } x_i \geq I^*(k_i, g_i, \xi, \eta), \\ r & \text{otherwise.} \end{cases}$$

The constrained first-best solution is characterized by the social planner problem, which objective function is to maximize the expected return of the agents in the economy (i.e., depositors and shareholders). We assume that the planner is able to (i) transfer the initial endowments of capital across banks, (ii) fix a financial network and (iii) suggest the type of project to invest in to the banks. Formally, the planner's problem is as follows.

Definition 1 *Given an economy (N, e), an allocation (K^*, x^*, s^*) is a constrained first-best (CFB) allocation if it maximizes*

$$\sum_{i \in N} [M_i(K, s) + m_i(K, x_i, s)] \tag{4}$$

subject to

$$x_i \geq 0 \text{ for all } i \in N, \tag{5}$$

$$\sum_{i \in N} x_i = \sum_{i \in N} e_i \equiv E, \tag{6}$$

$$s_i = \begin{cases} rf & \text{if } x_i \geq I^*(k_i, g_i, \xi, \eta) \\ r & \text{otherwise,} \end{cases} \quad \text{for all } i \in N, \tag{7}$$

$$m_i\left(K, x, s\right) \geq \max\{Re_i, \xi Re_i + B\} \text{ for all } i \in N, \tag{8}$$

$$p_i(K, s) f(k_i) R \geq 1. \tag{9}$$

The first two constraints represent the feasibility constraints. The planner cannot assign a negative amount of capital to each bank (constraint 5), and she can only reallocate by means of transfers the total amount of bank capital E in the economy (constraint 6). We allow banks to unilaterally deviate from investment decisions (constraint 7). The last constraint restricts the social planner problem in a way that moral hazard has to be taken into account. Finally, we impose the participation constraints for investors (constraint 8) and depositors (constraint 9). We first establish the existence of a CFB and a sufficient condition to have the (unconstrained) first-best network.

Proposition 1 *A CFB allocation always exists. Furthermore, assume that $E \geq E^*$. Then any CFB yields a unique network structure K^* such that $K_i^* = N \backslash \{i\}$ for all i and a unique strategy profile s^* such that $s_i^* = rf$ for all i.*

The proof is in the Appendix. The existence of a CFB is guaranteed even if the constraints in the planner's problem do not define a compact set on R^n because of constraint (7). Nevertheless, by modifying the planner's problem such that the constraints define a compact set on R^n, the solution to the modified maximization problem exists and it turns out to be also the solution of the planner's problem. Proposition 1 also establishes a condition on E that assures that the first best is an INE and all participation constraints are satisfied. The intuition for this result is quite simple: when total bank capital is sufficiently high the planner can achieve the first best avoiding the moral hazard problem. However, in a world where bank capital is scarce the planner problem is a constrained one.

We now establish the shape of the CFB network, which turns out to be characterized by a core-periphery structure. In this structure, the *core* banks choose the safe asset and are all connected to each other. The *periphery* banks choose the gambling asset and they can eventually be connected to some core banks and/or some periphery banks depending on the value of the parameters.

Proposition 2 *Let (K^*, x^*, s^*) be a CFB for a given economy (N, e). Then, for every pair of banks i and j such that $s_i^* = s_j^* = rf$ we have that $i \in K_j^*$ and $j \in K_i^*$.*

The proof is in the Appendix. The intuition behind the optimality of the core-periphery structure is as follows. When two banks are investing in the risk-free project, it is always better to have them connected than not connected. This is true since one more neighbor increases the possibility of liquidity coinsurance and therefore the expected payoff. Such additional link does not induce any bank to switch investment decision from the risk-free to the gambling project. Indeed, if a bank has enough bank capital to choose the safe project in a given interbank network and it prefers this allocation to autarky, then the same bank capital will be sufficient to avoid the gambling behavior if the bank has one more neighbor that invests in the risk-free project.

The next proposition determines the size of the core in a CFB network. Given an allocation (K, x, s), we denote by $C(K, x, s)$ the set of banks that choose the risk-free project. Note that if (K, x) is equal to $(\emptyset, e)$ then all the banks are in autarky. It is easy to verify that there is a unique INE in autarky, denoted $(\emptyset, e, s^A)$, such that for any bank i

$$s_i^A = \begin{cases} rf & \text{if } e_i \geq I^*(0, 0, \xi, \eta) = \frac{B}{(1-\xi)R}, \\ r & \text{otherwise.} \end{cases} \tag{10}$$

Proposition 3 *Let (K^*, x^*, s^*) be a CFB allocation. Then, $C(\emptyset, e, s^A) \subseteq C(K^*, x^*, s^*)$.*

The proof is in the Appendix. Proposition 3 implies that the size of the core in a CFB network can only increase as compared to the number of safe banks in autarky. More importantly, it also establishes that the core in a CFB network has to include the banks that invest in the risk-free project in autarky. The intuition is as follows. A bank that invests in the safe project in autarky is a bank with a relative high initial endowment of bank capital. But the higher the initial endowment of bank capital, the higher the capital the planner has to allocate to satisfy the investors' participation constraint of that bank. Precisely, the minimum bank capital the planner has to offer to make investors participate in the optimal network is sufficiently high to induce investors to choose the safe project in an INE for that network. Since the optimal allocation is an INE, Proposition 3 follows.

4.1 Links in the CFB Interbank Network

After establishing that the CFB interbank network has a core-periphery structure, we determine now the number of optimal links that each bank should have in the CFB network. This number will depend on the relationship between the benefit, captured by the function $f(k)$, and the cost, represented by the counterparty risk η, of participating in the interbank network. To characterize in a simple manner the number of optimal links, we assume the function $f(k)$ is such that the expression $\left(\frac{f(k_i)}{f(k_i+1)}\right)^{k_i+1}$ is increasing in k_i. We refer to this property as increasing power ratios (IPR) property.[5]

Under the assumption that the function $f(k)$ satisfies the IPR property, the relationship between $f(k)$ and η can be parameterized in terms of a number $k(\eta)$ that is defined as follows

$$k(\eta) = \begin{cases} 0 & \text{if } \eta < \frac{1}{f(1)}, \\ k^* & \text{if } \left(\frac{f(k^*-1)}{f(k^*)}\right)^{k^*} \leq \eta < \left(\frac{f(k^*)}{f(k^*+1)}\right)^{k^*+1} \text{ for some } k^* \in (0, n-1), \\ n-1 & \text{if } \eta \geq \left(\frac{f(n-2)}{f(n-1)}\right)^{n-1}. \end{cases} \tag{11}$$

The next proposition characterizes the number of optimal links in the CFB network.

Proposition 4 *Let (K^*, x^*, s^*) be a CFB allocation for a given economy (N, e). Assume $f(k)$ satisfies the IPR property. Let $k(\eta)$ be as defined in (11). Then:*

1. *If $k(\eta) = n - 1$ then $K_i^* = N \backslash \{i\}$ for all $i \in N$.*

2. *If $k(\eta) = 0$ then $g_i^* = 0$ for all i such that $s_i^* = r$.*

3. *If $k(\eta) \in (0, n-1)$ then :*

   (a) *$k_i^* < k(\eta)$ for some i implies that either $s_i^* = rf$ and $K_i^* = N \backslash \{i\}$ or $s_i^* = r$ and $k_j^* \geq k(\eta)$ for all $j \notin K_i^*$.*

   (b) *$k_i^* > k(\eta)$ and $s_i^* = r$ implies that for all $j \in G_i$:*

      i. *$k_j \leq k(\eta)$;*

[5] An increasing function like $f(k)$ satisfies the IPR property whenever it is also concave (i.e., $f''(k) < 0$), $f'''(k) > 0$ and $f'(0)$ is sufficiently high (result upon request). It is interesting to notice that most common concave functions, like $f(k) = 1 + k^A$ with $A < 1$ or $f(k) = 1 + \exp\{A - \frac{1}{k}\}$ with $A > 0$, satisfy the IPR property.

ii. there is no other bank b with $k_b^* < k(\eta)$ and $b \notin K_j$;

iii. for any other bank b with $k_b^* > k(\eta)$: $s_b^* = r$ and there is no bank $z \in G_b$ such that $z \notin K_j$.

The proof is provided in the Appendix. Statement 1 establishes that the CFB network is the complete network structure in which all banks are connected if $\eta \geq \left(\frac{f(n-2)}{f(n-1)}\right)^{n-1}$. The intuition is that when counterparty risk is small with respect to the benefits provided by the interbank network, connecting until the last gambling bank in the network increases the total expected payoff. Statement 2 determines that when $\eta < \frac{1}{f(1)}$ the CFB network is characterized by a sparse periphery in which the gambling banks are disconnected among each other. In this case counterparty risk is so high that it is not optimal to link two gambling banks. Notice that Statement 2 still allows risky banks to be connected to safe banks.

Statement 3 characterizes the CFB network for intermediates values of η. Statement 3a establishes that two disconnected banks cannot have less than $k(\eta)$ links. That is, whenever a bank has less than $k(\eta)$ links then all the banks that are not connected to it have already at least $k(\eta)$ links. Otherwise, the planner would connect two banks with less than $k(\eta)$ links, getting a higher expected payoff. Statement 3(b)i establishes that two periphery banks cannot have more than $k(\eta)$ links and be directly connected. That is, no further links have to be formed between two risky banks if they already have $k(\eta)$ links. Disconnecting two banks in this case yields a higher expected payoff than leaving them connected.

Statement 3(b)ii affirms that if two gambling banks i and j are connected, but bank i has more than $k(\eta)$ links and bank j has less than $k(\eta)$ links, the planner increases the expected payoff by severing their connection and connecting bank j to another gambling bank b that has less than $k(\eta)$ links. Finally, statement 3(b)iii says that if two gambling banks, called i and b, both with more than $k(\eta)$ links, have gambling neighbors, j and z, that are not linked with each other, the planner increases the expected payoff by connecting j and z while disconnecting them from i and b, respectively.

As already noticed, Proposition 4 does not preclude the possibility for the planner to find it optimal to link a gambling bank to a safe bank. The following Proposition establishes a sufficient condition under which the CFB network has an empty periphery, that is gambling banks are disconnected also from safe banks.

Proposition 5 *Let* (K^*, x^*, s^*) *be a CFB for a given economy* (N, e). *Then, if* $\eta < \left(\frac{1-\xi}{\rho[1+n\rho]}\right)^{n-1}$ *then* $K_i^* = \emptyset$ *for all* i *with* $s_i^* = r$.

The proof is in the Appendix. The intuition is as follows. The planner finds optimal to connect a gambling bank with a safe bank if the expected benefit for the former are high enough to outweigh the counterparty risk faced by the latter. The higher the counterparty risk is (i.e., the smaller η is) the larger the risk for the safe bank to be connected to the gambling bank is. At some point, counterparty risk becomes so high that the expected benefit for the gambling bank does not outweigh the risk taken by the safe bank.

5 Decentralized Interbank Network

In this section we study the decentralized interbank network formation. We assume first that bank capital transfers are not allowed at $t = 1$. Characterizing the efficiency properties without transfers makes it easier to analyze the effects of bank capital transfers.

5.1 Decentralized Interbank Networks without Transfers

We define an *economy without transfers* (N, e) in which the sequence of events in Table 1 does not include bank capital transfers across banks in $t = 1$. We solve the model backwards. Banks choose the network structure given their initial endowment of bank capital, anticipating the INE played in that network. For a given economy (N, e), an allocation without transfers (K, e, s) is an INE if there are no unilateral profitable deviations in the type of project that shareholders choose as investment.

Without bank capital transfers the network formation game is basically a static one, that is banks choose simultaneously to whom they want to connect. Accordingly, we adopt the equilibrium notion of *pairwise stability* introduced by Jackson and Wolinsky [30]. Given a network K, let $K \cup ij$ the network resulting from adding a link joining banks i and j to the existing network K. On the contrary, for any two banks i and j connected in K, let $K \backslash ij$ denotes the resulting network from severing the link joining banks i and j from K.

Definition 2 *An allocation without transfers* (K, e, s) *is pairwise stable (PSWT) if the following holds:*

1. *For all i and j directly connected in K: $m_i(K,e,s) \geq m_i(K\backslash ij, e, \tilde{s})$ and $m_j(K,e,s) \geq m_j(K\backslash ij, e, \tilde{s})$ for all allocations $(K\backslash ij, e, \tilde{s})$ that are INE.*

2. *For all i and j not directly connected in K: if there is an INE $(K \cup ij, e, \tilde{s})$ such that $m_i(K,e,s) < m_i(K \cup ij, e, \tilde{s})$, then $m_j(K,e,s) > m_j(K \cup ij, e, \tilde{s})$.*

The definition of PSWT captures two ideas that derive from the notion of pairwise stability. The first refers to the network's internal stability: no pair of banks directly connected in the current interbank network individually gain from severing their link. This implies that any of the two banks could sever the link unilaterally. The second establishes the network external stability: if one bank could gain from creating a link with another bank, it has to be that the other bank cannot gain from that link. This implies that both banks have to agree in order to create a link. Note that if one bank strictly gains with the creation of one link and the other bank is indifferent, it is assumed that the link is formed.

Definition 3 *An allocation without transfers (K,e,s) is a decentralized equilibrium (DEWT) if it is INE and PSWT.*

The following proposition establishes the set of decentralized equilibria for a given economy without transfers (N,e). It shows that also the decentralized network is characterized by a core-periphery structure.

Proposition 6 *Assume (N,e) define an economy without transfers. Then, a DEWT is a core-periphery structure, i.e., if (K^e, e, s^e) is a DEWT, then, for every pair of banks i and j such that $s_i^e = s_j^e = rf$, we have that $i \in K_j^e$ and $j \in K_i^e$.*

The proof is in the Appendix, while the intuition is as follows. On the one hand, a bank agrees to be connected to any neighbor that is choosing the risk-free project since this decision entails no counterparty risk. On the other hand, if a bank invests in the risk-free project any other bank would like to be connected to it for the same reason. Since links are expected to be beneficial for both participating banks, two banks choosing the safe project have the right incentive to be connected. A core-periphery structure emerges in which the safe banks are connected among themselves. Similar to the analysis of the CFB network, the connectivity in the decentralized network of the banks choosing the gambling project depends on the relationship between $f(k)$ and η. We have the following

Proposition 7 *Assume the function $f(k)$ satisfies the IPR property, and let $k(\eta)$ be as defined in (11). Then:*

1. *If (K^e, e, s^e) is a DEWT allocation, then $k_i^e < k(\eta)$ for some i implies that either $s_i^e = rf$ and $K_i^e = N\backslash\{i\}$ or $s_i^e = r$ and $k_j^e \geq k(\eta)$ for all $j \notin K_i^e$.*

2. *If (K^e, e, s^e) is a DEWT allocation, then $k_i^e > k(\eta)$ only if $G_i = \emptyset$.*

The proof is provided in the Appendix. Statement 1 indicates that in the decentralized interbank network two disconnected banks will not have less than $k(\eta)$ links. If there were two banks both having less than $k(\eta)$ neighbors, they would both gain by connecting to each other. This is also the same condition of the CFB network (given by Statement 3a in Proposition 4). Statement 2 indicates that $k(\eta)$ is also the highest number of neighbors that a bank will have, provided at least one of its neighbors is choosing the risky project. Note this is not the same condition of the CFB network, where $k(\eta)$ is the highest number of neighbors that two directly connected periphery banks can have (given by Statement 3b in Proposition 4). The reason is that it is not individually optimal to hold more than $k(\eta)$ links if at least one neighbor is risky, while for the planner it could be optimal to have a bank holding more than $k(\eta)$ links if the expected benefits of its neighbors compensate for its expected loss.

The next Corollary characterizes the decentralized network for the two extreme values of $k(\eta)$.

Corollary 1 *Assume the function $f(k)$ satisfies the IPR property, and let (K^e, e, s^e) be a DEWT for a given economy without transfers (N, e). Then:*

1. *If $k(\eta) = n - 1$ then $K_i^e = N\backslash\{i\}$ for all $i \in N$.*

2. *If $k(\eta) = 0$ then $g_i^e = 0$ for all i.*

Statement 1 shows that the decentralized interbank network is characterized by the complete structure when $\eta \geq \left(\frac{f(n-2)}{f(n-1)}\right)^{n-1}$. This is the same condition of the CFB network (given by Statement 1 in Proposition 4). When counterparty risk is very low, banks in the decentralized network take the optimal risk of being connected to the gambling banks. The incentives in the formation of the decentralized network are aligned with those of the planner, and there is no difference between the CFB and the decentralized networks. It is important to notice that Corollary 1, even if it establishes that the decentralized network structure could coincide with the CFB one, does not imply that investment decisions are the same in both networks. In the DEWT the investment decisions might be suboptimal (i.e., too many banks investing in the risky project). However, when η tends to one, the two type of projects yield (almost) the same expected payoff. As counterparty risk vanishes, the only factor that determines the expected payoff in the economy *is* the network structure.

Statement 2 in Corollary 1 establishes that the condition $\eta < 1/f(1)$ is sufficient to observe an empty periphery in the decentralized network (i.e., banks that invest in the risky project are not linked with any other bank including the safe ones). However, Proposition 4 states that the same condition is not sufficient for the empty periphery to be optimal. The planner allows in this case safe banks to have more risky links respect the decentralized network. The condition to have an empty periphery in the CFB network is given in Proposition 5. As a consequence, banks that invest in the risky project can be inefficiently under-connected in the decentralized interbank network whenever[6]

$$\eta \in \left[\left(\frac{1-\xi}{\rho\,[1+n\rho]} \right)^{n-1}, \frac{1}{f(1)} \right].$$

The decentralized interbank network shows fewer connections than the CFB network since banks that invest in the risk-free project consider engaging in bilateral insurance too risky. The benefit provided by the interbank network is neglected when counterparty risk is particularly high. Safe banks do not internalize the overall benefit of a higher interbank connection (i.e., an higher liquidity coinsurance).[7] The presence of inefficiency is due to the absence of a mechanism that would allow banks that invest in the risk-free project to internalize the positive network externality they create on risky banks. We are going to analyze this possibility in the next section.

[6]The interval is non empty since we have

$$\left(\frac{1-\xi}{\rho\,[1+n\rho]} \right)^{n-1} < \left(\frac{1-\xi}{\rho\,[1+n\rho]} \right) < \frac{1}{\rho\,[1+n\rho]} < \frac{1}{f(1)} < 1,$$

given that $1 > \xi > 0$, $n > 2$ and $1 < f(1) < \rho$.

[7]Clearly, since counterparty risk η is not the only parameter in determining the shape of a CFB network (bank capital endowments are important as well), the decentralized network is ‘underconnected’ whenever the more connected network is feasible given the initial capital endowments.

5.2 Decentralized Networks with Transfers

We consider now a sequential-move game in which banks can transfer bank capital to their neighboring banks. Bloch and Jackson [11] show that efficient networks are supported, although not uniquely, by pairwise stable equilibria when agents make transfers contingent on the network formed at the end. That is, they consider transfers made *before* the network is chosen in order to induce (because of positive externalities) or to deter (because of negative externalities) the creation of links to any other agent in the economy. Also in our model efficient networks could be achieved by assuming a system of contingent transfers. We depart from this assumption since such system of contingent transfers seems too demanding given that it would require a high level of coordination in the banking system. Following the sequence of events in our game in Table 1, transfers are made *after* the network structure is chosen but before the investment decisions take place. Transfers are not contingent on creating or destroying a link, but a bank making a transfer could induce the desired investment behavior of its neighbors reducing the exposure to counterparty risk associated with its links.

We again solve the model backwards. We analyze the INE given the transfers and the network. Then, given the network, we solve for the transfers anticipating the INE played in the last stage. Finally, we characterize the decentralized network anticipating the transfers made and the INE played in the following stages. As mentioned before, our concept of INE comes from a simultaneous-move game in which INE might not be unique. This can be problematic in a decentralized context where there is no coordination device.[8] However, we abstract from possible inefficiencies due to coordination failure in the investment decision. Our focus is on whether bank capital transfers can solve for the inefficiencies in the process of network formation.

We therefore consider a sequential-move game with perfect information in the transfer stage, with the following rule of order. We rank banks according to their bank capital endowment, and we start the order of transfers from the highest to the lowest capital endowed bank. Once the bank endowed with the lowest capital has taken his transfer decision, we move on to the investment stage. In this stage, the bank with the highest capital (which now might be different from the bank with the highest capital endowment because of the

[8]Note that the definition of PSWT does not preclude the possibility of coordination failure, that is the possibility of encountering more than one INE given network K and the initial endowment e.

transfers) decides the type of investment, and the other banks follow according to their level of bank capital. The backward solution allows us to select one INE and one profile of transfers for a given network, that will be a Subgame Perfect Equilibrium in the transfer and investment game. Once the transfer and investment profile are uniquely determined in equilibrium, we can apply pairwise stability to the network formation process.

We have the following

Proposition 8 *Let K be a given network and x be a vector of bank capital reallocation such that there are multiple INE following (K, x). Let $Q(s, s') \subseteq N$ be the group of banks for which investment decisions change from the risky to the risk-free project in two INE s, s' following (K, x). Then the backward induction argument in the sequential-move investment game does not select the INE where agents in Q choose the risky project, independently of the rule of order.*

The proof is in the Appendix. Proposition 8 states that with sequential investment decisions there is no coordination problem among banks. This means that if banks arrive at the investment stage after choosing the optimal network and having done the optimal transfers, the decentralized interbank network would coincide with the CFB network. We explore now whether bank capital transfers in a sequential move game lead to a CFB allocation, assuming that INE decisions in the last stage are optimal.

Proposition 9 *There exists a $\overline{\xi} \in [0, 1]$ such that if $\xi > \overline{\xi}$ then there are no transfers in the sequential-move transfer game that can induce a bank that invest in the risky project to switch its investment decision.*

The proof is in the Appendix. The intuition is the following. When the probability of success of the gambling project is sufficiently high, then no bank capital transfers occur. The high probability of success of the gambling project implies that counterparty risk is relatively low. Therefore the amount of bank capital that has to be transferred becomes too costly. Whenever ξ is sufficiently high, banks avoid to transfer bank capital to their neighboring gambling banks and prefer to face the (low) counterparty risk. It follows that if the decentralized network is not a CFB it remains an inefficient network even when transfers are allowed.

We are left to analyze the case in which the probability of success of the gambling project becomes lower (and counterparty risk becomes higher). In this case, is the decentralized allocation with bank capital transfers able to mimic the CFB interbank network? The answer is negative, even when transfers are correct (i.e., they give the right investment incentives to the neighboring banks) for each given network structure. We conclude this Section by giving the reason of why this occurs and a couple of simple examples to show the rationale behind it.[9]

Given that the incentive and the participation constraints are satisfied in a CFB network, banks prefer this network (with the implicit transfer of capital and the proposed investment decisions) to autarky. This property of the CFB network nevertheless does not prevent that at least one bank can improve upon the expected payoff provided by the CFB network by creating one new link or deleting an existing one. That is, the presence of bank capital transfers does not avoid the formation of under-connected decentralized network (like it was the case without such transfers) but actually induces also the creation of over-connected decentralized networks.

Let us illustrate the incentives to create an over-connected decentralized network. Consider four banks such that one of them, call it bank 1, has a positive amount of capital while the other three banks have an amount of it close to zero: $e_1 > 0$, and $e_2 = e_3 = e_4 = \varepsilon \approx 0$. Suppose e_1 is high enough so that in any CFB interbank network there is only one link connecting bank 1 to one of the other three banks. The two connected banks choose the risk-free project after a capital transfer is made from bank 1 to its neighbor. This CFB interbank network is consistent with $\eta < \frac{1}{f(1)}$. Indeed, in the decentralized network, bank 1 would transfer to its neighbor in the CFB exactly the amount of capital necessary to switch from the risky to the risk-free project.[10] What happens if bank 1's neighbor connects to one of the two risky banks that are disconnected in the CFB? For some values of the parameters, bank 1 still transfers to its neighbor exactly the amount of capital necessary to switch investment decision (so that transfers are correct). However, this makes bank 1's neighbor indifferent to create a link with one of the two disconnected banks.[11] And both disconnected banks would strictly gain from connecting to bank 1's neighbor.[12] Hence the CFB cannot be an equilibrium since two banks that should not be connected

[9] More elaborated numerical examples are available upon request.

[10] Bank 1's neighbor would then get $x_i = \frac{B}{(1-\xi)f(1)R}$ and obtain an expected payoff equal to $\frac{B}{(1-\xi)}$.

[11] Bank 1's neighbor would in this case get $x_i = \frac{B}{(1-\xi)\eta^{1/2}f(2)R}$, obtaining the same expected payoff $\frac{B}{(1-\xi)}$.

[12] The disconnected banks have an expected payoff equal to $\xi R f(1)\epsilon + B$ if they connect to bank 1's neighbor, as opposed to $\xi R\epsilon + B$ when disconnected.

would gain from doing so. The reason is that the transfer made by bank 1 represents a positive externality for the other banks since it induces a positive switch, from the risky to the safe project, in the investment strategy. Bank 1 does not internalize such externality and other banks have incentive to over-connect.

Let us now illustrate the incentives for under-connecting. Consider the same four banks, but assume that e_1 is high enough so that the CFB interbank network has three banks in the core. In the CFB bank 1 makes a transfer to its two neighbors that is sufficient to make them switch the investment decision from the risky to the risk-free project. What happens if bank 1 deletes one of his connections in the CFB? For some values of the parameters, bank 1 still transfers the exact amount of capital that induces to switch the investment decision of the remaining connected bank (so again transfers are correct) but it severs the connection and the relative transfer with the other bank. The reason is that in the CFB the planner guarantees that the expected gains of the two core banks receiving the transfers compensate for the expected loss that bank 1 suffers, such that the three banks are better off than in autarky. This is not incompatible with the fact that bank 1 individually prefers being connected to one bank (making only one transfer) than being linked to two banks (making two transfers), even though the latter situation is better than autarky. The CFB cannot be an equilibrium, as bank 1 has an incentive to delete one of its links.

6 Conclusion

We present a model of interbank network formation and characterize the set of optimal networks as core-periphery structures. The optimal networks get more and more connected as the counterparty risk becomes smaller and smaller. The decentralized interbank networks also show a core-periphery structure, although the size of the core may not coincide with the optimal one. On the contrary, when the counterparty risk is not so low an inefficient decentralized interbank network arises. This network externality in general is not internalized even if ex-post bank capital transfers are allowed. Some of our assumptions and results need some clarifications.

First, our exogenous assumptions on the benefits and costs of connecting to the interbank network allow us to sharp our analysis on the optimal network structure and the decentralized network formation. Even if we provide, in our opinion, a reasonable justification for these assumptions, a better understanding of the micro foundation that are able to rationalize such benefits and costs is clearly needed.

Second, we analyze the contagious effect of the links that *directly* connect banks (what we refer as counterparty risk), ruling out indirect contagion and systemic effects. Our results point out that systemic effects are not necessary in order to have inefficiencies in a decentralized interbank network. The paper highlights that the beneficial role of the interbank network is negatively affected when counterparty risk is not sufficiently low.

Third, we assume that all banks have access to the same coinsurance independently of the connections of their counterparts. For example, consider three banks and two structures. While in the first structure banks 1 and 3 are disconnected but both connected to bank 2, in the second structure all banks are directly connected. In our setting, bank 2 has the same access to coinsurance in both networks. However, it could be that bank 2 coinsurance becomes weaker or stronger when banks 1 and 3 are directly connected.[13] In our model, allowing for such possibility implies that the function $f(.)$ should have more than one argument. However, under the assumption that $f(.)$ is increasing in k_i, the results on the core-periphery structure still hold. Nevertheless, we consider this an interesting avenue for future research in network formation games.

Forth, establishing a connection in our model is a strategic decision that does not imply any capacity associated with the link. The establishment of a link is therefore a decision without any consequence on the level of exposure that a bank has with each of its linked banks (an issue that is analyzed for example by Cabrales et al. [16] and Castiglionesi and Eboli [17]). Clearly, joining the two strands of the literature represents a challenge for future research.

Finally, a crucial topic for future research concerns the implication of the network analysis for financial regulation. Our results suggest that decentralized networks form structures that resemble the CFB core-periphery structure but the decentralized network can be inefficiently under-connected when counterparty risk becomes sufficiently high. More work remains to be done to understand the appropriate instruments to address network inefficiencies.

[13] Castiglionesi and Wagner [19] provide a three-bank model where this issue is analyzed.

Appendix: Proofs

Proof of Proposition 1. The existence of a CFB can be proved in two steps. First, fixing each network K and each vector of investments s such that $p_i(K,s)f(k_i)R \geq 1$ (constraint 9) and maximizing the objective function subject to

$$x_i \geq 0 \text{ for all } i \in N \tag{12}$$

$$\sum_{i \in N} x_i = \sum_{i \in N} e_i \equiv E \tag{13}$$

$$x_i \geq I^*(k_i, g_i, \xi, \eta) \text{ if } s_i = rf \tag{14}$$

$$x_i < I^*(k_i, g_i, \xi, \eta) \text{ if } s_i = r. \tag{15}$$

Second, once the maximum total expected payoff is obtained given K and s, it suffices to choose the combination of K and s that has the highest expected total payoff. Notice that the second step does not create problem of existence as we are choosing the highest number on a discrete set of numbers. However, in the first step the problematic restriction is (15). Let us modify the planner problem by rewriting such constraint as

$$x_i \leq I^*(k_i, g_i, \xi, \eta) \text{ if } s_i = r.$$

In the modified problem, a maximum always exists. Furthermore, the solution of the modified problem is also a maximum of the social planner's problem. If this were not true, then there is an economy for which the solution of the modified problem, a triple (K^*, x^*, s^*), is such that $x_i^* = I^*(k_i, g_i, \xi, \eta)$ for at least one bank i with $s_i = r$. But if $x_i^* = I^*(k_i, g_i, \xi, \eta)$ for at least one bank i then the triple (K^*, x^*, s'), where s' and s^* differ only on the choice of investment by bank i, is also feasible and yields a higher expected payoff to at least bank i and its neighbors (and no bank gets lower expected payoff) than in s^*. Therefore, (K^*, x^*, s^*) could not have been a maximum of the modified problem, which is a contradiction.

Concerning the second statement, the participation and incentive constraints have to be satisfied. First, consider the incentive constraints, and note that if $\sum_{i \in N} e_i = E \geq \frac{n-1}{\rho-1} \frac{\rho B}{(1-\xi)R} \equiv E^*$ there is enough bank capital to satisfy them. This is so because

$$\frac{n-1}{\rho-1} \frac{\rho B}{(1-\xi)R} \geq \frac{nB}{(1-\xi)\,\rho R},$$

given that $1 < \rho \leq 2 \leq n$.

Regarding the participation constraints, note that the higher the initial capital endowment of a bank the harder it is for the planner to satisfy the participation constraint. The most difficult case is when one bank is endowed with all the bank capital in the economy E and the remaining banks have 0 bank capital. Since $n \geq 2 \geq \rho$, then $E \geq \frac{nB}{(1-\xi)\rho R}$ implies that $E \geq \frac{B}{(1-\xi)R}$ and the 'all-endowed' bank chooses the risk-free project in autarky. The rest of the banks choose the gambling project in autarky. Therefore, the planner needs to give $\max\{\frac{B}{(1-\xi)R}, \frac{E}{\rho}\}$ dollars of bank capital to the highly endowed bank and $\max\{\frac{B}{(1-\xi)R}, \frac{B}{\rho R}\}$ to each of the other banks. Note that $\frac{E}{\rho} \geq \frac{B}{(1-\xi)R} \geq \frac{B}{\rho R}$, for $1 < \rho \leq 2 \leq n$. Therefore, the planner needs E to be at least equal to $\frac{E}{\rho} + (n-1)\frac{B}{(1-\xi)R}$, or, rearranging $E \geq \frac{(n-1)}{(\rho-1)}\frac{\rho B}{(1-\xi)R}$. If the banks had the same bank capital endowment, it is easy to check that the condition for INE is sufficient to induce investors of all banks to participate. ■

To proof the following propositions it is useful to make use of transfers that are payoff-equivalent to the expected payoff in autarky. We indicate the expected payoffs in autarky as $m_i^A = max\{Re_i, \xi Re_i + B\}$ for each bank i. For a given network structure K and a strategy profile s, let $x_i^A(K,s)$ be such that $m_i\left(K, x_i^A(K,s), s\right) = m_i^A$. That is, $x_i^A(K,s)$ is a reallocation of bank capital that makes bank i indifferent between participating or not in the network K with strategy profile s. Note that the reallocation of bank capital $x_i^A(K,s)$ is unique given (K,s).

Proof of Proposition 2. The proof is made by contradiction. Assume that (K^*, x^*, s^*) is a CFB for (N,e) but there are two unconnected banks i and j such that $s_i^* = s_j^* = rf$. Take a new allocation $(\hat{K}, x^*, s^*)$ for the same economy (N,e), where the allocation of bank capital and strategies are the same but the network structure only adds the link between banks i and j. Formally, $\hat{K}_i = K_i^* \cup \{j\}$, $\hat{K}_j = K_j^* \cup \{i\}$, and $\hat{K}_b = K_b^*$ for all banks $b \neq i,j$. We show now that the allocation $(\hat{K}, x^*, s^*)$ is an INE, it satisfies the participation constraints for any bank and it yields a higher expected total payoff. Therefore, the initial allocation (K^*, x^*, s^*) cannot be a solution to the planner's problem and Proposition 2 follows.

Note that, by definition of $\hat{K}$, $p_b(\hat{K}, s^*) = p_b(K^*, s^*)$ for all $b \in N$, and

$$\hat{k}_b = \begin{cases} k_b^* + 1 & \text{if } b = i \text{ or } b = j \\ k_b^* & \text{otherwise.} \end{cases}$$

We first show that $(\hat{K}, x^*, s^*)$ is an INE. Note that (K^*, x^*, s^*) is an INE given that it is a CFB. This means, since both i and j are choosing the risk-free project, that $x_i^* \geq$

$I^*(k_i^*, g_i, \xi, \eta)$ and $x_j^* \geq I^*(k_j^*, g_j, \xi, \eta)$. By definition of the functions I^*, it is true then that $x_i^* \geq I^*(k_i^*+1, g_i, \xi, \eta)$ and $x_j^* \geq I^*(k_j^*+1, g_j, \xi, \eta)$. Therefore, the allocation $(\hat{K}, x^*, s^*)$ is an INE.

Second, we show that $(\hat{K}, x^*, s^*)$ satisfies the investors participation constraints in any bank. Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it is true that $x_i^* \geq x_i^A(K^*, x^*)$ and $x_j^* \geq x_j^A(K^*, x^*)$. Recall that: (i) $x_i^A(K^*, x^*) = \frac{m_i^A}{\eta^{g_i/k_i} f\left(k_i^*\right) R}$ and $x_j^A(K^*, x^*) = \frac{m_j^A}{\eta^{g_j/k_j} f\left(k_j^*\right) R}$; (ii) $x_i^A\left(\hat{K}, x^*\right) = \frac{m_i^A}{\eta^{g_i/(k_i+1)} f\left(k_i^*+1\right) R}$ and $x_j^A\left(\hat{K}, x^*\right) = \frac{m_j^A}{\eta^{g_j/(k_j+1)} f\left(k_j^*+1\right) R}$. Given that $f(k)$ is increasing in k and η a probability, it is true that $x_i^A(K^*, x^*) > x_i^A\left(\hat{K}, x^*\right)$ and $x_j^A(K^*, x^*) > x_j^A\left(\hat{K}, x^*\right)$. Therefore $x_i^* \geq x_i^A\left(\hat{K}, x^*\right)$ and $x_j^* \geq x_j^A\left(\hat{K}, x^*\right)$. This means that the allocation $(\hat{K}, x^*, s^*)$ satisfies the investors participation constraints in any bank.

Third, we see that the depositors' participation constraints are satisfied in $(\hat{K}, x^*, s^*)$. Note that, by assumption, (K^*, x^*, s^*) satisfies the participation constraints for depositors, given that it is a CFB. This means that $\eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) R \geq 1$ for all banks b. Since the function $f(k)$ is increasing in k and η is a probability, it has to be that $\eta^{\frac{g_i}{(k_i^*+1)}} f\left(k_i^*+1\right) R \geq 1$ and $\eta^{\frac{g_j}{(k_j^*+1)}} f(k_j^*+1) R \geq 1$.

Finally,

$$
\begin{aligned}
& \sum_{b \in N}\left[m_b\left(\hat{K}, x_b^*, s^*\right)+M_b\left(\hat{K}, s^*\right)\right]-\sum_{b \in N}\left[m_b(K^*, x^*, s^*)+M_b\left(K^*, s^*\right)\right] \\
= \; & R\left(x_i^*+1\right)[\eta^{\frac{g_i}{(k_i^*+1)}} f(k_i^*+1)-\eta^{\frac{g_i}{k_i^*}} f(k_i^*)]+R\left(x_j^*+1\right)[\eta^{\frac{g_j}{(k_j^*+1)}} f(k_j^*+1)-\eta^{\frac{g_j}{k_j^*}} f(k_j^*)]>0
\end{aligned}
$$

since $f(k)$ is increasing in k and η is a probability. Given the last inequality, the allocation $(\hat{K}, x^*, s^*)$ yields higher expected payoff in the economy and therefore (K^*, x^*, s^*) was not a CFB. ■

Proof of Proposition 3. We prove that if an allocation is an INE and it satisfies the investors participation constraints in any bank, then it has to be that any bank choosing the risk-free project in autarky chooses the same project in this allocation as well. Take any bank i such that $\max\{Re_i, \xi Re_i + B\} = Re_i$, i.e. it chooses the safe project in autarky. Assume by contradiction that there exists an allocation (K, x, s) that is an INE, it satisfies the participation constraints for any bank and $s_i = r$. This implies then:

1. $x_i < I^*(k_i, g_i, \xi, \eta)$ since (K, x, s) is an INE, and
2. $\xi \eta^{g_i/k_i} f\left(k_i\right) R x_i + B \geq Re_i$ since (K, x, s) satisfies the participation constraint.

Given that bank i chooses the risk-free project in autarky, we have $Re_i \geq \frac{B}{(1-\xi)}$. The last condition together with the participation constraint for bank i (item 2 above), implies that

$$\xi \eta^{g_i/k_i} f(k_i) R x_i + B \geq \frac{B}{(1-\xi)},$$

and rearranging terms, we have

$$x_i \geq \frac{B}{(1-\xi)\eta^{g_i/k_i} f(k_i) R} = I^*(k_i, g_i, \xi, \eta)$$

a contradiction with (K, x, s) being an INE (item 1 above). Therefore, any bank i choosing the risk-free project in autarky chooses the safe project in any INE satisfying the participation constraint (for at least that bank i). ■

In order to prove Proposition 4, it is useful to prove the following lemma.

Lemma 1 *Let $\underline{k}(\eta)$ be the highest $k \in \{1, ..., n-1\}$ such that $\eta \geq \left(\frac{f(k-1)}{f(k)}\right)^k$ and let $\overline{k}(\eta)$ be the lowest $k \in \{0, 1, ..., n-2\}$ such that $\eta < \left(\frac{f(k)}{f(k+1)}\right)^{k+1}$. Then:*

1. *If (K^*, x^*, s^*) is a CFB allocation, then, $k_i^* < \underline{k}(\eta)$ for some i implies that either $s_i^* = rf$ and $K_i^* = N \backslash \{i\}$ or $s_i^* = r$ and $k_j^* \geq \underline{k}(\eta)$ for all $j \notin K_i^*$.*

2. *If (K^*, x^*, s^*) is a CFB allocation, then, $k_i^* > \overline{k}(\eta)$ and $s_i^* = r$ implies that for all $j \in G_i$:*

   (a) $k_j \leq \overline{k}(\eta)$;

   (b) there is no other bank b with $k_b^* < \overline{k}(\eta)$ and $b \notin K_j$;

   (c) for any other bank b with $k_b^* > \overline{k}(\eta)$, $s_b^* = g$ there is no bank $z \in G_b$ such that $z \notin K_j$.

Proof of Lemma 1. We prove all statements by contradiction.

Proof of Statement 1. Assume that (K^*, x^*, s^*) is a CFB allocation where there are two banks i and j not directly connected and such that $k_i < \underline{k}(\eta)$ and $k_j < \underline{k}(\eta)$. Take a new allocation $(\hat{K}, x^*, s^*)$ for the same economy (N, e), where the allocation of capital and strategies are the same but the network structure adds the link between banks i and j. Formally, $\hat{K}_i = K_i^* \cup \{j\}$, $\hat{K}_j = K_j^* \cup \{i\}$, and $\hat{K}_b = K_b^*$ for all $b \neq i, j$.

We show first that the participation constraint is satisfied by $(\hat{K}, x^*, s^*)$ for all banks. Later, we show that $(\hat{K}, x^*, s^*)$ yields a higher total expected payoff than (K^*, x^*, s^*).

Therefore, the initial allocation (K^*, x^*, s^*) cannot be a solution to the planner's problem and Statement 1 follows. We separate cases depending whether bank i chooses the risk-free or the risky project in the investment profile s^*.

Note that if (K^*, x^*, s^*) is a CFB then it has to be an INE and it has to satisfy the participation constraints for depositors and investors in any bank. If at least one of them, for example bank i, is choosing the risk-free project, then $x_i^* \geq I^*(k_i^*, g_i, \xi, \eta)$. The other bank, say j, is choosing the risky project. We know from Proposition 2 that both banks cannot invest in the risk-free project because in such a case (K^*, x^*, s^*) would not be a CFB allocation.

We note that $I^*(k_i^*, g_i, \xi, \eta) \geq I^*(k_i^* + 1, g_i + 1, \xi, \eta)$ if and only if $\eta^{\frac{g_i+1}{k_i^*+1}} f(k_i^* + 1) \geq \eta^{\frac{g_i}{k_i^*}} f(k_i^*)$, or, equivalently, $\eta^{\frac{k_i^*-g_i}{k_i^*(k_i^*+1)}} \geq \frac{f(k_i^*)}{f(k_i^*+1)}$. Given that the ratio $\left(\frac{f(k_i)}{f(k_i+1)}\right)^{k_i+1}$ is increasing in k_i we have that

$$\left(\frac{f(\underline{k}(\eta)-1)}{f(\underline{k}(\eta))}\right)^{\underline{k}(\eta)} \geq \left(\frac{f(k_i)}{f(k_i+1)}\right)^{k_i+1} \quad \text{for any } k_i \leq \underline{k}(\eta) - 1.$$

Moreover, as $\eta < 1$ and $\frac{k_i^*-g_i}{k_i^*(k_i^*+1)} < \frac{1}{(k_i^*+1)}$. Hence, $\eta > \left(\frac{f(\underline{k}(\eta)-1)}{f(\underline{k}(\eta))}\right)^{\underline{k}(\eta)}$ implies that $\eta > \left(\frac{f(k_i^*)}{f(k_i^*+1)}\right)^{k_i^*+1}$ because $k_i^* \leq \underline{k}(\eta) - 1$. The latter inequality implies in turn that $\eta^{\frac{k_i^*-g_i}{k_i^*(k_i^*+1)}} \geq \frac{f(k_i^*)}{f(k_i^*+1)}$. Therefore, the allocation $(\hat{K}, x^*, s^*)$ is an INE as far as $x_j < I^*(k_i + 1, g_i, \xi, \eta)$. If it were not, there is an allocation $(\hat{K}, x^*, \hat{s})$ in which bank j chooses the risk-free project instead of the risky one. This would be an INE for bank i since $I^*(k_i^* + 1, g_i + 1, \xi, \eta) > I^*(k_i^* + 1, g_i, \xi, \eta)$.

Participation constraints. Assume that bank i chooses the risk-free project in (K^*, x^*, s^*). If $\eta \geq \left(\frac{f(\underline{k}(\eta)-1)}{f(\underline{k}(\eta))}\right)^{\underline{k}(\eta)}$ then $\eta^{\frac{1}{k_i^*+1}} f(k_i^* + 1) > f(k_i^*)$ for $k_i < \underline{k}(\eta)$. As $\frac{g_i+1}{k_i^*+1} - \frac{g_i}{k_i^*} = \frac{k_i^*-g_i}{k_i^*(k_i^*+1)} < \frac{1}{k_i^*+1}$, the latter inequality implies that

$$\eta^{\frac{g_i+1}{k_i^*+1}} f(k_i^* + 1) R x_i^* > \eta^{\frac{g_i}{k_i^*}} f(k_i^*) R x_i^*$$

and

$$\eta^{\frac{g_i+1}{k_i^*+1}} f(k_i^* + 1) R > \eta^{\frac{g_i}{k_i^*}} f(k_i^*) R$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\eta^{\frac{g_i}{k_i^*}} f(k_i^*) R x_i^* \geq m_i^A$ and $\eta \frac{g_i}{k_i^*} f(k_i^*) R \geq 1$, and therefore, $(\hat{K}, x^*, s^*)$ satisfies the participation constraints for bank i.

If bank i chooses the risky project in (K^*, x^*, s^*), the argument is equivalent. Note that if $\eta \geq \left(\frac{f(\underline{k}(\eta)-1)}{f(\underline{k}(\eta))}\right)^{\underline{k}(\eta)}$ then $\eta^{\frac{1}{k_i^*+1}} f\left(k_i^*+1\right) > f\left(k_i^*\right)$ for $k_i < \underline{k}\left(\eta\right)$. As $\frac{g_i+1}{k_i^*+1} - \frac{g_i}{k_i^*} = \frac{k_i^*-g_i}{k_i^*(k_i^*+1)} < \frac{1}{k_i^*+1}$, the latter inequality implies that

$$\xi \eta^{\frac{g_i+1}{k_i^*+1}} f\left(k_i^*+1\right) R x_i^* + B \geq \xi \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R x_i^* + B,$$

and

$$\xi \eta^{\frac{g_i+1}{k_i^*+1}} f\left(k_i^*+1\right) R \geq \xi \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R.$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\xi \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R x_i^* + B \geq m_i^A$ and $\xi \eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R \geq 1$, and therefore, $(\hat{K}, x^*, s^*)$ satisfies the participation constraint for bank i.

Checking that the participation constraint for bank j, which chooses the gambling project in (K^*, x^*, s^*), is satisfied in $(\hat{K}, x^*, s^*)$ works the same way as in the previous case and it is therefore omitted.

Total expected payoff. If bank i chooses the risk-free project in (K^*, x^*, s^*) we have:

$$\sum_{b \in N} \left[m_b\left(\hat{K}, x_i^*, s^*\right) + M_b\left(\hat{K}, s^*\right)\right] - \left[m_b\left(K^*, x_i^*, s^*\right) - M_b\left(K^*, s^*\right)\right] =$$

$$= \eta^{\frac{g_i}{k_i^*}} R\left(x_i^*+1\right)\left[\eta^{\frac{k_i^*-g_i}{k_i^*+1}} f(k_i^*+1) - f(k_i^*)\right] + \xi \eta^{g_j} R\left(x_j^*+1\right)\left[\eta^{\frac{1}{k_j^*+1}} f(k_j^*+1) - \eta^{\frac{1}{k_j^*}} f(k_j^*)\right]. \quad (16)$$

If banks i and j both choose the risky project in (K^*, x^*, s^*) we have:

$$\sum_{b \in N} \left[m_b\left(\hat{K}, x_i^*, s^*\right) + M_b\left(\hat{K}, s^*\right)\right] - \left[m_b\left(K^*, x_i^*, s^*\right) + M_b\left(K^*, s^*\right)\right] =$$

$$= \xi \eta^{\frac{g_i}{k_i^*}} R\left(x_i^*+1\right)\left[\eta^{\frac{k_i^*-g_i}{k_i^*+1}} f(k_i^*+1) - f(k_i^*)\right] + \xi \eta^{\frac{g_j}{k_j^*}} R\left(x_j^*+1\right)\left[\eta^{\frac{k_j^*-g_j}{k_j^*+1}} f(k_j^*+1) - f(k_j^*)\right]. \quad (17)$$

Expressions (16) and (17) are greater than zero since $\eta \geq \left(\frac{f\left(k_i^*\right)}{f\left(k_i^*+1\right)}\right)^{k_i^*+1}$ and $\eta \geq \left(\frac{f(k_j^*)}{f(k_j^*+1)}\right)^{k_j^*+1}$ for $k_i^* < \underline{k}\left(\eta\right)$ and $k_j^* < \underline{k}\left(\eta\right)$. Therefore, the allocation $(\hat{K}, x^*, s^*)$ yields a higher total expected payoff and therefore (K^*, x^*, s^*) is not a CFB.

Furthermore, equation (16) implies that if $s_i^* = rf$ any bank $j \notin K_i^*$ would be better off connecting to bank i. Hence, there is no bank j such that $j \notin K_i^*$ if $s_i^* = rf$ and $k_i^* < k\left(\eta\right)$. But then $k_i^* = n-1$, a contradiction. Therefore, if $k_i^* < \underline{k}\left(\eta\right)$ then $s_i^* = rf$ and $k_j^* > \underline{k}(\eta)$ for any bank $j \notin K_i^*$. Finally, if $(\hat{K}, x^*, s^*)$ is not an INE it is because bank j would choose the safe project as well, once $\hat{K}$ is given, or, when both banks i and j

choose the risky project in (K^*, x^*, s^*), an INE would select at least one of the two banks to choose the safe project. In all these cases, the new INE will yield an higher expected payoffs for both banks i and j. This implies that the participation constraint would be satisfied, and therefore a higher total expected payoff than in $(\hat{K}, x^*, s^*)$.

Proof of Statements 2a, 2b and 2c. Assume that (K^*, x^*, s^*) is a CFB allocation where there is at least one bank i with $s_i^* = r$ and $g_i^* \neq 0$. As before, we show that for all the three cases 2a, 2b, and 2c, there is an allocation $(\hat{K}, x^*, s^*)$ that satisfies the participation constraints for any bank and yields a higher total expected payoff. In particular, the allocation $(\hat{K}, x^*, s^*)$ yields both individual and total expected payoffs higher than the allocation (K^*, x^*, s^*), which then cannot be a solution to the planner's problem.

First, assume that statement 2a is not true and $k_j > \overline{k}(\eta)$ for $j \in G_i^*$. Take a new allocation $(\hat{K}, x^*, s^*)$ where the bank capital and strategies are the same but the network structure removes the link between banks i and j. Formally, $\hat{K}_i = K_i^* \backslash \{j\}$, $\hat{K}_j = K_j^* \backslash \{i\}$, and $\hat{K}_b = K_b^*$ for all $b \neq i, j$.

Second, assume that statement 2b is not true and there is a bank $b \neq j$ with $k_b < \overline{k}(\eta)$ and b and j are not directly connected. Take a new allocation $(\hat{K}, x^*, s^*)$ where the bank capital and strategies are the same but the network structure removes the link between banks i and j and creates a link between banks j and b. Formally, $\hat{K}_i = K_i^* \backslash \{j\}$, $\hat{K}_j = K_j^* \backslash \{i\} \cup \{b\}$, and $\hat{K}_b = K_b^* \cup \{j\}$ and for $\hat{K}_l = K_l^*$ for all $l \neq i, j, b$.

Finally, assume that statement 2c is not true and there is a bank $b \neq j$ with $k_b > \overline{k}(\eta)$ with $s_b^* = r$ such that one of its direct gambling neighbors z is not directly connected to j. Take a new allocation $(\hat{K}, x^*, s^*)$ where the bank capital and strategies are the same but the network structure severs the links between banks i and j, and between b and z, and it creates a link between banks j and z. Formally, $\hat{K}_i = K_i^* \backslash \{j\}$, $\hat{K}_j = K_j^* \backslash \{i\} \cup \{z\}$, $\hat{K}_b = K_b^* \backslash \{z\}$ and $\hat{K}_z = K_z^* \backslash \{b\} \cup \{j\}$ and for $\hat{K}_l = K_l^*$ for all $l \neq i, j, b, z$.

Participation constraints. If $\eta < \left(\dfrac{f\left(\overline{k}(\eta)\right)}{f\left(\overline{k}(\eta)+1\right)} \right)^{\overline{k}(\eta)+1}$ then $\eta^{\frac{1}{k_i^*}} f(k_i^*) < f(k_i^* - 1)$ because $k_i^* > \overline{k}(\eta)$. This implies that

$$\xi \eta^{\frac{g_i}{k_i^*}} f(k_i^*) R x_i^* + B < \xi \eta^{\frac{g_i - 1}{k_i^* - 1}} f(k_i^* - 1) R x_i^* + B,$$

or

$$\xi \eta^{\frac{g_i}{k_i^*}} f(k_i^*) < \xi \eta^{\frac{g_i - 1}{k_i^* - 1}} f(k_i^* - 1).$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\xi\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) Rx_i^* + B \geq m_i^A$ and $\xi\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R \geq 1$, and therefore $(\hat{K}, x^*, s^*)$ satisfies the participation constraints for bank i. The proof is equivalent for bank j if statement 2a is considered and for bank b if statement 2c is considered. Notice that bank j in statements 2b and 2c and bank z in statement 2c are indifferent between the allocations (K^*, x^*, s^*) and $(\hat{K}, x^*, s^*)$. Consider now the participation constraint for bank b in statement 2b. Note that given the definition of $\overline{k}(\eta)$, it has to be that $\eta^{\frac{1}{k_b^*+1}} f\left(k_b^*+1\right) > f\left(k_b^*\right)$ for $k_b^* < \overline{k}(\eta)$. This implies that

$$\xi\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) Rx_b^* + B > \xi\eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) Rx_b^* + B,$$

or

$$\xi\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) > \xi\eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right),$$

if b chooses the risky project. If bank b chooses the risk-free project, we have

$$\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) Rx_b^* > \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) Rx_b^*,$$

or

$$\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) > \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right).$$

Given that (K^*, x^*, s^*) satisfies the participation constraints for any bank, it has to be that $\xi\eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) Rx_b^* + B \geq m_b^A$ and $\xi\eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) R \geq 1$ if bank b is gambling, or $\eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) Rx_b^* \geq m_b^A$ and $\eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right) R \geq 1$, otherwise. Therefore the allocation $(\hat{K}, x^*, s^*)$ satisfies the participation constraints for bank b as well.

Total expected payoff. Let

$$\sum_{l\in N} m_l\left(\hat{K}, x_l^*, s^*\right) - \sum_{l\in N} m_l\left(K^*, x_l^*, s^*\right) \equiv \Delta,$$

and, conditional on which of the three statements we consider, we have:

in Statement 2a

$$\Delta = \xi R\left(x_i^*+1\right)\left[\eta^{\frac{g_i}{k_i^*-1}} f\left(k_i^*-1\right) - \eta^{\frac{g_i+1}{k_i^*}} f\left(k_i^*\right)\right] + \xi R\left(x_j^*+1\right)\left[\eta^{\frac{g_j}{k_j^*-1}} f\left(k_j^*-1\right) - \eta^{\frac{g_j+1}{k_j^*}} f\left(k_j^*\right)\right],$$

where both $k_i^* > \overline{k}(\eta)$ and $k_j^* > \overline{k}(\eta)$;

in Statement 2b

$$\Delta = \xi R\left(x_i^*+1\right)\left[\eta^{\frac{g_i}{k_i^*-1}} f\left(k_i^*-1\right) - \eta^{\frac{g_i+1}{k_i^*}} f\left(k_i^*\right)\right] + \xi R\left(x_b^*+1\right)\left[\eta^{\frac{g_b+1}{k_b^*+1}} f\left(k_b^*+1\right) - \eta^{\frac{g_b}{k_b^*}} f\left(k_b^*\right)\right],$$

where $k_i^* > \overline{k}(\eta)$ and $k_b^* < \overline{k}(\eta)$;

and in Statement 2c

$$\Delta = \xi R\left(x_i^*+1\right)\left[\eta^{\frac{g_i}{k_i^*-1}} f\left(k_i^*-1\right)-\eta^{\frac{g_i+1}{k_i^*}} f\left(k_i^*\right)\right]+\xi R\left(x_b^*+1\right)\left[\eta^{\frac{g_b}{k_b^*-1}} f\left(k_b^*-1\right)-\eta^{\frac{g_b+1}{k_b^*}} f\left(k_b^*\right)\right],$$

where both $k_i^* > \overline{k}(\eta)$ and $k_b^* > \overline{k}(\eta)$.

Using the same arguments we discussed above, it follows that $\Delta > 0$ in each of the three statements. ■

Proof of Proposition 4. Assume first that $\underline{k}(\eta) = n-1$. According to Statement 1 of Lemma 1, if there is any bank i with $k_i^* < n-1$ then, for all bank j not directly connected to bank i, we have $k_j^* = n-1$. But this is a contradiction, since any bank j with $n-1$ connections has to be directly connected to bank i. Assume now that $\overline{k}(\eta) = 0$. According to Statement 2a of Lemma 1, if there is any bank i that invest in the risky project and it is directly connected to another gambling bank j, it has to be that $k_j^* = 0$. But again this is a contradiction with the fact that bank j is directly connected to bank i. Finally, assume that $\underline{k}(\eta) \neq n-1$ and $\overline{k}(\eta) \neq 0$. By definition, $\overline{k}(\eta)$ is the minimum number k such that $\eta^{\frac{1}{k+1}} f(k+1) \leq f(k)$. Then it has to be that $\eta^{\frac{1}{k+1}} f(k+1) > f(k)$ for $k \leq \overline{k}(\eta) - 1$. This follows because f satisfies the IPR property. ■

Proof of Proposition 5. Assume that bank i is choosing the risk-free project in the CFB allocation (K^*, x^*, s^*), with $g_i^* > 0$. As shown, if $\eta < \dfrac{1}{f(1)}$ we have $g_j = 0$ for any $j \in G_i$. Take a new allocation $(\hat{K}, \hat{x}, s^*)$ where the investment strategies are the same but (i) the bank capital given to every $j \in G_i$ is the autarky-equivalent payoff $x_j^A\left(\hat{K}, s^*\right)$, and (ii) the network structure severs all the risky links of bank i. Formally, (i) $\hat{x}_j = x_j^A\left(\hat{K}, s^*\right)$ for any $j \in G_i$, $\hat{x}_i = x_i^* + \sum_{j \in G_i}\left(x_j^* - \hat{x}_j\right)$ and $\hat{x}_b = x_b^*$ for all $b \notin G_i$, $b \neq i$, and (ii) $\hat{K}_i = K_i^* \backslash \{G_i\}$, $\hat{K}_j = K_j^* \backslash \{i\}$, for all $j \in G_i$ and $\hat{K}_b = K_b^*$ for all $b \notin G_i$, $b \neq i$. We show that the new allocation $(\hat{K}, \hat{x}, s^*)$ satisfies the participation constraints for any bank, it is an INE and it yields an higher total expected payoff. Therefore, the initial allocation (K^*, x^*, s^*) cannot be a solution to the planner's problem.

Participation constraints. We consider only bank i, since for any bank $j \in G_i$ the investor participation constraints are satisfied by definition. For depositors in any bank $j \in G_i$, recall that $g_j^* = 0$ since $\eta < \frac{1}{f(1)}$. This means that depositors in any bank $j \in G_i$ obtain $\xi f(k_j^* - 1)R$, which is greater than 1 given that $\xi R \geq 1$. For any other bank, the participation constraints are satisfied since the allocation (K^*, x^*, s^*) satisfies the participation constraints. Depositors in bank i obtain $f\left(k_i^* - g_i\right) R > 1$, given that

$f(k_i^* - g_i) > 1$ and $R > 1$. For investors in bank i, we need to show that

$$x_i^* + \sum_{j \in G_i} \left(x_j^* - \hat{x}_j\right) \geq x_i^A \left(\hat{K}, s^*\right) \text{ for } \eta < \left(\frac{1-\xi}{1+n\rho}\right)^{n-1}.$$

Recall that, given that (K^*, x^*, s^*) satisfies the participation constraint, it has to be that $x_i^* \geq x_i^A (K^*, s^*)$ and $x_j^* \geq x_j^A (K^*, s^*)$ for any $j \in G_i$. Therefore,

$$x_i^* + \sum_{j \in G_i} \left(x_j^* - \hat{x}_j\right) \geq x_i^A (K^*, s^*) + \sum_{j \in G_i} \left(x_j^A (K^*, s^*) - \hat{x}_j\right).$$

Note that, given that (K^*, x^*, s^*) is an INE satisfying the participation constraint, by Proposition 3 any $j \in G_i$ chooses the risky project in autarky. This means that $x_j^A (K^*, s^*) = \frac{e_j}{f\left(k_j^*\right)}$ and $x_j^A \left(\hat{K}, s^*\right) = \frac{e_j}{f\left(k_j^*-1\right)}$ for any $j \in G_i$. Since bank i chooses the risk-free project, we have $x_i^A (K^*, s^*) = \frac{m_i^A}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R}$ and $x_i^A \left(\hat{K}, s^*\right) = \frac{m_i^A}{\eta^{\frac{g_i}{k_i^*-g_i}} f\left(k_i^*-g_i\right) R}$, with $m_i^A = \max\{R\ e_i, \xi R\ e_i + B\}$.

Consider first $x_j^A (K^*, s^*)$ for any $j \in G_i$. Since (K^*, x^*, s^*) is an optimal allocation it has to be a core-periphery structure. Furthermore, $g_j = 0$ for any $j \in G_i$ given that $\eta < \frac{1}{f(1)}$. Then, $k_j^* \leq k_i^*$ for any $j \in G_i$. This is so given that (i) bank i is connected to all other banks choosing the safe project and to all banks in G_i, that is $k_i^* = c^* - 1 + g_i$ (with c^* being the number of banks in the core); (ii) any bank j choosing the gambling project can be connected at most to all the banks choosing the safe project (and no bank choosing the gambling project), that is $k_j^* \leq c^*$. Then, since $g_i \geq 1$, we have $k_j^* \leq k_i^*$ and therefore

$$x_j^A (K^*, s^*) = \frac{e_j}{f\left(k_j^*\right)} \geq \frac{e_j}{f\left(k_i^*\right)} \text{ for any } j \in G_i. \tag{18}$$

Consider now $x_j^A \left(\hat{K}, s^*\right)$ for any $j \in G_i$. Note that

$$x_j^A \left(\hat{K}, s^*\right) = \frac{e_j}{f\left(k_j^* - 1\right)} \leq e_j \text{ for any } j \in G_i,$$

given that $f\left(k_j^* - 1\right) \geq 1$ for any $k_j^* \geq 1$. Then we have

$$x_i^A (K^*, s^*) + \sum_{j \in G_i} \left(x_j^A (K^*, s^*) - \hat{x}_j\right) \geq \frac{m_i^A}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} + \sum_{j \in G_i} \left(\frac{e_j}{f\left(k_i^*\right)} - e_j\right). \tag{19}$$

Note that

$$\frac{m_i^A}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} + \sum_{j \in G_i} \left(\frac{e_j}{f\left(k_i^*\right)} - e_j\right) \geq \frac{m_i^A}{f\left(k_i^* - g_i\right) R} = x_i^A \left(\hat{K}, s^*\right),$$

or

$$\frac{m_i^A}{R}\left[\frac{1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)}-\frac{1}{f\left(k_i^*-g_i\right)}\right] \geq \frac{f\left(k_i^*\right)-1}{f\left(k_i^*\right)} \sum_{j \in G_i} e_j,$$

given that (i) $\frac{m_i^A}{R}\left[\frac{1}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right)}-\frac{1}{f\left(k_i^*-g_i\right)}\right] \geq \frac{B}{R} \times \frac{1-\eta^{\frac{1}{n-1}} \rho}{\eta^{\frac{1}{n-1}} \rho}$, because $m_i^A \geq B$, (ii) $1 \leq f(.) \leq \rho$, (iii) $\frac{f\left(k_i^*\right)-1}{f\left(k_i^*\right)} \sum_{j \in G_i} e_j \leq (n-1) I^*(0,0, \xi, \eta)=\frac{B}{R} \times \frac{n-1}{1-\xi}$, given that by Proposition 3 we know that any $j \in G_i$ chooses the risky project in autarky and $f\left(k_i^*\right)-1 \leq f\left(k_i^*\right)$, and (iv) $\eta^{\frac{1}{n-1}} \rho<\frac{1-\xi}{n-\xi}$ if $\eta^{\frac{1}{n-1}}<\frac{1}{\rho(1+n \rho)}$ because $\frac{1}{\rho(1+n \rho)}<\frac{1-\xi}{n-\xi}$.

The allocation $(\hat{K}, \hat{x}, s^*)$ is an INE. If (K^*, x^*, s^*) is a CFB then it has to be an INE. Consider any player $j \in G_i$. Given that (K^*, x^*, s^*) is an INE it has to be that

$$x_j^*<\frac{B}{(1-\xi) f(k_j^*) R},$$

for any $j \in G_i$, since $s_j^* \in G_i$ and $g_j=0$. From equation (18), $x_j^* \geq \frac{e_j}{f\left(k_j^*\right)}$. The two last inequalities imply that

$$e_j<\frac{B}{(1-\xi) R}. \tag{20}$$

By definition of $\hat{x}$, we have $\hat{x}_j=x_j^A\left(\hat{K}, s^*\right)=\frac{e_j}{f\left(k_j^*-1\right)}$. From (20), $\frac{e_j}{f\left(k_j^*-1\right)}<\frac{B}{(1-\xi) f\left(k_j^*-1\right) R}$, and therefore $\hat{x}_j<I^*\left(k_j^*-1,0, \xi, \eta\right)$. Consider now bank i and recall that $\hat{x}_i=x_i^*+\sum_{j \in G_i}\left(x_j^*-\hat{x}_j\right)$. From equation (19) we know that

$$x_i^*+\sum_{j \in G_i}\left(x_j^*-\hat{x}_j\right) \geq \frac{m_i^A}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R}+\sum_{j \in G_i}\left(\frac{e_j}{f\left(k_i^*\right)}-e_j\right).$$

We need to prove that

$$\frac{m_i^A}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R}+\sum_{j \in G_i}\left(\frac{e_j}{f\left(k_i^*\right)}-e_j\right) \geq I^*(0,0, \xi, \eta)=\frac{B}{(1-\xi) R}. \tag{21}$$

Rearranging terms, equation (21) is equivalent to

$$\frac{m_i^A}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} \geq \frac{B}{(1-\xi) R}+\frac{f\left(k_i^*\right)-1}{f\left(k_i^*\right)} \sum_{j \in G_i} e_j.$$

Note that $m_i^A \geq B$ and $\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R \leq \eta^{\frac{1}{n-1}} \rho R$. Thus,

$$\frac{m_i^A}{\eta^{\frac{g_i}{k_i^*}} f\left(k_i^*\right) R} \geq \frac{B}{\eta^{\frac{1}{n-1}} \rho R} \geq n \frac{B}{(1-\xi) R},$$

the last inequality being true for $\eta^{\frac{1}{n-1}}\rho < \frac{1-\xi}{n}$. By assumption, $\eta^{\frac{1}{n-1}}\rho < \frac{1-\xi}{1+n\rho}$, with $\frac{1-\xi}{1+n\rho} < \frac{1-\xi}{n}$. Finally, we have

$$n\frac{B}{(1-\xi)R} \geq \frac{B}{(1-\xi)R} + \frac{f(k_i^*)-1}{f(k_i^*)}\sum_{j\in G_i} e_j,$$

given that $\frac{f(k_i^*)-1}{f(k_i^*)}\sum_{j\in G_i} e_j < g_i\frac{B}{(n-1)R}$, since $e_j < I^*(0,0,\xi,\eta)$ for any $j \in G_i$, and that $g_i \leq (n-1)$.

Expected total payoff. We need to prove that

$$\begin{aligned}\sum_{b\in N}\left[m_b(\hat{K},\hat{x},s^*)+M_b(\hat{K},s^*)\right]-\sum_{b\in N}\left[m_b(K^*,x^*,s^*)+M_b(K^*,s^*)\right] &= \\ = Rf(k_i^*-g_i)(\hat{x}_i+1)-\eta^{\frac{g_i}{k_i^*}}f(k_i^*)R(x_i^*+1)+ \\ +\xi\sum_{j\in G_i} f(k_j^*-1)R(\hat{x}_j+1)-\xi\sum_{j\in G_i} f(k_j^*)R\left(x_j^*+1\right) > 0,\end{aligned}$$

for $\eta < \left(\frac{1-\xi}{\rho(1+n\rho)}\right)^{n-1}$. Recall that $\hat{x}_i = x_i^* + \sum_{j\in G_i}\left(x_j^*-\hat{x}_j\right)$, where $\hat{x}_j = \frac{e_j}{f\left(k_j^*-1\right)}$. Rearranging terms, we have to prove that

$$\begin{aligned}&\left[f(k_i^*-g_i)-\eta^{\frac{g_i}{k_i^*}}f(k_i^*)\right](x_i^*+1)+\sum_{j\in G_i}\left[f(k_i^*-g_i)-\xi f\left(k_j^*\right)\right]x_j^*- \\ &-\sum_{j\in G_i}\left[f(k_i^*-g_i)-\xi f\left(k_j^*-1\right)\right]\hat{x}_j > \xi\sum_{j\in G_i}\left[f\left(k_j^*\right)-f\left(k_j^*-1\right)\right].\end{aligned}$$

First note that $k_i^* = c^*-1+g_i \geq c^* \geq k_j^*$, where c^* is the number of core banks. Given that $\xi < \eta < \frac{1}{f(1)}$, and that the ratio $\frac{f(k)}{f(k+1)}$ is increasing in k -because f is concave-, we have

$$\sum_{j\in G_i}\left[f(k_i^*-g_i)-\xi f\left(k_j^*\right)\right]x_j^* \geq \sum_{j\in G_i}\left[f\left(k_j^*-1\right)-\xi f\left(k_j^*\right)\right]x_j^* \geq 0.$$

Furthermore, $f(k_i^*-g_i)-\eta^{\frac{g_i}{k_i^*}}f(k_i^*) > \xi\sum_{j\in G_i}\left[f\left(k_j^*\right)-f\left(k_j^*-1\right)\right]$ because $\xi < \eta^{\frac{1}{n-1}} < \frac{1-\xi}{\rho(1+n\rho)}$. So it suffices to show that

$$\left[f(k_i^*-g_i)-\eta^{\frac{g_i}{k_i^*}}f(k_i^*)\right]x_i^* > \sum_{j\in G_i}\left[f(k_i^*-g_i)-\xi f\left(k_j^*-1\right)\right]\hat{x}_j$$

to prove our claim.

We have (i) $[f(k_i^*-g_i)]-\eta^{\frac{g_i}{k_i^*}}f(k_i^*) \geq \left(1-\eta^{\frac{1}{n-1}}\rho\right)$; (ii) $x_i^* \geq \frac{B}{(1-\xi)\eta^{\frac{1}{n-1}}\rho R}$, since x^* satisfies the incentive constraint; (iii) $f(k_i^*-g_i)-\xi f\left(k_j^*-1\right) < \rho$ for any $j \in G_i$; (iv)

$\hat{x}_j = \frac{e_j}{f\left(k_j^*-1\right)} \leq e_j < \frac{B}{(1-\xi)R}$, given that by Proposition 3, bank j chooses the risky project in autarky. Then the following inequality holds

$$\begin{aligned}
[f(k_i^* - g_i) - \eta^{\frac{g_i}{k_i^*}} f(k_i^*)]x_i^* &\geq \left(1 - \eta^{\frac{1}{n-1}}\rho\right) \frac{B}{(1-\xi)\,\eta^{\frac{1}{n-1}}\rho R} > \\
&> (n-1)\rho \frac{B}{(1-\xi)\,R} > \sum_{j \in G_i} \left[f(k_i^* - g_i) - \xi f\left(k_j^* - 1\right)\right] \hat{x}_j,
\end{aligned}$$

where the second inequality follows from the assumption $\eta^{\frac{1}{n-1}} < \frac{1-\xi}{\rho(1+n\rho)}$. ■

Proof of Proposition 6. Assume by contradiction that (K^e, e, s^e) is a DEWT, but there are two banks i and j such that $s_i^e = s_j^e = rf$ with $i \notin K_j^e$, and therefore $j \notin K_i^e$. We prove that $(K^e \cup ij, e, s^e)$ is an INE and that both $m_i(K^e \cup ij, e, s^e) > m_i(K^e, e, s^e)$ and $m_j(K^e \cup ij, e, s^e) > m_j(K^e, e, s^e)$, contradicting the fact that (K^e, e, s^e) is a PSWT, and therefore it cannot be a DEWT.

Note that, since (K^e, e, s^e) is a DEWT, it has to be an INE. This means that $e_i \geq I^*(k_i, g_i, \xi, \eta)$ and $e_j \geq I^*(k_j, g_j, \xi, \eta)$. Furthermore, it has to be $I^*(k_i, g_i, \xi, \eta) \geq I^*(k_i + 1, g_i, \xi, \eta)$ and $I^*(k_j, g_j, \xi, \eta) \geq I^*(k_j + 1, g_j, \xi, \eta)$ given that $f(k)$ and $\eta^{\frac{g}{k}}$ are increasing in k. This implies that $e_i \geq I^*(k_i + 1, g_i, \xi, \eta)$ and $e_j \geq I^*(k_j + 1, g_j, \xi, \eta)$. Therefore, $(K^e \cup ij, e, s^e)$ is also an INE. Finally, note that

$$m_i(K^e \cup ij, e, s^e) = \eta^{\frac{g_i}{k_i+1}} f(k_i + 1)Re_i > \eta^{\frac{g_i}{k_i}} f(k_i)Re_i = m_i(K^e, e, s^e)$$

and

$$m_j(K^e \cup ij, e, s^e) = \eta^{\frac{g_j}{k_j+1}} f(k_j + 1)Re_j > \eta^{\frac{g_j}{k_j}} f(k_j)Re_j = m_j(K^e, e, s^e)$$

since the functions $f(k)$ and $\eta^{\frac{g}{k}}$ are increasing in k. Therefore, (K^e, e, s^e) cannot be a DEWT. ■

Proof of Proposition 7. The proof is based on Lemma 1. Assume that $k_i^e < \underline{k}(\eta)$. This means that $k_i^e \leq \underline{k}(\eta) - 1$, and by definition of $\underline{k}(\eta)$ and the fact that the ratio $\left(\frac{f(k)}{f(k+1)}\right)^{k+1}$ is increasing in k, we have $\eta \geq \left(\frac{f\left(k_i^e\right)}{f(k_i^e+1)}\right)^{k_i^e+1}$. Hence, bank i will gain if connecting to any other bank $j \notin K_i^e$, no matter j's strategy. Therefore, K^e can be a pairwise stable structure only if there is no other bank j willing to connect to bank i. If bank i is choosing the risk-free project, any bank not yet connected to bank i would be better off by connecting to it. Therefore, if $s_i^e = rf$ then $K_i^e = N \backslash \{i\}$. Otherwise, if $s_i^e = r$ it has to be that no other bank j not connected to bank i could be better off from connecting to bank i. This could only happen if $k_j^e \geq \underline{k}(\eta)$ for any bank $j \notin K_i^e$, given that any bank j would like to connect with bank i if $k_j^e < \underline{k}(\eta)$.

Assume that $k_i^e > \overline{k}(\eta)$ for some bank i. This means that $k_i^e \geq \overline{k}(\eta) + 1$, and by definition of $\overline{k}(\eta)$ and the fact that the ratio $\left(\frac{f(k)}{f(k+1)}\right)^{k+1}$ is increasing in k, we know that $\eta < \left(\frac{f\left(k_i^e-1\right)}{f(k_i^e)}\right)^{k_i^e}$. This means that bank i is better off if it unilaterally disconnects any of its links with banks investing in the risky project. So the allocation (K^e, e, s^e) can only be PSWT if $G_i = \emptyset$. ■

Proof of Corollary 1. The proof follows the same steps of Proposition 4 and therefore omitted. ■

Proof of Proposition 8. Given the definition of Q we have

$$Q = \{i \text{ such that } I^*(k_i, \widehat{g}_i, \xi, \eta) \leq x_i < I^*(k_i, \widehat{g}_i + q_i, \xi, \eta)\}$$

where $\widehat{g}_i$ is the number of banks in $K_i \backslash Q$ that choose the gambling project, and q_i is the minimum number of banks in Q connected to bank i that, by choosing the gambling project, would make bank i switch from the risk-free project to the gambling project. Note that $1 \leq q_i \leq |K_i \cap Q|$. Namely, q_i has to be at least one, otherwise bank i will always choose the gambling project and would not belong to $Q(s, s')$. Furthermore, q_i has to be at most $Q \cup K_i$, i.e., the number of banks in Q that are connected to bank i. Otherwise, bank i will always choose the risk-free project and would not belong to $Q(s, s')$. Assume that the sequential-move investment game calls bank i to make the investment decision (given the choices of banks Q.) If history in the game is such that q_i banks in Q connected to bank i choose the gambling project, bank i chooses the gambling project as well. If history in the game is such that there are less than q_i banks in Q connected to bank i choosing the gambling project, bank i chooses the safe project. But if history is such that if by choosing the safe project there are less than q_i, say, c_1 banks choosing the gambling project in Q and if by choosing the gambling project there are more than q_i, say, c_2 banks choosing the gambling project, bank i chooses the gambling asset only if

$$\eta^{\frac{\widehat{g}_i+c_1}{k_i}} f(k_i) R x_i < \xi \eta^{\frac{\widehat{g}_i+c_2}{k_i}} f(k_i) R x_i + B, \tag{22}$$

where $c_1 < q_i \leq c_2$. The previous inequality implies that

$$x_i < \frac{B}{\left(1 - \xi \eta^{\frac{c_2-c_1}{k_i}}\right) \eta^{\frac{\widehat{g}_i+c_1}{k_i}} f(k_i) R}, \tag{23}$$

a contradiction with the fact that

$$x_i \geq \frac{B}{(1-\xi)\,\eta^{\frac{\widehat{g}_i + c_1}{k_i}} f(k_i) R} = I^*(k_i, \widehat{g}_i, \xi, \eta),$$

given that c_1 banks in $|Q \cap K_i|$ are not enough to make bank i switching from the safe project to the gambling one.

This implies that every time a bank is critical in the rule of order to decide among different continuation paths, it will choose the safe path. Then we can conclude that for any rule of order, the sequential-move investment game selects the INE profiles where the highest number of banks choose the risk-free project. ■

Proof of Proposition 9. Recall that $x_i = e_i + t_i \geq 0$. Assume bank i is considering to make a transfer t_i. Since $t_i \geq 0$ it has to be that $t_i \leq x_i$. Denote by j the bank that receives a transfer t_j to be induced to choose the risk-free project. Therefore $x_j + t_j \geq I^*(k_j, g_j, \xi, \eta)$. Given that $t_i \geq t_j$ it has to be that $x_i + x_j \geq I^*(k_j, g_j, \xi, \eta) > \frac{B}{(1-\xi)\rho R}$. Clearly, $x_i + x_j \leq E = \sum_{i \in N} e_i$. Thus, for $E < \frac{B}{(1-\xi)\rho R}$ there is no transfer t_i that can induce bank j to invest in the risk-free project, no matter the values of x_i or x_j. This happens when $\xi > 1 - \frac{B}{\rho R E}$. We have $\overline{\xi} = max\{1 - \frac{B}{\rho R E}, 0\}$. ■

References

[1] Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2015), 'Systemic Risk and Stability in Financial Networks', *American Economic Review*, **105** (2): 564-608.

[2] Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2014), 'Systemic Risk in Endogenous Financial Networks', working paper, MIT and Columbia Business School.

[3] Afonso, G., A. Kovner and A. Schoar (2011), 'Stressed, not Frozen: The Federal Funds Market in the Financial Crisis', *Journal of Finance,* **66** (4): 1109-1139.

[4] Allen, F. and A. Babus (2009), 'Networks in Finance' in P. Kleindorfer, J. Wind and R. E. Gunther (eds), '*The Network Challenge*', Wharton School Publishing.

[5] Allen, F., A. Babus, and E. Carletti (2012), 'Asset Commonality, Debt Maturity and Systemic Risk' *Journal of Financial Economics,* **104** (3): 519-534.

[6] Allen, F. and D. Gale (2000), 'Financial Contagion', *Journal of Political Economy*, **108**: 1-33.

[7] Babus, A. (2015), 'The Formation of Financial Networks', *RAND Journal of Economics*, forthcoming.

[8] Bank of England (2009), *Financial Stability Report*, Issue No. 25.

[9] Bech, M. L. and E. Atalay (2010), 'The Topology of the Federal Funds Market', *Physica A*, **389** (22): 5223-5246.

[10] Bloch, F., G. Genicot, and D. Ray (2008), 'Informal Insurance in Social Networks', *Journal of Economic Theory*, **143**: 36-58.

[11] Bloch, F. and M. O. Jackson (2007), 'The Formation of Networks with Transfers among Players', *Journal of Economic Theory*, **133**: 83-110.

[12] Boss, M., H. Elsinger, M. Summer and S. Thurner (2004), 'Network Topology of the Interbank Market', *Quantitative Finance*, **4**: 1-8.

[13] Bramoullé, Y. and R. Kranton (2007), 'Risk Sharing Networks', *Journal of Economic Behavior and Organization* , **64**: 275 - 294.

[14] Brusco, S. and F. Castiglionesi (2007), 'Liquidity Coinsurance, Moral Hazard, and Financial Contagion', *Journal of Finance*, **62**: 2275 - 2302.

[15] Caballero, R. J. and A. Simsek (2013), 'Fire Sales in a Model of Complexity', *Journal of Finance*, **68** (6): 2549-2587.

[16] Cabrales, A., P. Gottardi and F. Vega-Redondo (2014), 'Risk-sharing and Contagion in Networks', working paper, UCL, EUI and Bocconi.

[17] Castiglionesi, F. and M. Eboli (2015), 'Liquidity Flows in Interbank Networks', working paper, Tilburg University.

[18] Castiglionesi, F., F. Feriozzi and G. Lorenzoni (2012), 'Financial Integration and Liquidity Crises', working paper, Tilburg and Northwestern University.

[19] Castiglionesi, F. and W. Wagner (2013), 'On the Efficiency of Bilateral Interbank Insurance', *Journal of Financial Intermediation*, **22** (2): 177-200.

[20] Cocco, J. F., F. J. Gomes and N. C. Martins (2009), 'Lending Relationship in the Interbank Market', *Journal of Financial Intermediation*, **18** (1): 24-48.

[21] Craig, B. and G. von Peter (2014), 'Interbank Tiering and Money Center Banks', *Journal of Financial Intermediation*, **23** (3): 322–347.

[22] Elliott, M., B. Golub, and M. O. Jackson (2014), 'Financial Networks and Contagion', *American Economic Review*, **104** (10): 3115-3153.

[23] Embree, L. and T. Roberts (2009), 'Network Analysis and Canada's Large Value Transfer System', Bank of Canada, *Discussion Paper* No. 2009-13.

[24] Farboodi, M. (2015), 'Intermediation and Voluntary Exposure to Counterparty Risk', working paper, Princeton University.

[25] Freixas, X., B. Parigi and J. C. Rochet (2000), 'Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank', *Journal of Money, Credit and Banking*, **32**: 611-638.

[26] Furfine, C. (2001), 'Banks as Monitor of Other Banks: Evidence from the Overnight Federal Funds Market', *Journal of Business*, **74** (1): 33-57.

[27] Furfine, C. (2002), 'Interbank Markets in a Crisis', *European Finance Review*, **46** (4-5): 809-820.

[28] Heider, F., M. Hoerova, and C. Holthausen (2015), 'Liquidity Hoarding and Interbank Market Spreads: the Role of Counterparty Risk', *Journal of Financial Economics*, **118**: 336-354.

[29] Inaoka, H. , T. Ninomiya, T. Shimizu, H. Takayasu, and K. Taniguchi (2004), 'Fractal Network Derived from Banking Transaction – An Analysis of Network Structures Formed by Financial Institutions', Bank of Japan, *Working Paper* No. 04-E-04.

[30] Jackson, M. O. and A. Wolinsky (1996), 'A Strategic Model of Social and Economic Networks', *Journal of Economic Theory*, **71**: 44-74.

[31] King, T. B. (2008), 'Discipline and Liquidity in the Interbank Market', *Journal of Money, Credit, and Banking*, **40** (2-3): 295-317.

[32] Leitner, Y. (2005), 'Financial Networks: Contagion, Commitment, and Private Sector Bailouts', *Journal of Finance*, **6**: 2925-2953.

[33] Malherbe, F. (2014), 'Self-Fulfilling Liquidity Dry-Ups', *Journal of Finance*, **69** (2): 947-970.

[34] Morrison, A. and L. White (2005), 'Crises and Capital Requirements in Banking', *American Economic Review*, **95** (5): 1548-1572.

[35] Soramäki, K., M. L. Bech, J. Arnolda, R. J. Glass, and W. E. Beyeler (2007), 'The Topology of Interbank Payment Flows', *Physica A*, **379** (1): 317-333.

[36] Zawadowski, A. (2013), 'Entangled Financial Systems', *Review of Financial Studies,* **26** (5): 1291-1323.

Cahiers du GREThA
Working papers of GREThA

GREThA UMR CNRS 5113

Université de Bordeaux

Avenue Léon Duguit
33608 PESSAC - FRANCE
Tel : +33 (0)5.56.84.25.75
Fax : +33 (0)5.56.84.86.47

http://gretha.u-bordeaux.fr/

Cahiers du GREThA (derniers numéros – last issues)

2015-32 : ODUOR Jacob, MABROUK Fatma, SHIMELES Abebe *: Remittances and Youth Labor Market Participation in Africa*

2015-33 : CHOUNET François, MAVEYRAUD Samuel *: Corrélation des taux de change et régime d'étalon-or durant la Première Guerre mondiale*

2015-34 : CAPARROS Alejandro, PEREAU Jean-Christophe *: Multilateral versus sequential negotiations over climate change*

2015-35 : BECUWE Stéphane, BLANCHETON Bertrand, ONFROY Karine *: Données du commerce extérieur de la France entre 1836 et 1938 - Base Montesquieu*

2016-01 : CHENAF-NICET Dalila, ROUGIER Eric : *The effect of macroeconomic instability on FDI flows: A gravity estimation of the impact of regional integration in the case of Euro-Mediterranean agreements*

2016-02 : COMBARNOUS François, DEGUILHEM Thibaud *: Urban labor market revisited: Why quality of employment matters in Bogota*

2016-03 : DEGUILHEM Thibaud, FRONTENAUD Adrien *: Régimes de qualité de l'emploi et diversité des pays émergents*

2016-04 : BENABDEJLIL Nadia, LUNG Yannick, PIVETEAU Alain *: L'émergence d'un pôle automobile à Tanger (Maroc)*

2016-05 : MAICHANOU Ahamadou, *La micro-assurance agricole indicielle: raisons et conditions d'exercice au Niger*

2016-06 : MAICHANOU Ahamadou*, Déterminants d'emprunt et de risque de crédit des ménages ruraux au Niger*

*2016-07 :*BERTHE Alexandre*, Mesurer les inégalités d'accès à l'eau et à l'assainissement dans le Nord et le Nordeste du Brésil : quels enseignements en matière de justice sociale ?*

2016-08 : LORENZON Emmanuel, *Collusion with a Greedy Center in Position Auctions*

2016-09 : DELFAUD Pierre, Retour sur une expérience unique d'interventionnisme en économie de marché: La Politique agricole commune européenne (1955-2015)

2016-10 : MUTASCU Mihai, PEREAU Jean-Christophe, URSU Eugen : A Wavelet Analysis of the Environmental Kuznets Curve in France

2016-11 : LISSONI Francesco : Migration and Innovation Diffusion: An Eclectic Survey

2016-12 : BONIN Hubert : Les paradoxes de l'économie maritime française au début du XXIe siècle

La coordination scientifique des Cahiers du GREThA est assurée par Emmanuel PETIT.
La mise en page est assurée par Julie VISSAGUET