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How to negotiate with Coase?

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Comment négocier avec Coase ?

Résumé

Cet article analyse la portée du théorème de Coase dans le cadre d'une négociation stratégique entre 3 agents. Nous considérons plusieurs protocoles de négociations bilatérales entre 2 firmes qui polluent un ménage. Nos résultats montrent que lorsque les droits de propriété sont donnés au ménage, ce dernier préfère négocier simultanément et séparément avec les deux firmes. En revanche quand les firmes détiennent les droits, elles préfèrent négocier de manière séquentielle avec le ménage. Cette dernière configuration se révèle non optimale pour l'économie, ce qui invalide le théorème de Coase.

Mots-clés : Théorème de Coase, délégation, négociation non coopérative, droits de propriété, négociation séquentielle, négociation simultanée

How to negotiate with Coase?

Abstract

This article analyses the bargaining side of the Coase theorem in a 3-player strategic framework. We consider several bilateral bargaining protocols between two firms polluting one household (the victim). Our results show that when property rights are given to the victim, he prefers to bargain separetely and simultaneously with the two firms. However when property rights are given to the firms, they prefer to bargain sequentially with the household. This last configuration yields a non optimal outcome and invalidates the bargaining Coase theorem.

Keywords: Coase theorem, delegation, non cooperative bargaining, property rights, sequential negotiation, simultaneous negotiation

JEL : C78, K41

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1 Introduction

The Coase theorem has motivated an extensive literature, partly due to the numerous variants of the Coase theorem one can find (Veljanovski, 1982; Medena, 1994; Usher, 1998). This paper focuses[1] on the *bargaining Coase theorem* which states: *If rights are fully specified and transaction costs are zero, voluntary bargaining between agents will lead to an efficient outcome, regardless of how rights are initially assigned.* The traditional analysis of Coasian negotiations has been to determine the conditions under which property rights trading can fix externalities. Calabresi (1968), Regan (1972), Cooter (1982) and Veljanovski (1982) argue that the bargaining Coase theorem actually reflects the optimistic belief of Coase himself, namely that bargaining between rational agents is sufficiently cooperative to exhaust all mutual advantages.[2]

Consequently, analyzing the validity of this theorem becomes equivalent to analyzing the efficiency of the bargaining process itself. Since most of the literature on the Coase bargaining theorem focuses on bilateral externalities, the bargaining process considered is also bilateral. As pointed out by Farrell (1987), this question of the efficiency of the bargaining is only relevant in non cooperative bargaining models since the Nash cooperative solution is by definition based on the axiom of efficiency. In complete information, bargaining leads to an efficient outcome, no matter how property rights are allocated, if bargaining takes the form of a take-it-or-leave-it offer (e.g. an ultimatum game) as in Mas-Colell *et al.* (1995) or the form of the Rubinstein (1982) alternating-offers model (Farrell, 1987).[3] However it is well known that the Rubinstein model is identical to the Nash solution when the discount rates of the bargainers tend towards one (Binmore *et al*, 1986). In incomplete information, the inefficiency of bargaining invalidates the bargaining Coase theorem (Samuelson, 1985; Matsuo, 1989; McKelvey and Page, 2002). Inefficiency can take the form of delay in bargaining model (Kenan and Wilson, 1993). The issue is then to determine the conditions under which the Coase

[1] We do not mention the competitive market theorem (Arrow, 1969) and the so-called invariance Coase theorem (Hurwicz, 1995).

[2] Calabresi (1968, p.68) wrote: "If people are rational, bargains are costless, and there are no legal impediments to bargains, transactions will *ex hypothesis* occur to the point [...] of optimal resource allocation. We can, therefore, state as an axiom the proposition that all externalities can be internalized and all misallocations, even those created by legal structures, can be remedied by the market, except to the extent that transactions cost money or the structure itself creates some impediments to bargaining."

[3] In a "modified" Rubinstein model, Lee and Sabourian (2007) show how a Coasian failure can occur even under complete information when transaction and/or complexity costs are introduced.

conjecture[4] is still verified (Muthoo, 1999).

When restricting our analysis to complete information, the bargaining Coase theorem holds true whenever one considers face-to-face externalities. Things become more interesting when externalities involving more than two individuals are considered (Aivazian and Callen, 1981, 1987; Dixit and Olson, 2000). Based on the cooperative concept of core, Aivazian and Callen (1981) consider an example with two factories polluting a neighborhood laundry. If the factories are not liable for environmental damages, these authors show that the core of the game is empty, assuming the two factories obtain economies of scale when they merge. Aivazian and al. (1987) uses a similar example to show that the bargaining set, e.g., the set of imputations and coalition structures which are individually rational and coalitionally stable, either is not reduced to the grand coalition or does not contain it. However their results (emptiness of the core and instability of the grand coalition) only hold when the two factories are more efficient when they merge (due to increasing returns to scale). These two examples show that the bargaining Coase theorem is also questionable from the perspective of cooperative game theory when we depart from the pure bargaining problem. This problem occurs when only the grand coalition formed by the two factories and the laundry can create a positive surplus with respect to what each player can achieve if he does not cooperate with anyone. It implies that the agreement of all parties involved is necessary to the creation of a surplus. Allowing a sub-coalition to get power to create a surplus thus adds strategic interaction.

Dixit and Olson (2000) consider the case of one factory polluting many agents which is assumed not to be liable for the environmental damages. The efficient outcome requires that the factory ceases its activity. The authors assumed that in a first step, the agents simultaneously decide whether to participate (or not) to the bargaining and then in the second step, the group of volunteers bargains with the factory. This setting creates incentives for each agent to free-ride. On the one hand, if an agent participates, he will have to compensate for the loss of the factory, in case the outcome of the bargaining is to cease its activity. On the other hand, most of the time, an agent's participation will not modify the outcome of the bargaining, which mostly depends on the group's overall propensity to pay. These examples show that relaxing face-to-face externalities emphasizes new strategic behavior in the bargaining process. As suggested by Veljanoski (1982, p. 58): *"Coase did not rule-out strategic behavior but assumed that when it did occur*

[4]The Coase conjecture states that in the limit all potential gains from trade are realized without any costly delay and the profit of the proposer uniformed player is arbitrarily close to zero (Muthoo, p.280, 1999).

it would be transitory and have no effects on the long run efficient allocation of resources." But to our knowledge, no analysis of the efficiency of the bargaining process with more than two players has been studied even by Coase himself. This paper aims at tackle this issue.

Adapting Aivazian and Callen (1981, 1987), we consider a situation with two firms polluting a household. We also exclude the case of the pure bargaining problem in which the agreement of the three players involved is needed to create the surplus. However, our analysis differs with respect to the game solution concept under consideration. Instead of using cooperative game theory, we assume that bargaining always takes the form of a sequence of strategic bilateral bargaining. We then assume that face-to-face bargaining is efficient, that players have equal bargaining power and that the bargaining game stops as soon as one agent has met every other agent once in a bilateral bargaining. The two first assumptions are taken into consideration by using the Rubinstein Bargaining Solution to predict the outcome of any bilateral negotiation. The latter assumption is considered through the use of the following types of game trees. The household can bargain with the two firms simultaneously or the household can bargain with one firm first, and then with the other. The two firms can also decide to merge before bargaining with the household and the two firms can bargain together and delegate one of them to bargain with the household. However, contrary to Aivazian and Callen (1981, 1987), we do not assume merger efficiency gains. As in Dixit and Olson (2000), we pay attention to the impact of the game tree on the efficiency of the outcome of the game and the sharing of the surplus between players. Our results show that the bargaining agenda matters and that agents may disagree with respect to the choice of the game tree. This proves that, even if agents are rational and are able to bargain efficiently, agenda conflicts and inefficiencies may result.

The paper is organized as follows: In Section 2 the model is presented. Sections 3 and 4 solves the equilibrium transfer and the expected payoffs that result from each of these negotiations according to the liability system. In Section 3 the household have the property rights and in Section 4 the firms have the property rights. Section 5 considers which one of the procedures each player would prefer. Section 6 concludes.

2 The model

We consider one household - denoted 0 - suffering from the pollutant emissions made by two identical firms - denoted 1 and 2 -. The benefit functions of the firms are assumed increasing and concave in their own emissions, e_1

and e_2, while the damage function of the household is assumed increasing and convex with respect to the aggregate emission, $e_1 + e_2$. We note $D(e_1 + e_2)$ the damage function and $B(e_i)$ $i = 1, 2$ the benefit function. We set $D' > 0$, $D'' > 0$, $B' > 0$ and $B'' < 0$.

We assume that a mechanism of utility transfers occurs between the firms and the household to offset the impact of the environmental damage according to the liability rules in place in the economy. These liability rules determine the initial status quo point for the firms and the household. When firms are responsible for the damage they create, we assume that without an agreement with the household, their emissions are nil, $e_i = 0$. However when firms are not liable for the damage they create, their emissions are solution of $\overline{e_i} = \arg\max_{e_i} B(e_i)$.

The objective functions of the household and the firms are:

$$U_0 = t_{01} + t_{02} - D(e_1 + e_2) \quad (1)$$

$$U_1 = B(e_1) - t_{01} + t_{12} \quad (2)$$

$$U_2 = B(e_2) - t_{02} - t_{12} \quad (3)$$

The terms t_{ij} represent utility transfers between player i and player j with $i = 0, 1, 2$ and $j \neq i$.

Definition 1. *Optimality.*

Emissions (e_1^o, e_2^o) *are said optimal if they maximizes the social welfare* $B(e_1) + B(e_2) - D(e_1 + e_2)$. *Optimal emissions satisfy*

$$B'(e_1^o) = B'(e_2^o) = D'(e_1^o + e_2^o).$$

Definition 2. *Efficiency.*

Emissions (e_1, e_2) *are said efficient if they maximize the joint benefit* $B(e_1) + B(e_2)$ *subject to* $e_1 + e_2 \leq e_{12}$ *where* e_{12} *is any given maximum emission. Efficient emissions satisfy*

$$B'(e_1) = B'(e_2).$$

As in Aivazian and Callen (1981) we assume that firms can decide to merge. However, in order to isolate the effect of the bargaining process from the other efficiency effects which characterize a merger, the payoff of the merger firm is defined such that

$$B(e_{12}) = \max \{B(e_1) + B(e_2); e_1 + e_2 \leq e_{12}\} \quad (4)$$

Emissions and transfers are obtained by considering bilateral bargaining protocols. Figure 1 shows the protocols that are considered. The *simultaneous bargaining* (case a)) refers to a protocol in which the household simultaneously and separately bargains with each firm to determine a pair (e_1, t_{01})

and (e_2, t_{02}). The *sequential bargaining* (case a)) differs from the previous protocol by assuming that the household bargains firstly with firm 1 a pair (e_1, t_{01}) and secondly with firm 2 a pair (e_2, t_{02}).[5] Since no transfer occurs between the firms, we set $t_{12} = 0$. The *merger bargaining* (case b)) refers to a negotiation between the household and the two firms which has formed a merger -denoted 12- over a pair $(e_{12}, t_{(12)0})$. Lastly the *delegation bargaining* (case c)) consists in an internal negotiation between the two firms followed by an external negotiation between one firm and the household. It corresponds to a protocol in which firm 2 bargains with firm 1 in a first round a pair (e_2, t_{12}) and delegates firm 1 to bargain with the household a pair (e_1, t_{01}) in a second round.[6] In that case no transfer occurs between firm 2 and the household, $t_{02} = 0$.

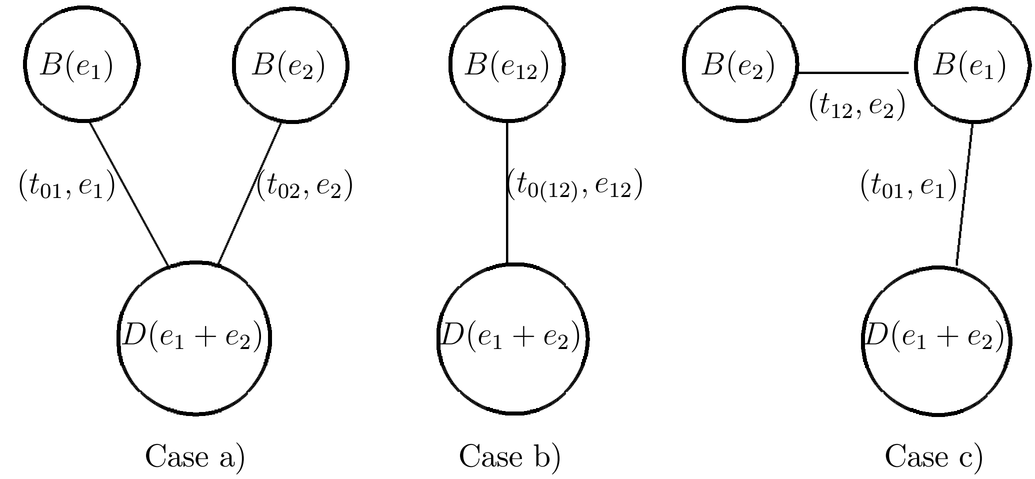

Figure 1: Bargaining protocols

In all cases, we use the strategic Rubinstein Bargaining Solution (RBS hereafter). Appendix A provides a complete derivation for a two-round bargaining. To remove the first mover advantage of the Rubinstein model and ensure that players have equal bargaining power, we assume that the discount rate of all players tends towards one. These bilateral protocols are

[5] Examples of simultaneous and sequential bilateral negotiation in a multilateral framework can be found in Clark and Pereau (2008, 2009).

[6] This delegation procedure differs from the literature in which one principal appoints an intermediary to bargain on his behalf with another principal (Bester and Sakovics, 2001; Segendorff, 1998). Here, we assume a negotiation between a monolithic party (the household) and pluralistic parties (the firms). The intermediary is assumed to be one of the firms and we allow the possibility of making internal compensation on one side of the table (Raiffa, 2002).

compared to a *Coasian benchmark* corresponding to the 3-players Nash Bargaining solution (NBS hereafter). This multilateral cooperative bargaining refers to a pure bargaining game in which each player gets a veto right.

3 Firms are liable for environmental damage

In this section, we assume that the legal system enforces a strict liability for damages. Hence, unless another contract is signed between the household and the firms, the household can obtain a compensation by bringing a suit to a court. The status quo (SQ hereafter) refers to a non polluted economy: $\overline{e_i} = 0$, $i = 1, 2$.

3.1 The Coasian benchmark

Using (1)-(3), the agreement payoffs of the household and firms are $U_0 = t_{01} + t_{02} - D(e_1 + e_2)$, $U_1 = B(e_1) - t_{01}$ and $U_2 = B(e_2) - t_{02}$. The disagreement payoffs are nil. The 3-players NBS is obtained by maximizing the product of the net rent for each player

$$\max_{(t_{01},e_1),(t_{02},e_2)} U_0 \times U_1 \times U_2.$$

The NBS implies a equal payoff for each player, corresponding to a equal share of the total welfare

$$U_0 = U_1 = U_2 = \frac{1}{3}\left(B(e_1) + B(e_2) - D(e_1 + e_2)\right) \tag{5}$$

Welfare in (5) is positive when the benefits derived from the emissions exceed the total damage ($B(e_1) + B(e_2) > D(e_1 + e_2)$). From (6) emissions are optimal (according to Definition 1) but not efficient (according to Definition 2):

$$B'(e_1) = B'(e_2) = D'(e_1 + e_2) \tag{6}$$

Transfers between the household and the firms are given by:

$$t_{01} = \frac{2}{3}B(e_1) - \frac{1}{3}B(e_2) + \frac{1}{3}D(e_1 + e_2) > 0 \tag{7}$$

$$t_{02} = -\frac{1}{3}B(e_1) + \frac{2}{3}B(e_2) + \frac{1}{3}D(e_1 + e_2) > 0 \tag{8}$$

Transfers given by (7) and (8) are positive. When firms are responsible for their pollutant emissions, the NBS states that each of them has to pay a positive transfer t_{0i} to the household to get the right to pollute e_i.

3.2 The household opens first

We consider the cases where the household can initiate simultaneous or sequential negotiation.

In the *simultaneous protocol,* the household bargains at the same time and separately with firms 1 and 2 under the belief that a failure of an agreement with one firm does not break down collapse the agreement with the other firm.[7] For instance, as long as the household takes for granted that its agreement with firm 2 is secured , his higher SQ gives him more bargaining power *vis-à-vis* firm 1. To get the net surplus of each player, we have to consider what happens in the case of an agreement and in the case of a disagreement. When bargaining with firm 1, the household gets the utility $U_0 = t_{01} + t_{02} - D(e_1 + e_2)$ in the case of an agreement and $\overline{U_0} = t_{02} - D(e_2)$ in the case of a failure of the negotiation. The payoffs of firm 1 are respectively $U_1 = B(e_1) - t_{01}$ and $\overline{U_1} = 0$. It gives the net surplus $U_0 - \overline{U_0} = t_{01} - D(e_1 + e_2) + D(e_2)$ and $U_1 - \overline{U_1} = B(e_1) - t_{01}$. Since the program is symmetric with firm 2, the RBS gives the following payoffs:

$$U_0 = \frac{1}{2}(B(e_1) - D(e_1)) + \frac{1}{2}(B(e_2) - D(e_2)) \tag{9}$$

$$U_i = \frac{1}{2}B(e_i) - \frac{1}{2}(D(e_i + e_j) - D(e_j)) \text{ for } i = 1, 2, \ i \neq j \tag{10}$$

where emissions are solution of

$$B_1'(e_1) = B_2'(e_2) = D'(e_1 + e_2) \tag{11}$$

and transfers are given by for $i = 1, 2$, $i \neq j$

$$t_{0i} = \frac{1}{2}B(e_i) + \frac{1}{2}(D(e_i + e_j) - D(e_j)) > 0 \tag{12}$$

The household utility given by (9) is positive when the benefit associated to individual emissions exceeds the corresponding individual damage for each firm ($B(e_i) > D(e_i)$ for $i = 1, 2$). However the rent of firm i given in (10) is positive when its benefit from polluting is higher than its marginal contribution to the damage ($B(e_i) > (D(e_i + e_j) - D(e_j))$). We can show that this protocol favors the household $U_0 > U_i$ since $B(e_i) > D(e_i)$ and $D(e_1 + e_2) > D(e_1) + D(e_2)$. Equation (11) show that emissions satisfy Definition 1 (optimality) but not Definition 2 (efficiency). Since the damage

[7] In each bilateral negotiation, we assume that parties has passive beliefs (see McAfee and Schwartz (1994) for a definition). Here it implies that after receiving an unexpected offer from the household, one firm assumes that the household keeps its equilibrium offer in its negociation with the other firm.

function is increasing, it turns out that when firms are responsible for their pollutant emissions, they have to pay a positive transfer t_{0i} given by (12) to the household to get the right to pollute e_i.

The household can decide to bargain sequentially with firm 1 over a pair (e_1, t_{01}) in a first round and with firm 2 over a pair (e_2, t_{02}) in a second round. By backward induction, we begin with round 2. This round remains the same as the simultaneous case. In particular, firm 2 takes as given the outcome obtained in the first round and firm 2 assumes that a failure of its bargain will not impact the first-round outcome between the household and firm 1. The first-round agreement is non-renegotiable. Agreement payoffs are given by $U_0 = t_{01} + t_{02} - D(e_1 + e_2)$ and $U_2 = B(e_2) - t_{02}$ while disagreement payoffs are $\overline{U_0} = t_{01} - D(e_1)$ and $\overline{U_2} = 0$. Then in round 1, the household and firm 1 bargain taking into account the impact of their decision on the second round outcome. This initial round also differs from the simultaneous case because by definition a failure will break down all the bargain process. Hence, agreement payoffs are equal to $U_0 = t_{01} + t_{02} - D(e_1 + e_2)$ for the household and $U_1 = B(e_1) - t_{01}$ for firm 1. Disagreement payoffs are given by $\overline{U}_0 = \overline{U_1} = 0$. Equations (13) and (14) gives the equilibrium payoffs

$$U_0 = U_1 = \frac{1}{2}\left(B(e_1) - D\left(e_1\right)\right) + \frac{1}{2}U_2 \tag{13}$$

$$U_2 = \frac{1}{2}B(e_2) - \frac{1}{2}\left(D(e_1 + e_2) - D\left(e_1\right)\right) \tag{14}$$

where emissions are solution of

$$2B'(e_1) - D'(e_1) = B'(e_2) = D'(e_1 + e_2) \tag{15}$$

Emissions do not satisfy the optimality and efficiency conditions. Moreover, (15) implies that the firm in position of leader emits more than the follower[8], that is $e_1 > e_2$. Transfers are such that

$$t_{01} = \frac{1}{4}\left(2B(e_1) - B(e_2)\right) + \frac{1}{4}D(e_1 + e_2) + \frac{1}{4}D\left(e_1\right) > 0$$

$$t_{02} = \frac{1}{2}B(e_2) + \frac{1}{2}\left(D(e_1 + e_2) - D\left(e_1\right)\right) > 0.$$

When firms are liable for their pollutant emissions, they have to pay a positive transfer t_{0i} to the household to get the right to pollute e_i. This

[8] (15) can be rewritten as (a) $B'(e_1) + [B'(e_1) - B'(e_2)] = D'(e_1)$ and (b) $B'(e_2) = D'(e_1 + e_2)$. By contradiction, assume $e_1 \leq e_2$. Hence, $B'(e_1) \geq B'(e_2)$ since B' is decreasing. From (a), it turns out that $B'(e_1) \leq D'(e_1)$ since the term in brackets is positive or null. As $D'(e_1) < D'(e_1 + e_2)$ since $e_2 > 0$ and D' is increasing, we obtain $B'(e_1) < D'(e_1 + e_2)$. Using (b), we have $B'(e_1) < B'(e_2)$, implying $e_1 > e_2$. Hence it contradicts our assumption and thus $e_1 > e_2$.

protocol shows that players (the household and firm 1) involved in round 1 get the same net surplus. Here, firm 2 obtains the same share of the aggregate surplus as in the simultaneous bargaining. However, as the negotiated outcome is not optimal (e.g. e_1 and e_2 are different here), its payoff is smaller.

3.3 Firms open first

We consider the cases where firms can decide to form a merger or to reach an internal agreement and send one of the firm for an external bargain with the household.

The *merger bargaining* gives a negotiation between the merger firm - denoted 12- and the household. The payoff of the merger firm is denoted by $B(e_{12})$. In the case of an agreement, payoffs are respectively for the household and the merger firm $U_0 = -D(e_{12}) + t_{0(12)}$ and $U_{12} = B(e_{12}) - t_{0(12)}$. It also gives the net payoffs since $\overline{U_0} = \overline{U_{12}} = 0$.

This protocol yields the same equilibrium payoff

$$U_0 = U_{12} = \frac{1}{2}\left(B(e_{12}) - D(e_{12})\right)$$

with

$$B'(e_{12}) = D'(e_{12}) \tag{16}$$

and

$$t_{0(12)} = \frac{1}{2}B(e_{12}) + \frac{1}{2}D(e_{12}) > 0.$$

By definition of the merger, emissions in (16) satisfy Definition 2 of efficiency. The merger firm pays a positive transfer $t_{0(12)}$ to the household to get the right to pollute e_{12}.

The *delegation bargaining* is a sequential protocol combining an internal negotiation between the firms and an external negotiation between one firm and the household. We consider an external negotiation (in round 2) between firm 1 and the household over the pair (e_1, t_{01}), taking as given the outcome of the internal negotiation over (e_2, t_{12}). In the case of an agreement, the household gets $U_0 = t_{01} - D(e_1 + e_2)$ and firm 1 gets $U_1 = B(e_1) - t_{01} + t_{12}$. In the case of a disagreement, the internal agreement does not hold since the household can break down the internal agreement if he decides to bring the case to court. It implies nil payoffs $\overline{U_0} = \overline{U_1} = 0$. In round 1, agreement payoffs for firms 1 and 2 are $U_1 = B(e_1) - t_{01} + t_{12}$ and $U_2 = B(e_2) - t_{12}$. In the case of a disagreement, we have $\overline{U_1} = \overline{U_2} = 0$. The RBS gives the equilibrium payoffs

$$\begin{aligned} U_0 &= U_1 = \frac{1}{4}\left(B\left(e_1\right)+B\left(e_2\right)-D\left(e_1+e_2\right)\right) && (17) \\ U_2 &= 2U_1 && (18) \end{aligned}$$

where the emissions are solution of the optimality condition

$$B'(e_1) = B'(e_2) = D'\left(e_1+e_2\right).$$

Transfers are given by

$$\begin{aligned} t_{01} &= \frac{3}{4}D\left(e_1+e_2\right)+\frac{1}{4}B\left(e_1\right)+\frac{1}{4}B\left(e_2\right)>0 \\ t_{12} &= \frac{1}{2}D\left(e_1+e_2\right)-\frac{1}{2}B\left(e_1\right)+\frac{1}{2}B\left(e_2\right)>0. \end{aligned}$$

The household and firm 1 get the same payoff (17) while the payoff of firm 2 giving in (18) is twice higher. In this protocol, firm 1 gives a positive transfer to the household $t_{01}>0$ and receives a positive transfer $t_{12}>0$ from firm 2. Although firms 1 and 2 get the same benefit, the transfer paid by firm 1 is more than twice higher than the transfer it gets from firm 2, implying a higher utility for firm 2.

4 Firms have the right to pollute

The SQ situation refers to positive emissions $\overline{e_i}$, $i=1,2$. It changes the disagreement payoffs of the players in the negotiation and the amount of transfers. Emissions e_i are defined on $[0,\overline{e_i}]$. They differ in the sequential and delegation negotiation, which proves that the choice of the liability rule modifies the emissions in the economy.

4.1 The Coasian benchmark

According to the outcome of the negotiation, the household and firms 1 and 2 get respectively $U_0=t_{01}+t_{02}-D(e_1+e_2)$ and $\overline{U_0}=-D(\overline{e_1}+\overline{e_2})$, $U_1=B(e_1)-t_{01}$ and $\overline{U_1}=B(\overline{e_1})$, $\overline{U_2}=B(\overline{e_2})$. Let be $V_i=U_i-\overline{U_i}$ the net surplus. The cooperative solution implies an equal payoff for all players:

$$V_i=\frac{1}{3}\left(\left(B(\overline{e_1})-B(e_1)\right)+\left(B(\overline{e_2})-B(e_2)\right)+\left(D(\overline{e_1}+\overline{e_2})-D(e_1+e_2)\right)\right).$$

Emissions are optimal and transfers are given by for $i=1,2$ and $i\neq j$

$$t_{0i}=-\frac{2}{3}\left(B(\overline{e_i})-B(e_i)\right)+\frac{1}{3}\left(B_j(\overline{e_j})-B_j(e_j)\right)-\frac{1}{3}\left(D(\overline{e_i}+\overline{e_j})-D(e_i+e_j)\right).$$

We can show that $t_{0i} < 0$. When firms are not responsible for their pollutant emissions, they receive positive transfers t_{0i} from the household to reduce their emissions by the amount $\overline{e_i} - e_i$.

4.2 The household opens first

In the *simultaneous protocol,* when the household bargains with firm 1, it gets the utility $U_0 = t_{01} + t_{02} - D(e_1 + e_2)$ and $\overline{U_0} = t_{02} - D(\overline{e_1} + e_2)$ depending on the outcome of the negotiation. The payoff of firm 1 is then $U_1 = B(e_1) - t_{01}$ and $\overline{U_1} = B(\overline{e_1})$. It gives the net surplus $V_0 = t_{01} - D(e_1 + e_2) + D(\overline{e_1} + e_2)$ and $V_1 = B(e_1) - B(\overline{e_1}) - t_{01}$. The RBS gives the following payoffs

$$\begin{aligned} V_0 &= -\frac{1}{2}\left(B(\overline{e_1}) - B(e_1)\right) + \frac{1}{2}\left(D\left(\overline{e_1} + \overline{e_2}\right) - D\left(e_1 + \overline{e_2}\right)\right) \\ &\quad -\frac{1}{2}\left(B(\overline{e_2}) - B(e_2)\right) + \frac{1}{2}\left(D\left(\overline{e_1} + \overline{e_2}\right) - D\left(\overline{e_1} + e_2\right)\right) \end{aligned} \tag{19}$$

and for $i = 1, 2,\ i \neq j$

$$V_i = -\frac{1}{2}\left(B(\overline{e_i}) - B(e_i)\right) + \frac{1}{2}\left(D\left(\overline{e_i} + e_j\right) - D\left(e_i + e_j\right)\right) \tag{20}$$

Payoff (19) is positive when the marginal gain in terms of damage for a firm i (from switching from $\overline{e_i}$ to e_i) when the other firm emits its non cooperative amount ($\overline{e_j}$) exceeds the loss of benefit $(B(\overline{e_i}) - B(e_i))$. This condition holds for the two firms. For a firm i, the marginal gain concerns its contribution from switching from $\overline{e_i}$ to e_i for a given equilibrium emissions of the other firm e_j. The corresponding transfers are for $i = 1, 2$

$$t_{0i} = -\frac{1}{2}\left(B(\overline{e_i}) - B(e_i)\right) - \frac{1}{2}\left(D\left(\overline{e_i} + e_j\right) - D\left(e_i + e_j\right)\right) < 0.$$

When firms are not responsible for their pollutant emissions, they receive positive transfers t_{0i}^* from the household to reduce their emissions by the amount $\overline{e_i} - e_i$.

The *sequential bargaining* consists in a first round negotiation between the household and firm 1 over a pair (e_1, t_{01}) followed by a round between the household and firm 2. In round 2, agreement payoffs are given by $U_0 = t_{01} + t_{02} - D(e_1 + e_2)$ and $U_2 = B(e_2) - t_{02}$ while disagreement payoffs are $\overline{U_0} = t_{01} - D(e_1 + \overline{e_2})$ and $\overline{U_2} = B(\overline{e_2})$. In round 1, agreement payoffs are $U_0 = t_{01} + t_{02} - D(e_1 + e_2)$ for the household and $U_1 = B(e_1) - t_{01}$ for firm 1 while disagreement payoffs are given by $\overline{U_0} = -D(\overline{e_1} + \overline{e_2})$ and $\overline{U_1} = B(\overline{e_1})$.

Equations (21) and (22) gives the equilibrium payoffs

$$V_0 = V_1 = -\frac{1}{2}\left(B(\overline{e_1}) - B(e_1)\right) + \frac{1}{2}V_2 + \frac{1}{2}\left(D(\overline{e_1}+\overline{e_2}) - D(e_1+\overline{e_2})\right) \tag{21}$$

$$V_2 = -\frac{1}{2}\left(B(\overline{e_2}) - B(e_2)\right) + \frac{1}{2}\left(D(e_1+\overline{e_2}) - D(e_1+e_2)\right) \tag{22}$$

Emissions are solution of

$$2B'(e_1) - D'(e_1+\overline{e_2}) = B'(e_2) = D'(e_1+e_2) \tag{23}$$

and do not satisfy Definition 1 (optimality) and Definition 2 (efficiency). We can show that this protocol implies that the firm in position of follower emits more than the leader[9], that is $e_1 < e_2$. This differs with the case in which firms were liable for their emissions. It shows that the choice of the liability rule affects the emissions.

Transfers are given by

$$t_{01} = -\frac{1}{2}\left(B(\overline{e_1}) - B(e_1)\right) - \frac{1}{2}\left(D(\overline{e_1}+\overline{e_2}) - D(e_1+e_2)\right) - \frac{1}{2}t_{02}$$

$$t_{02} = -\frac{1}{2}\left(B(\overline{e_2}) - B(e_2)\right) - \frac{1}{2}\left(D(e_1+\overline{e_2}) - D(e_1+e_2)\right) < 0.$$

When firm 2 is not liable for its pollutant emissions, the transfer t_{02} corresponds to a subsidy paid by the household to firm 2 for an amount of emissions reduction $\overline{e_2} - e_2$. The sign of t_{01} is *a priori* ambiguous but we expect $t_{01} < 0$. However, this protocol shows that players involved in the first round get the same payoff.

4.3 Firms open first

In the *merger bargaining*, the SQ level of emissions of the merger firm is given by $\overline{e_{12}}$, yielding the disagreement payoffs $\overline{U_0} = -D\left(\overline{e_{12}}\right)$ for the household and $\overline{U_{12}} = B(\overline{e_{12}})$ for the merger firm. It implies the net payoffs $V_0 =$

[9] (23) can be rewritten as (a) $B'(e_1) + [B'(e_1) - B'(e_2)] = D'(e_1+\overline{e_2})$ and (b) $B'(e_2) = D'(e_1+e_2)$. By contradiction, assume $e_1 \geq e_2$. Hence, $B'(e_1) \leq B'(e_2)$ since B' is decreasing. From (a), it turns out that $B'(e_1) \geq D'(e_1+e_2)$ since the term in brackets is positive or null. As $D'(e_1+\overline{e_2}) > D'(e_1+e_2)$ since $\overline{e_2} > e_2$ and D' is increasing, we obtain $B'(e_1) > D'(e_1+\overline{e_2})$. Using (b), we have $B'(e_1) < B'(e_2)$, implying $e_1 < e_2$. Hence it contradicts our assumption and thus $e_1 < e_2$.

$-D(e_{12}) + t_{0(12)} + D(\overline{e_{12}})$ and $V_{12} = B(e_{12}) - B(\overline{e_{12}}) - t_{0(12)}$. The RBS yields the payoff

$$V_0 = V_{12} = \frac{1}{2}\left(\left(D\left(\overline{e_{12}}\right) - D\left(e_{12}\right)\right) - \left(B(\overline{e_{12}}) - B(e_{12})\right)\right)$$

with the transfer

$$t_{0(12)} = -\frac{1}{2}\left(B(\overline{e_{12}}) - B(e_{12})\right) - \frac{1}{2}\left(D\left(\overline{e_{12}}\right) - D\left(e_{12}\right)\right) < 0$$

showing that the merger firm receives a positive transfer $t_{0(12)}$ from the household in exchange of a lower amount of emissions $\overline{e_{12}} - e_{12}$.

The second round of the *delegation bargaining* refers to the external negotiation between firm 1 and the household over the pair (e_1, t_{01}), taking as given the outcome of the internal negotiation over (e_2, t_{12}). In the case of an agreement, the household gets $U_0 = t_{01} - D(e_1 + e_2)$ and firm 1 gets $U_1 = B(e_1) - t_{01} + t_{12}$. In the case of a disagreement, payoffs are $\overline{U_0} = -D(\overline{e_1} + e_2)$ and $\overline{U_1} = B(\overline{e_1}) + t_{12}$. In round 1, agreement payoffs for firms 1 and 2 are $U_1 = B(e_1) - t_{01} + t_{12}$ and $U_2 = B(e_2) - t_{12}$ while disagreement payoffs are respectively $\overline{U_1} = B(\overline{e_1})$ and $\overline{U_2} = B(\overline{e_2})$. The RBS gives the emissions (24) and (25) which are solution of the following equations[10] for $e_1 \in]0, \overline{e_1}[$:

$$B'(e_1) = D'\left(e_1 + \overline{e_2}\right) \tag{24}$$
$$e_2 = \overline{e_2} \tag{25}$$

which states that firm 2 sets its emission at its maximum level $e_2 = \overline{e_2}$ implying $e_1 < e_2$. It turns out that the equilibrium payoffs are

$$V_0 = \frac{1}{2}\left(D\left(\overline{e_1} + \overline{e_2}\right) - D\left(e_1 + \overline{e_2}\right)\right) - \frac{1}{2}\left(B(\overline{e_1}) - B(e_1)\right) \tag{26}$$
$$V_1 = V_2 = \frac{1}{2}V_0 \tag{27}$$

Equation (26) shows that V_0 is positive when the marginal contribution of firm 1 to the reduction in the global damage exceeds the associated lost benefit. Firms 1 and 2 get the same surplus which is the half of the household rent.

Transfers are

$$t_{01} = -\frac{1}{2}\left(B(\overline{e_1}) - B(e_1)\right) - \frac{1}{2}\left(D\left(\overline{e_1} + \overline{e_2}\right) - D\left(e_1 + \overline{e_2}\right)\right) < 0 \tag{28}$$
$$t_{12} = \frac{1}{4}\left(B(\overline{e_1}) - B(e_1)\right) - \frac{1}{4}\left(D\left(\overline{e_1} + \overline{e_2}\right) - D\left(e_1 + \overline{e_2}\right)\right) \tag{29}$$

[10] The proof is shown in appendix B.

It shows that in the internal negotiation, the emissions of firm 2 remain at the SQ level ($e_2 = \overline{e_2}$) giving more power to firm 1 in the external negotiation since emissions reduction can only come from firm 1 in exchange of the transfer t_{01}. Let us remark that the rent of firm 2 only comes from the surplus firm 1 and the household will create in the second round. In exchange firm 2 didn't bargain with the household to get a transfer on its own.

5 The choice of bargaining protocol

Based on specified functional forms for the damage and benefit functions, we analyze which one of the bargaining protocols each player would prefer.

Consider the following quadratic functions

$$\begin{aligned} B(e_i) &= ae_i - \frac{b}{2}e_i^2,\ i = 1, 2 \\ D(E) &= \frac{d}{2}(e_1 + e_2)^2 \end{aligned}$$

with the coefficients $a, b, d > 0$. The SQ emissions are equal to $\overline{e} = a/b$. By construction (see (4)) the payoff of the merger firm is given by

$$B(e_{12}) = ae_{12} - \frac{b}{4}e_{12}^2.$$

It implies $\overline{e_{12}} = 2\overline{e}$ and $e_{12} = e_1 + e_2$.

Tables in appendix C give a complete picture of the emissions, transfers, individual and aggregate payoffs that can be achieved by each player from the different types of negotiations.

When firms are responsible for their emissions, all bargaining protocols except the sequential bargaining imply the same amount of emissions for both firms. These emissions are optimal according to Definition 1 (e^o) They maximize the total welfare. It turns out that the welfare of the sequential protocol is lower that the former protocols. In the sequential bargaining, the leader (denoted by the subscript L) emits more pollution than the follower (denoted by F) with $e^L > e^o > e^F$. We introduced the parameter $\gamma = d/b > 0$ which measures the ratio of the slope of the marginal cost of damage and the slope of the marginal benefit of the emissions. For $\gamma \to 0$ we have $e \to \overline{e}$ and we show that emissions decrease with γ (for a high value of d for a given b or a low value of b for a given d).

When firms have the right to pollute, emissions are optimal for the simultaneous and the merger protocols. Emissions are ranked as follows

$e^I > e^F > e^o > e^L > e^{IE}$ where e^I stands for the emissions of the firm involved in the internal negotiation and e^{IE} for the firm involved in the internal and in the external negotiation. As expected (see (15)), the follower emits more pollution than the leader. We also show that emissions are decreasing with γ except for the emissions of the firm only involved in the internal negotiation which remains at $\overline{e}$. The sequential bargaining implies a higher aggregate welfare than the delegation bargaining. They are both lower than the optimal welfare.

As we have seen, the bargaining protocols in two rounds imply different level of utilities for firms. Since firms do not know in which position they will bargain, we assume that they get the same probability to be leader or follower in the sequential bargaining or only involved in the internal or in both internal and external negotiation. This defines an average expected utility. For the merger bargaining, we assume that firms share in two equal parts the aggregate utility. The following propositions show what will be the bargaining protocol choice by each player.

Proposition 1 *When firms are responsible for their emissions:*

1. *The household prefers to bargain simultaneously with the two firms.*

2. *Firms prefer to be only involved in the internal negotiation.*

3. *On average, firms prefer the delegation bargaining.*

The household prefers to bargain separately and simultaneously with the firms since it gives him the highest payoff. Firms prefer to be leader in the sequential procedure rather than be the delegate firm who has to conduct the external negotiation. However they prefer to be only involved in the internal negotiation. In average, the delegation protocol gives a higher expected payoff than the sequential procedure. It turns out that whatever the choice of the bargaining protocols the household and firms can make, emissions are optimal according to Definition 1. It means that the Coase theorem holds true when strategic bargaining is considered. In all cases the outcome is efficient and differences between the protocols only depend on the way the optimal welfare is shared between the household and the firms. We can also show that compared to the Coasian benchmark given by the Nash Solution, all players are better off.

We consider now the next case.

Proposition 2 *When firms have the right to pollute:*

1. *The household prefers to bargain simultaneously with the two firms.*

2. *Firms prefer to act as a leader in the sequential procedure.*

3. *On average, firms prefer the sequential bargaining.*

This proposition shows that the household always prefer the simultaneous negotiation whoever holds the property right. Firms prefer the sequential negotiation when they have the right to pollute rather than the delegation negotiation as in the previous proposition. However, this proposition states that inefficiencies may result from the bargaining. This occurs when firms have the right to pollute and have the right to choose the protocol they prefer. In that case, the choice of sequential bargaining implies non optimal emissions according to Definition 1. Strategic negotiation leads to an inefficient outcome and invalidates the bargaining Coase theorem. However we can show that if we compare this situation with the Coasian benchmark, firms will prefer the cooperative Nash Solution. Our result contradicts Veljanoski (1982) since strategic behavior has an effect on the allocation of resources.

6 Conclusion

In this paper we have considered strategic bargaining between three players: two firms which pollute a household. Depending on who has the property right to pollute, we have consider four kind of bilateral bargaining procedures and we have managed to identify the ones that each player would prefer to see implemented. In doing this, we identify an important result by presenting conditions under which the bargaining Coase theorem is invalidated. When firms have the right to pollute and are those who can choose the bargaining protocol, their choice leads to an inefficient outcome: emissions are not optimal and the aggregate welfare is not maximized. Our result contradicts Veljanoski (1982, p. 58) who wrote that *"Coase did not rule-out strategic behavior but assumed that when it did occur it would (..) have no effects on the (..) efficient allocation of resources."* This paper shows that such inefficiencies can appear in a non cooperative bargaining framework. When we consider externalities involving more than two players, the validity of the bargaining Coase theorem only holds with the axiomatic Nash Solution. Generalizing this framework to n players on both sides of the negotiation table through a network is a topic of our ongoing research.

References

Aivazian, V.A. and J.L. Callen (1981), "The Coase Theorem and the Empty Core", J*ournal of Law and Economics*, 24(1): 175-181.

Aivazian, V.A., J.L. Callen and I. Lipnowski (1987), "The Coase Theorem and Coalitional Stability", *Economica*, 54(216): 517-520.

Arrow, J.K. (1969), The organization of economic activity: issues pertinent to the choice of market versus nonmarket allocation, in The Analysis and evaluation of public expenditures, Congress of the United States, Vol. I, Part 1.

Binmore, K., Rubinstein, A., and A. Wolinsky (1986), "The Nash Bargaining Solution in Economic Modelling", *RAND Journal of Economics*, 17:176-188.

Bester, H. and Sakovics J; (2001), "Delegated bargaining and Renegotiation", *Journal of Economic Behavior and Organization*, 45(4):459-473.

Calabresi, G. (1968), "Transaction Costs, Resource Allocation and Liability Rules: A Comment", *Journal of Law and Economics*, 11(1): 67-73.

Clark, D. and Pereau J-C. (2008), "Passing the buck in sequential bargaining", *The B.E. Journal of Theoretical Economics*, Vol. 8(1) (Topics), Art 26.

Clark, D. and Pereau J-C. (2009), "Fragmented property rights and royalty bargaining", *Journal of Economic Behavior and Organization*, 72:546-553.

Coase, R.H. (1960), "The Problem of Social Cost", *Journal of Law and Economics*, 3 : 1-44.

Coase, R.H. (1981), "The Coase Theorem and the Empty Core: A Comment", *Journal of Law and Economics*, 24(1): 183-187.

Cooter, R. (1982), "The Cost of Coase", *Journal of Legal Studies*, 11(1): 1-33.

Dixit, A. and M. Olson (2000), "Does voluntary participation undermine the Coase Theorem?", *Journal of Public Economics* 76: 309–335.

Farrell, J. (1987), "Information and the Coase Theorem", *Journal of Economic Perspectives*, 1(2): 113-129.

Hurwicz, L. (1995), "What is the Coase Theorem", *Japan and the World Economy*, 7: 49-74.

Kenan J. and Wilson R (1993) "Bargaining with Private Information", *Journal of Economic Litterature*, 31: 45-104.

Lee, J and Sabourian H. (2007), "Coase theorem, complexity and transaction costs", *Journal of Economic Theory*, 135:214-235

Mas-Colell, A. and al. (1995), *Microeconomy Theory*, New York, Oxford University Press.

Matsuo, T. (1989), "On Incentive Compatible, Individually Rational, and Ex Post Efficient Mechanisms for Bilateral Trading », *Journal of Economic Theory*, 49: 189-194.

Muthoo, A (1999) *Bargaining Theory with Applications*, Cambridge University Press.

McAfee, P. and Schwartz, M. (1994) "Opportunism in Multilateral Contracting: Nondiscrimination, Exclusivity and Uniformity", *American Economic Review*, 84:210-230.

McKelvey, R.D. and T. Page (2002), "Status Quo Bias in Bargaining: An Extension of the Myerson–Satterthwaite Theorem with an Application to the Coase Theorem", *Journal of Economic Theory*, 107: 336–355.

Myerson, R.B. et M.A. Satterthwaite (1983), "Efficient Mechanism for Bilateral Trading", *Journal of Economic Theory*, 29: 265-281.

Medena, S.G. (1994), "The Myth of Two Coases: What Coase Is Really Saying", *Journal of Economic Issues*, 28(1): 208-217.

Medema S.G. and Zerbe R.O. Jr (2000), "The Coase Theorem", in B. Bouckaert and G. De Geest (eds), The Encyclopedia of Law and Economics, vol. I: The History and Methodology of Law and Economics, Cheltenham, Edward Elgar Publishing, p. 836-892.

Raiffa, H. (2002), *Negotiation Analysis*, The Belknap press of Harvard University Press, Cambridge, Massachusetts, and London, England.

Regan, D.H. (1972), "The Problem of Social Cost Revisited", *Journal of Law and Economics*, 15(2): 427-437.

Rubinstein A. (1982), "Perfect equilibrium in a bargaining model", *Econometrica,* 50, 97-109.

Samuelson, P. (1995), "Some uneasiness with the Coase Theorem", *Japan and the World Economy*, 7: 1-7.

Segendorff, B. (1998), "Delegation and Threat in Bargaining", *Games and Economic Behavior*, 23:266-283.

Usher, D. (1998) The Coase theorem is tautological, incoherent or wrong", *Economic Letters*, 61:3-11

Varian, H.R. (2000), "Coase, competition, and compensation", *Japan and the World Economy*, 7: 13-27.

Veljanovski, C.G. (1982), "The Coase Theorems and the Economic Theory of Markets and Law", *Kyklos*, 35(1): 53-74.

Appendix A: The Rubinstein bargaining solution

The proof of the uniqueness of a subgame perfect equilibrium for the two-round bargaining is based on Muthoo (1999, p.55). The RBS is determined for the sequential protocol when firms have the right to pollute. The other cases can be easily derived from this one. We assume that player 0 is the proposer and makes the offers (t_{01}, e_1) to player 1 in round 1 and (t_{02}, e_2) to player 2 in round 2. By backward induction, we begin with the negotiation between 0 and 2 over (t_{02}, e_2) based on an agreement in round 1 over (t_{01}, e_1). The impasse points are $\overline{U_0} = t_{01}^* - D(e_1^*, \overline{e_2})$ and $\overline{U_2} = B(\overline{e_2})$. Let be $\left(t^{(i)}, e^{(i)}\right)$ an offer made by player i and $V_i^* = U_i\left(t_i^*, e_i^*\right) - \overline{U_i}$. The payoff of player 2 from rejecting any offer from 0 is $\delta_2 V_2^*$.

Perfection requires that player 1 accepts (refuses) the offer from 0 $\left(t_{02}^{(0)}, e_2^{(0)}\right)$ st $U_2(t_{02}^{(0)}, e_2^{(0)}) - \overline{U_2} > (<) \delta_2 V_2^*$. Hence the offer from 0 has to be such that $U_2(t_{02}^{(0)}, e_2^{(0)}) - \overline{U_2} = \delta_2 V_2^*$.

Optimality requires that (t_{02}^*, e_2^*) maximises its utility $U_0(t_{02}^{(0)}, e_2^{(0)}) - \overline{U_0}$ subject to $U_2(t_{02}^{(0)}, e_2^{(0)}) - \overline{U_2} = \delta_2 V_2^*$. It gives $e_2^{(0)} = e_2^*$ the unique interior solution of $D'(e_1 + e_2) = B'(e_2)$. By symmetry, we have $U_0(t_{02}^{(2)}, e_2^{(2)}) - \overline{U_0} = \delta_0 V_0^*$ and $e_2^{(2)} = e_2^*$. Let be $\pi = (B(e_2^*) - B(\overline{e_2})) - (D(e_1^* + e_2^*) - D(e_1^*, \overline{e_2}))$ the net surplus that is created. We have $V_0^* = \pi - \delta_2 V_2^*$ and $V_2^* = \pi - \delta_0 V_0^*$. It gives

$$V_2^* = \left(\frac{1-\delta_0}{1-\delta_0\delta_2}\right)\left((B(e_2^*) - B(\overline{e_2})) - (D(e_1^* + e_2^*) - D(e_1^*, \overline{e_2}))\right)$$

and the transfer offer made by the proposer

$$t_{02}^* = \left(\frac{1-\delta_2}{1-\delta_0\delta_2}\right)(B(e_2^*) - B(\overline{e_2})) + \left(\frac{\delta_2(1-\delta_0)}{1-\delta_0\delta_2}\right)(D(e_1^* + e_2^*) - D(e_1^*, \overline{e_2})).$$

To determine the rent of player 0, we have to solve the next bargaining round. In round 1, player 0 bargains with player 1 over (t_{01}, e_1). Impasse points are $\overline{U_0} = -D(\overline{e_1} + \overline{e_2})$ and $\overline{U_1} = B(\overline{e_1})$, we get

$$\begin{aligned} U_0 - \overline{U_0} &= t_{01} + t_{02}^* - (D(e_1 + e_2^*) - D(\overline{e_1} + \overline{e_2})) \\ U_1 - \overline{U_1} &= (B(e_1) - B(\overline{e}_1)) - t_{01}. \end{aligned}$$

Using the same method as before, the payoff of player 1 from rejecting any offer from 0 is $\delta_1 V_1^*$. Hence the offer of 0 denoted by $\left(t_{01}^{(0)}, e_1^{(0)}\right)$ has to be such that $U_1\left(t_{01}^{(0)}, e_1^{(0)}\right) = \delta_1 V_1^*$. Optimality requires that (t_{01}^*, e_1^*) maximises its utility $U_0(t_0, e_0)$ subject to $U_1\left(t_{01}^{(0)}, e_1^{(0)}\right) = \delta_1 V_1^*$. It gives $e_1^* = e^*$ the unique interior solution of

$$\frac{\partial t_{02}^*}{\partial e_1} - \left(1 + \frac{\partial e_2}{\partial e_1}\right) D'(e_1 + e_2) + B'(e_1) = 0.$$

From the expression of t_{02}^*, we get

$$\begin{aligned} \frac{\partial t_{02}^*}{\partial e_1} &= \left(\frac{1-\delta_2}{1-\delta_0\delta_2}\right)\frac{\partial e_2}{\partial e_1}B'(e_1^*) + \frac{\delta_2(1-\delta_0)}{1-\delta_0\delta_2}\left(1+\frac{\partial e_2}{\partial e_1}\right)D'(e_1^*+e_2^*) \\ &\quad -\frac{\delta_2(1-\delta_0)}{1-\delta_0\delta_2}D'(e_1^*, \overline{e_2}). \end{aligned}$$

After substitution and using the optimality condition of the second round $D'(e_1 + e_2) = B'(e_2)$, we obtain

$$B'(e_1^*) - \left(\frac{\delta_2(1-\delta_0)}{1-\delta_0\delta_2}\right)D'(e_1^*, \overline{e_2}) = \left(\frac{1-\delta_2}{1-\delta_0\delta_2}\right)D'(e_1^*+e_2^*).$$

By symmetry, we have $U_0\left(t_{01}^{(1)}, e_1^{(1)}\right) = \delta_0 V_0^*$ and $e_1^{(1)} = e_1^*$. Let $\pi = (B(e_1^*) - B(\overline{e_1})) - (D(e_1^* + e_2^*) - D(\overline{e_1}, \overline{e_2})) + t_{02}^*$ be the net surplus that is created. We have

$$\begin{aligned} U_0 - \overline{U_0} &= \left(\frac{1-\delta_1}{1-\delta_0\delta_1}\right)\left((B(e_1^*) - B(\overline{e}_1)) - (D(e_1^*+e_2^*) - D(\overline{e_1}, \overline{e_2})) + t_{02}^*\right) \\ U_1 - \overline{U_1} &= \left(\frac{1-\delta_0}{1-\delta_0\delta_1}\right)\left((B(e_1^*) - B(\overline{e}_1)) - (D(e_1^*+e_2^*) - D(\overline{e_1}, \overline{e_2})) + t_{02}^*\right). \end{aligned}$$

and the transfer

$$\begin{aligned} t_{01}^* &= \left(\frac{1-\delta_1}{1-\delta_0\delta_1}\right)(B(e_1^*) - B(\overline{e}_1)) + \left(\frac{\delta_1(1-\delta_0)}{1-\delta_0\delta_1}\right)(D(e_1^*+e_2^*) - D(\overline{e_1}, \overline{e_2})) \\ &\quad -t_{02}^*. \end{aligned}$$

Substituting the expression of t_{02}^* gives

$$\begin{aligned}
U_0 - \overline{U_0} &= \left(\frac{1-\delta_1}{1-\delta_0\delta_1}\right)((B(e_1^*) - B(\bar{e}_1)) - (D(e_1^* + e_2^*) - D(\overline{e_1}, \overline{e_2}))) \\
&+ \left(\frac{1-\delta_1}{1-\delta_0\delta_1}\right)\left(\frac{1-\delta_2}{1-\delta_0\delta_2}\right)(B(e_2^*) - B(\overline{e_2})) \\
&+ \left(\frac{\delta_2(1-\delta_0)}{1-\delta_0\delta_2}\right)\left(\frac{1-\delta_1}{1-\delta_0\delta_1}\right)(D(e_1^* + e_2^*) - D(e_1^*, \overline{e_2})) \\
U_1 - \overline{U_1} &= \left(\frac{1-\delta_0}{1-\delta_0\delta_1}\right)((B(e_1^*) - B(\bar{e}_1)) - (D(e_1^* + e_2^*) - D(\overline{e_1}, \overline{e_2}))) \\
&+ \left(\frac{1-\delta_0}{1-\delta_0\delta_1}\right)\left(\frac{1-\delta_2}{1-\delta_0\delta_2}\right)(B(e_2^*) - B(\overline{e_2})) \\
&+ \left(\frac{\delta_2(1-\delta_0)}{1-\delta_0\delta_2}\right)\left(\frac{1-\delta_0}{1-\delta_0\delta_1}\right)(D(e_1^* + e_2^*) - D(e_1^*, \overline{e_2}))
\end{aligned}$$

and

$$\begin{aligned}
t_{01}^* &= \left(\frac{1-\delta_1}{1-\delta_0\delta_1}\right)(B(e_1^*) - B(\overline{e_1})) + \left(\frac{\delta_1(1-\delta_0)}{1-\delta_0\delta_1}\right)(D(e_1^* + e_2^*) - D(\overline{e_1}, \overline{e_2})) \\
&- \left(\frac{1-\delta_2}{1-\delta_0\delta_2}\right)(B(e_2^*) - B(\overline{e_2})) - \left(\frac{\delta_2(1-\delta_0)}{1-\delta_0\delta_2}\right)(D(e_1^* + e_2^*) - D(e_1^*, \overline{e_2})).
\end{aligned}$$

Assume that $\delta_0 = \delta_1 = \delta_2 = \delta \to 1$, we get the expressions of the text (13) and (14) when $\overline{e_1} = \overline{e_2} = 0$ and in the general case (21) and (22).

Appendix B: Emissions in the delegation negotiation

For the delegation protocol, we explicit the conditions under which interior and corner solutions can exist when $\overline{e_1} = \overline{e_2} \neq 0$. We assume that $\delta_0 = \delta_1 = \delta_2 = \delta \to 1$.

In the external negotiation, the total surplus created between players 0 and 1 is

$$V_0 + V_1 = (B(e_1) - D(e_1 + e_2^*)) - (B(\overline{e_1}) - D(\overline{e}_1 + e_2^*)).$$

Let E_2 be the amount of emissions such that

$$B'(0) - D'(E_2) = 0.$$

We consider two cases:

- If $e_2^* \geq E_2$ then $e_1^* = 0$. It implies a positive surplus $V_0 + V_1 = D(\overline{e}_1 + e_2^*) - D(e_2^*) - B(\overline{e_1}) > 0$ and the associated transfer

  $$t_{01} = \frac{1}{2} D(e_2^*) - \frac{1}{2}(B(\overline{e_1}) + D(\overline{e}_1 + e_2^*)).$$

  It turns out that $\mathrm{d}e_1^*/\mathrm{d}e_2 = 0$ and $\mathrm{d}t_{01}^*/\mathrm{d}e_2 = \frac{1}{2}(D'(e_2^*) - D'(\overline{e}_1 + e_2^*))$.

- if $e_2^* < E_2$ then $e_1^* \in [0, \overline{e_1}[$ satisfies[11] $B'(e_1^*) - D'(e_1^* + e_2^*) = 0$ (see (24) in the text). It gives the surplus and the associated transfer (see (28)):

  $$\begin{aligned} V_0 + V_1 &= (B(e_1^*) - D(e_1^* + e_2^*)) - (B(\overline{e_1}) - D(\overline{e}_1 + e_2^*)) > 0 \\ t_{01} &= \frac{1}{2}(B(e_1^*) + D(e_1^* + e_2^*)) - \frac{1}{2}(B(\overline{e_1}) + D(\overline{e}_1 + e_2^*)). \end{aligned}$$

  It implies

  $$\begin{aligned} \frac{\mathrm{d}e_1^*}{\mathrm{d}e_2} &= \frac{D''(e_1^* + e_2^*)}{B''(e_1^*) - D''(e_1^* + e_2^*)} \\ \frac{\mathrm{d}t_{01}^*}{\mathrm{d}e_2} &= \frac{1}{2}(B'(e_1^*) + D'(e_1^* + e_2^*))\frac{\mathrm{d}e_1^*}{\mathrm{d}e_2} + \frac{1}{2}(D'(e_1^* + e_2^*) - D'(\overline{e}_1 + e_2^*)). \end{aligned}$$

Consider now the internal negotiation between 1 and 2. The total surplus is

$$V_1 + V_2 = (B(e_1^*) - t_{01} - B(\overline{e_1})) + (B(e_2) - B(\overline{e_2})).$$

To determine the value of e_2 that maximise the total surplus, we compute $\mathrm{d}(V_1 + V_2)/\mathrm{d}e_2$ according to the value of e_1^* obtained in the external negotiation.

[11] The case $e_1 = \overline{e_1}$ when $B'(\overline{e_1}) - D'(\overline{e_1} + e_2^*) > 0$ is impossible since $B'(\overline{e_1}) = 0$ and $D'' > 0$ implies $-D'(\overline{e_1} + e_2^*) < 0$.

- For $e_1^* = 0$, we have

$$\frac{\mathrm{d}\left(V_1+V_2\right)}{\mathrm{d}e_2} = B'(e_2) - \frac{\mathrm{d}t_{01}^*}{\mathrm{d}e_2} = B'(e_2) + \frac{1}{2}\left(D'\left(\overline{e}_1+e_2\right) - D'\left(e_2\right)\right).$$

- For $e_1^* \in [0, \overline{e_1}[$, we have

$$\frac{\mathrm{d}\left(V_1+V_2\right)}{\mathrm{d}e_2} = B'\left(e_1^*\right)\frac{\mathrm{d}e_1^*}{\mathrm{d}e_2} - \frac{\mathrm{d}t_{01}^*}{\mathrm{d}e_2} + B'\left(e_2\right).$$

Substitute the value of $\mathrm{d}t_{01}^*/\mathrm{d}e_2$ gives

$$\frac{\mathrm{d}\left(V_1+V_2\right)}{\mathrm{d}e_2} = \left(B'\left(e_1^*\right) - D'\left(e_1^*+e_2\right)\right)\frac{\mathrm{d}e_1^*}{\mathrm{d}e_2} - \left(D'\left(e_1^*+e_2\right) - D'\left(\overline{e}_1+e_2\right)\right) + 2B'\left(e_2\right)$$

which can simplify as

$$\frac{\mathrm{d}\left(V_1+V_2\right)}{\mathrm{d}e_2} = B'\left(e_2\right) + \frac{1}{2}\left(D'\left(\overline{e}_1+e_2\right) - D'\left(e_1^*+e_2\right)\right).$$

Since $D'\left(\overline{e}_1+e_2\right) > D'\left(e_1^*+e_2\right)$ for $e_1^* \in [0, \overline{e_1}[$ and $B'\left(e_2\right) > 0$, the total surplus is maximized for $e_2^* = \overline{e}_2$ (see (25)). We can show that the RBS of this negociation gives the following transfer

$$t_{12} = \frac{1}{2}\left(B(\overline{e_1}) - B(e_1^*) + t_{01}\right).$$

Substitute gives the expressions of the payoffs (26) and (27) in the text.

Appendix C: Expected payoffs

The following tables give a complete picture of the expected payoffs that can be achieved by each player in the different procedures. We note $\overline{e} = a/b$ and $\gamma = d/b$.

When firms are responsible for their emissions, we have:

	Cooperative	Simultaneous	Sequential	Merger	Delegation
e_1	$\frac{1}{1+2\gamma}\overline{e}$	$\frac{1}{1+2\gamma}\overline{e}$	$e_1^L = \frac{\gamma+2}{\gamma^2+4\gamma+2}\overline{e}$		$e_1^{IE} = \frac{1}{1+2\gamma}\overline{e}$
e_2	e_1	e_1	$e_2^F = \frac{2}{\gamma^2+4\gamma+2}\overline{e}$		$e_2^E = e_1$
e_{12}				$\frac{2}{1+2\gamma}\overline{e}$	
t_{01}	$\frac{(8\gamma+1)}{6(2\gamma+1)^2}b\overline{e}^2$	$\frac{(1+7\gamma)}{4(2\gamma+1)^2}b\overline{e}^2$	$\frac{\left(3\gamma^3+15\gamma^2+18\gamma+2\right)}{4(\gamma^2+4\gamma+2)^2}b\overline{e}^2$		$\frac{(10\gamma+1)}{4(2\gamma+1)^2}b\overline{e}^2$
t_{02}	t_{01}	t_{01}	$\frac{\left(2\gamma^2+7\gamma+1\right)}{(\gamma^2+4\gamma+2)^2}b\overline{e}^2$		
$t_{0(12)}$				$\frac{(6\gamma+1)}{2(2\gamma+1)^2}be^2$	
t_{12}					$\frac{\gamma}{(2\gamma+1)^2}b\overline{e}^2$
U_0	$\frac{1}{6\gamma+3}b\overline{e}^2$	$\frac{(3\gamma+1)}{2(2\gamma+1)^2}b\overline{e}^2$	$\frac{\gamma+3}{4(\gamma^2+4\gamma+2)}b\overline{e}^2$	$\frac{1}{2(2\gamma+1)}be^2$	$\frac{1}{4(2\gamma+1)}b\overline{e}^2$
U_1	U_0	$\frac{\gamma+1}{4(2\gamma+1)^2}b\overline{e}^2$	$U_1^L = U_0$		$U_1^{IE} = U_0$
U_2	U_1	U_1	$U_2^F = \frac{\gamma+1}{(\gamma^2+4\gamma+2)^2}b\overline{e}^2$		$U_1^E = 2U_1$
U_{12}				U_0	
$\sum U_i$	$\frac{1}{2\gamma+1}b\overline{e}^2$	$\frac{1}{2\gamma+1}b\overline{e}^2$	$\frac{\gamma^3+7\gamma^2+16\gamma+8}{2(\gamma^2+4\gamma+2)^2}b\overline{e}^2$	$\frac{1}{2\gamma+1}b\overline{e}^2$	$\frac{1}{2\gamma+1}b\overline{e}^2$

We can show that

- $U_0^{sim} > U_0^{mer} > U_0^{\text{seq}} > U_0^{del}$ for the household.
- $U_i^I > U_i^L > U_i^{mer} = U_i^{IE} > U_i^{sim} > U_i^F$ for a firm i.
- $U^{del} > U^{\text{seq}} > U^{mer} > U^{sim}$ for an average firm.
- The cooperative solution is such that $U_0^{\text{seq}} > U_0^{coop} > U_0^{del}$, $U_i^L > U_i^{coop} > U_i^{mer}$ and $U^{del} > U^{coop} > U^{\text{seq}}$.

When firms have to right to pollute, we have:

	Cooperative	Simultaneous	Sequential	Merger	Delegation
e_1	$\frac{1}{1+2\gamma}\overline{e}$	$\frac{1}{1+2\gamma}\overline{e}$	$e_1^L = \frac{2-\gamma^2}{\gamma^2+4\gamma+2}\overline{e}$		$e_1^{IE} = \frac{1-\gamma}{1+\gamma}$
e_2	e_1	e_1	$e_2^F = \frac{\gamma^2+2}{\gamma^2+4\gamma+2}\overline{e}$		$e_2^I = \overline{e}$
e_{12}				$\frac{2}{1+2\gamma}\overline{e}$	
t_{01}	$\frac{2\gamma^2(4\gamma+5)}{3(2\gamma+1)^2}b\overline{e}^2$	$\frac{\gamma^2(\gamma+3)}{(2\gamma+1)^2}b\overline{e}^2$	$\frac{-\gamma^2\left(\gamma^3+9\gamma^2+18\gamma+6\right)b\overline{e}^2}{(\gamma^2+4\gamma+2)^2}$		$-\frac{\gamma^2(\gamma+3)}{(\gamma+1)^2}b\overline{e}^2$
t_{02}	t_{01}	t_{01}	$-\frac{4\gamma^2(\gamma+3)}{(\gamma^2+4\gamma+2)^2}b\overline{e}^2$		
$t_{0(12)}$				$\frac{-2\gamma^2(2\gamma+3)b\overline{e}^2}{(2\gamma+1)^2}$	
t_{12}					$-\frac{\gamma^2}{2(\gamma+1)}b\overline{e}^2$
V_0	$\frac{4\gamma^2}{6\gamma+3}b\overline{e}^2$	$\frac{2\gamma^2(3\gamma+1)}{(2\gamma+1)^2}b\overline{e}^2$	$\frac{(\gamma+3)\gamma^2}{\gamma^2+4\gamma+2}b\overline{e}^2$	$\frac{2\gamma^2}{2\gamma+1}be^2$	$\frac{\gamma^2}{\gamma+1}b\overline{e}^2$
V_1	V_0	$\frac{\gamma^2(\gamma+1)}{(2\gamma+1)^2}b\overline{e}^2$	$V_1^L = V_0$		$V_1^{IE} = \frac{1}{2}V_0$
V_2	V_0	V_1	$V_2^F = \frac{4\gamma^2(\gamma+1)}{(\gamma^2+4\gamma+2)^2}b\overline{e}^2$		$V_2^I = \frac{1}{2}V_0$
V_{12}				V_0	
$\sum V_i$	$\frac{4\gamma^2}{2\gamma+1}b\overline{e}^2$	$\frac{4\gamma^2}{2\gamma+1}b\overline{e}^2$	$\frac{2\gamma^2\left(\gamma^3+7\gamma^2+16\gamma+8\right)}{(\gamma^2+4\gamma+2)^2}b\overline{e}^2$	$\frac{4\gamma^2}{2\gamma+1}b\overline{e}^2$	$\frac{2\gamma^2}{\gamma+1}b\overline{e}^2$

Let us remark that $e_1^L > 0$ for $\gamma < \sqrt{2}$, $e_1^{IE} > 0$ for $\gamma < 1$.
We can show that:

- $V_0^{sim} > V_0^{mer} > V_0^{\text{seq}} > V_0^{del}$ for the household.
- $V_i^L > V_i^{mer} > V_i^{sim} > V_i^{del} > V_i^F$ for a firm i for $\gamma < 0.71$ and the order between V_i^{sim} and V_i^{del} is reversed for $\gamma > 0.71$.
- $V^{\text{seq}} > V^{mer} > V^{sim} > V^{del}$ for an average firm.
- The cooperative solution is such that $V_0^{\text{seq}} > V_0^{coop} > (<) V_0^{del}$ for $\gamma > (<) 0.5$ and $V^{coop} > V^{\text{seq}}$.

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