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Dominance relations when both quantity and quality matter, and applications to the comparison of US research universities and worldwide top departments in economics

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Relations de dominance lorsque la quantité et la qualité importent, et applications à la comparaison des universités de recherche aux USA et des meilleurs départements mondiaux en économie

Résumé

Dans cet article, nous proposons une extension du concept de dominance stochastique à des comparaisons de productions composites dont la quantité et la qualité sont importantes. Notre théorie nous permet en outre de requérir l'unanimité de jugement au sein de nouvelles classes de fonctions. Nous appliquons cette théorie au classement des universités de recherche US, procurant ainsi un nouvel outil aux scientomètres (et aux communautés académiques) qui ont pour objectif de comparer les institutions de recherche en prenant en considération à la fois le volume de publication et l'impact des articles. L'autre application proposée concerne les comparaisons et les classements des départements académiques lorsque l'on prend en compte à la fois la taille du département et le prestige de chacun de ses membres.

Mots-clés : classements, relations de dominance, citations.

Dominance relations when both quantity and quality matter, and applications to the comparison of US research universities and worldwide top departments in economics

Abstract

In this article, we propose an extension of the concept of stochastic dominance intensively used in economics for the comparison of composite outcomes both the quality and the quantity of which do matter. Our theory also allows us to require unanimity of judgement among new classes of functions. We apply this theory to the ranking of US research universities, thereby providing a new tool to scientometricians (and the academic communities) who typically aim to compare research institutions taking into account both the volume of publications and the impact of these articles. Another application is provided for comparing and ranking academic departments when one takes into account both the size of the department and the prestige of each member.

Keywords: Ranking, dominance relations, citations.

JEL: D63, I23

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1 Introduction

Since the seminal contributions of Quirk and Saposnick (1962), Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970), economists have intensively applied the notion of stochastic dominance to the theory of choice under uncertainty in which one basically compares lotteries (or density distributions) in an unambiguous manner among given classes of utility functions. Another application pointed out by Atkinson (1970) concerns the unanimous comparison of income distributions among all social welfare functions which are symmetric and increasing concave in all its arguments (equivalent in this context to the anonymity condition and the Pareto principle). In these two contexts the value judgements do not or should not take size into account. Indeed, in the theory of choice under uncertainty, when comparing lotteries, the sum of probabilities is clearly always equal to the unity. Only the probability of occurrence of each possible state of the world and its associated returns do matter, which, in more general terms, we associate here to the notion of quality. Things are slightly different in the context of income distribution comparisons, where value judgements could in principle take quantity into account but are designed not to do so - that is here population size - but only quality, which relates to incomes and their distribution. Size obviously does not matter when comparing two income distributions within the same population (any distribution can then be obtained from any other through a finite set of transfers). However, when comparing the income distributions of two different countries with different population sizes, then, as stated by Dasgupta et Al. (1973), using an average social welfare criterion allows them to exclude any influence of population size per se.

There are however numerous contexts in which both quality and quantity do matter. A typical example we will discuss in this article is related to scientific production. The community of scientometricians (or, equivalently, infometricians) has for a long time been concerned with the necessary quantification of the scientific production of various academic institutions and actors. Two main approaches were developed for this purpose. The first one aims to measure the volume of publications, or basically the number of articles published.[1] The second one is mostly concerned with the quality of these publications (since obviously not all papers are equal) and mainly relies on the number of citations[2,3] articles receive. However, a unique measurement of scientific production

[1] Various corrections need however to be introduced. extensive applied literature in scientometrics discusses which list of journals should be retained, how to account for the length of articles, how to (and should we) correct for co-authorship.

[2] Peer judgement is an obvious alternative (though more time consuming) way of measuring quality.

[3] One may also consider the journal's attributes as an appropriate (though more indirect) measurement

taking into account both the volume of publication (quantity) and the quality of each article is often needed. A basic way of computing such a measurement would simply be to sum up the number of citations all articles receive, a solution which is far from satisfactory since it implies that quality increases proportionally with citations.[4] Interest in this question has recently been spurred by Hirsch's introduction (2005) of the so-called h-index, an index precisely designed to simultaneously account for both quality and quantity in a specific manner.[5] Though this index has been highly discussed,[6] few papers have sought to explain the implicit value judgements of the h-index and of its variants.[7] To our knowledge no academic article yet has attempted to derive comparisons of scientific productions from explicit value judgements taking into consideration both their quantity and impact. This is a question we can and shall address using the more general theory developed in this article.

Thus, the first contribution of this article consists in providing a rationale for comparing the outcome of various institutions (or individuals) when both quality and quantity matter, while rendering completely explicit its associated value judgements. Our contribution can be interpreted as a generalization of the stochastic dominance theory, the standard applications of this theory to choice under uncertainty and income distributions being particular cases in which only quality is taken into consideration. Though most of the theorems we introduce in this article have analogs in the stochastic dominance literature, our problem is more general and the mathematical proofs follow different lines and are therefore new. Moreover, several applications of the theory prompted us to explore the consequences of assuming that the social value function is convex (rather than concave) in each of its arguments, an assumption which is quite exotic from the point of

of its scientific impact. For instance, the journal's Impact Factor (the average number of citations received within a fixed window period by the articles published in the journal) is a widely used statistics.

[4]The first (and recently rediscovered) synthetic measurement of both the quantity and quality of articles was proposed by Lindsay (1978). It is computed as the average number of citations times the squared root of the total number of citations.

[5]One authors'h is the maximum number of articles he authored which received at least h citations. This measure has the specific characteristic that it neither takes into account the citations received by the articles having less than h citations nor it accounts for the citations received by papers above the threshold of their h first citations. These citations are said to fall outside the h-core. Hirsch (2005) argues that highly cited papers shall not be taken into account proportionnally to the citations recieved because some papers attract an anomalous amount of citations.

[6]Many contributions have aimed to overcome such shortcomings (especially of the latter kind), the g-index (Egghe, 2006), the tapered h-index (Anderson et al., 2008), w-index (Woeringer, 2008)...

[7]That very interesting line of inquiry, followed by Woeringer (2008) and Marchant (2009), consists in picking the desired index and building an axiomatic which explicits its value judgements.

view of the theory of choice under uncertainty or the theory of income distribution.[8]

In line with the example developed above, this theory is first applied for establishing dominance relations between the research productions of universities within a given period of time. We assume that the social value of universities' research is additively separable in all its arguments, namely the quality of each article recorded. The quality of the articles can be assessed from precise measurements of their impact, which in turn can be computed through several procedures: using the number of direct citations received by the papers within a given period of time or the average impact of the journals in which the papers were published.[9] It seems acceptable to assume that publishing a higher impact paper or publishing an additional article can never decrease the total value of any scientific production. Assumptions on the second derivative are more debatable. However, it appears, implicitly or explicitly, in most of the interviews conducted by the authors with rectors or presidents of several universities, that convexity is a widely accepted assumption. Indeed, it is important in the eyes of most university representatives that their institution appear on papers that reach high scores in terms of impact, enabling the institution to increase its visibility within a community characterized by very high scientific standards. Typically, if one assumes that the number of citations is the accepted quality measure, convexity would mean for instance that the value of two articles with fifty citations is never higher than the value of one article with an hundred citations. One can also give different focuses to the assumptions, focuses which we hope can guide the comparisons: *volume*, when the only assumption is that the value is positive; *quality*, when the value is positive and does not decrease with impact or *excellence*, when the value is positive, non decreasing and convex with impact. Our theory is first applied for establishing dominance relations between US research universities[10] both in the disciplinary and in the interdisciplinary contexts. We also show and discuss to what extent rankings can be inferred from such dominance relations.

In this article we propose a second application of the extended stochastic dominance theory, namely the comparison of academic departments' degrees of prestige. The academic prestige of a department may be measured by using several variables, but most

[8]This issue was examined in a recent study (Bazen and Moyes, 2011) which was spurred by a previous version of our work.

[9]Basically, this involves the same measure as the direct citations but averaged over the total number of papers published in the journal within the same period. As we will see below, this measurement can also be normalized at the sub-field level.

[10]We shall here mention Lubrano and Protopopescu (2004) who were the first to apply the notion of dominance to the academic sphere. They are concerned with the distribution of articles among the population of economists in various countries.

would agree that it should be calculated using the information on the prestige of each of its current members. Again, the most prestigious research departments are those that employ the highest number of researchers with the highest degree of prestige. This is another typical example of a situation in which quantity (or just size here) and quality do matter. However, there may be some debate as to how to evaluate each since merely summing up the total amount of prestige enjoyed by individuals would not be a convincing solution. Again, this issue is implied in the assumptions that can be made about the form of the function of evaluation of the prestige of each member. For instance, assuming that individual prestige contribution to the prestige of the department is convex, seems to capture the intuition that research departments need to hire a number of academic leaders likely to contribute more than proportionally (through their own prestige) to the prestige of the department. We develop this application using the Rep-Ec dataset and by comparing the economics research departments positioned among the world's top-five percent. Quantity is measured through the number of (registered) members. The prestige of each member is proxied by the number of citations their papers receive. This use of citation data here represents a difference from the previous application in that it takes into account all the papers authored by the scholars currently employed in each institution considered (articles often written before their authors were actually employed in the institution in question)[11] and all the citations these papers received.

One may think of several other contexts in which both quality and quantity matter. For instance, social clubs care about both the number of members and the latter's social status. Schools also care about the numbers of students they train and their future income. Museums value both the number of paintings and their importance in the history of arts (of course making the strong assumption that such a quality index can be unambiguously assessed). Here, convexity would capture the intuition that museums need to acquire some high quality paintings that provide a more than proportional contribution to the reputation of the museum. For instance, Da Vinci's Mona Lisa's contribution to the reputation of the Louvre museum may be more than proportional to its real importance in the history of arts.

The article is organized as follows. Our basic theory of extended dominance relations is developed in the next section. The third section presents how two-by-two relations of dominance can be turned into dominance networks and rankings. In the fourth section, we show how this theory can be used to compare the scientific production of research institutions and apply it to top US research universities. The fifth section is dedicated to

[11] This cumulative way of computing production was used by Combes and Linnemer (2003) in their study on the ranking of European economic departments.

the comparison and the ranking of the world's top research departments in economics. The last section concludes.

2 The extended theory of dominance relations

2.1 Notations

Let us define a set I of n agents $i = 1, ..., n$, which can denote either individuals or institutions. Each item produced by any of these agents is denoted by an index $a = 1, ...n_i$, with n_i the total number of items produced by agent i. Each item a is characterized by an associated quality measure $s_a \in S$, with S ($\subseteq \mathbb{R}^+$) the set of all possible quality measures. The outcomes of agent i is described by a $1 \times n_i$ vector $s_i := (s^i_1, s^i_2, ..., s^i_a, ..., s^i_{n_i})$. Let us now define $f_i(s)$ the production performance of i with quality s:

$$f_i(s) := \sum_{a=1,...,n_i} 1\left\{s^i_a = s\right\}. \tag{1}$$

with $1\{.\}$ the indicator function which is equal to one if the condition into brackets is verified and zero otherwise. The conditional distribution $\{f_i(s), \forall s \in S\}$ describes the production of agent i.

The valuation function $v(\cdot) : S \to \mathbb{R}$ gives the "value" of any unit item as a function of its quality. The value of the whole production performance of agent i is given by :

$$V_i = \sum_{s \in S, s \leq \bar{s}} v(s) f_i(s). \tag{2}$$

with $\bar{s} = \min s > \max_{i \in I} \max_{j=1,...,n_i} s^i_j$, the lowest quality, no item produced by any agent in set I reached (it provides a strict upper bound to quality production in I).

2.2 Dominance relations

We now introduce four dominance relations: strong dominance, dominance, convex-weak dominance and concave-weak dominance. To each specification of the value function can be associated a value judgement, that is a particular assessment of the quantity and quality. And each dominance relation requires unanimity within a particular category of judgement. Thus, Definitions 1 to 3 require that the total value of an institution's production be superior to that of another institution for any function $v(s)$, within clearly defined classes, for it to be dominant. Theorems 1 to 3 establish the necessary and sufficient conditions for each dominance relation to hold.

Let us define the notion of strong dominance over the set of agents I. This dominance relation only requires that the function $v(\cdot)$ be non negative, i.e. no item will contribute negatively to the performance of any agent.

Definition 1 *The production of agent i strongly dominates that of agent j, noted $i \blacktriangleright j$, if, for any non negative function $v(\cdot)$ over set S, $V_i \geq V_j$.*

Theorem 1 *$i \blacktriangleright j$ if and only if $\forall x \in S$ and $x \in [0, \bar{s}[$, $f_i(x) - f_j(x) \geq 0$.*

Proof. *See Appendix A.*

Theorem 1 simply means that the necessary and sufficient condition for there to be a strong dominance of one institution over another is that it does not perform less for any possible level of quality. This condition is intuitive since strong dominance requires unanimity of judgement for any non negative value function, which may arbitrarily increase the value of any positive level of quality.

We now turn to the notion of dominance which requires unanimity among any non negative and now also non decreasing functions $v(\cdot)$, that is to say that articles of a higher quality shall never have a lower value.

Definition 2 *The production of agent i dominates that of agent j, noted $i \rhd j$, if, for any non negative and non decreasing function $v(\cdot)$ over set S, $V_i \geq V_j$.*

Theorem 2 *$i \rhd j$ if and only if $\forall x \in S$ and $x \in [0, \bar{s}[$, $\sum_{s \in S, x \leq s \leq \bar{s}} (f_i(s) - f_j(s)) \geq 0$.*

Proof. *See Appendix A.*

Two additional hypotheses can be introduced relative to the second derivative of the value function. Definition 3 introduces a notion of dominance which requires the convexity of $v(\cdot)$ while Definition 4 alternatively requires concavity.

Definition 3 *The production of agent i convex-weakly dominates that of agent j, noted $i \unrhd j$, if, for any non negative, non decreasing and weakly convex function $v(\cdot)$ over set S, $V_i \geq V_j$.*

Definition 4 *The production of agent i concave-weakly dominates that of agent j, noted $i \overline{\rhd} j$, if, for any non negative, non decreasing and weakly concave function $v(\cdot)$ over set S, $V_i \geq V_j$.*

We now have two statements synthesized in the following theorem.

Theorem 3 *The two following statements hold:*

i) $i \trianglerighteq j$ *if and only if* $\forall x \in S$ *and* $x \in [0, \bar{s}[, \sum_{s \in S, x \leq s \leq \bar{s}} s \left[f_i(s) - f_j(s)\right] \geq 0.$

ii) $i \overline{\triangleright} j$ *if and only if* $\forall x \in S$ *and* $x \in [0, \bar{s}[, \ \sum_{s \in S, 0 \leq s \leq x} s \left[f_i(s) - f_j(s)\right] \geq 0.$

Proof. *See Appendix A.*

Since in the applications, the convexity of the value functions is more admissible than concavity, we shall only use, in Sections 4 and 5, convex-weak dominance which for ease of reading we will call weak dominance.

With these three theorems in hands, we will, as is usually done in the stochastic dominance literature, be able to assess comparisons between pairs of institutions.

2.3 Some basic properties of dominance relations

We describe here some simple properties of the dominance relations that will prove to be useful in the next sections: all dominance relations have in common the transitivity property and there are some natural causal relations between dominance relations. Before doing so, we need to define a comparison principle between dominance relations. A dominance relation $\succ$ is stronger than any other dominance relation $\succ'$, noted $\succ \gg \succ'$, if, $\forall i, j$, $i \succ j$ implies $i \succ' j$. The symbols $\succ$ and $\succ'$ account for any one of the dominance relations introduced above ($\succ, \succ' \in \{\blacktriangleright, \triangleright, \trianglerighteq, \overline{\triangleright}\}$). A dominance relation is stronger than an other if a dominance assessment of the former type between any two agents necessarily implies a dominance of the latter type between these two agents. The proposition below sums up the properties.

Proposition 1 *The two following statements hold:*

i) if $i \succ j$ *and* $j \succ h$*, then* $i \succ h$, $\forall \succ \in \{\blacktriangleright, \triangleright, \trianglerighteq, \overline{\triangleright}\}$

ii) $\blacktriangleright \gg \triangleright, \triangleright \gg \trianglerighteq$, *and* $\triangleright \gg \overline{\triangleright}$.

Proof. *The proofs derive straightforwardly from definitions 1 to 4.*

Part i) of the proposition simply establishes that all the dominance relations introduced are transitive. Part ii) of the proposition means that the weaker a dominance relation the more dominance relations it is possible to establish between the agents of any given set of agents I.

3 Dominance networks and rankings

We now focus on the systematic use of the dominance relations for the implementation of ranking procedures. We first define dominance networks which make it possible to formally describe and represent the architecture of the dominance relations between the agents. We shall afterwards introduce two types of dominance ranking : complete dominance ranking and pseudo dominance ranking.

3.1 Directed dominance networks

Let us consider $\succ$, which could be any one of the dominance relations examined above. Let us build the (directed) dominance network $\vec{g}_{\succ}$ associated to dominance relation $\succ$ and agents set I by establishing a directed link from any agent $i \in I$ to an agent $j \in I$ ($j \neq i$) if i dominates j according to $\succ$. That is formally: $\forall i, j \in I, ij \in \vec{g}_{\succ}$ iff $i \succ j$.

In this network, the strictly transitive dominance triplets are uninformative since we know that transitivity always holds. Therefore, for clarity purposes, it is convenient to define the adjusted dominance networks constructed from the dominance networks by eliminating such triplets. To build such a network $\vec{g}'_{\succ}$, we begin by assigning a link from i to j in $\vec{g}'_{\succ}$ if $ij \in \vec{g}_{\succ}$. But some links are deleted according to the following rule: $\forall i, j, h \in I$, if $ij, jh \in \vec{g}_{\succ}$ and $hj \notin \vec{g}_{\succ}$ then $ih \notin \vec{g}'_{\succ}$. The condition whereby h must not dominate j enables us to avoid eliminating the link from i to h when j and h dominate each other (which basically means they have identical productions).

3.2 Complete dominance rankings

A ranking over a set I is a vector of dimension $n = \#I$, $r_I = (r_i)_{i=1,\ldots n}$, each unitary element $r_i \in \mathbb{N}$ of which is the rank of agent i.[12] A ranking r_I is said to be a $\succ$-complete ranking if $\forall i, j \in I$, $r_i < r_j$ iff $i \succ j$. This means that a difference observed in the ranks of any pair of institutions can only be based on a dominance relation between these institutions, and vice versa. Nevertheless not all dominance relations can constitute the basis of a complete ranking. We shall show that such complete ranking over set I can only be constructed on the basis of an I-complete dominance relation defined below.

Definition 5 *A dominance relation $\succ$ is said to be I-complete if $\forall i, j \in I$, if $i \not\succ j$ then $j \succ i$.*

When a dominance relation is complete over a set I, then one can always establish (at least) one such dominance relation between any two agents of I. Note that this definition is equivalent to saying that relation $\succ$ is a total preorder over set I. Then, ranking these agents becomes an easy and unambiguous task on the basis of that dominance relation.

[12] If $i \succ j$ and $j \succ i$ then i and j are *ex aequos*, that is $r_i = r_j$. The rank of *ex aequos* is set to the minimum among all the possible ranks.

Lemma 4 *A complete $\succ$ ranking $r_I = (r_i)_{i=1,\ldots n}$ can be constructed over agents set I iff the dominance relation $\succ$ is I-complete.*

Proof. *If $\succ$ is an I-complete dominance relation then $r_i < r_j$ or (and, when i and j are ex aequos) $r_j < r_i, \forall i, j \in I$ and thus a complete ranking can be established. Concerning the reverse implication, if a ranking can be established on agents set I, it can be inferred that $r_i < r_j$ or (and, when i and j when ex aequos) $r_j < r_i, \forall i, j \in I$ which is equivalent to $i \succ j$ or (and) $j \succ i$, and thus relation $\succ$ is an I-complete dominance relation.*$\square$

The dominance network $\vec{g}_{\succ}$ associated with any I-complete dominance relation $\succ$ is such that $\forall i, j \in I$, with $i \neq j$, then $ij \in \vec{g}_{\succ}$ or $ji \in \vec{g}_{\succ}$. Then the associated adjusted dominance network $\vec{g}'_{\succ}$ is a chain (if there are no mutual domination, that is $i \succ j$ and $j \succ i$, which occur when two institutions have identical productions) which permits a natural and unambiguous ranking.

3.3 Pseudo dominance rankings

A more indirect procedure can be established to build a ranking from bilateral dominance relations when the associated dominance relation $\succ$ is not I-complete. It relies on some scores that are computed thanks to that dominance relation $\succ$ on set I. In the pseudo dominance ranking defined below, we propose two criteria. The dominant criterion is the number of agents within the population (excluding itself) the considered agent dominates (by decreasing order). The second criterion is the number of agents (excluding itself) that dominate this institution (by increasing order).

Definition 6 *A ranking $\sigma_I = (\sigma_i)_{i=1,\ldots n}$ is a pseudo $\succ$-ranking over a given set of agents I if firstly, it is based on the number of dominance relations of type $\succ$ which emanate from each agent over the remaining other agents in I (dominant criterion). That is, the ranks are such that $\sigma_i < \sigma_j$ if $n_i > n_j$, with $n_i = \# \{ j \in I \,| j \neq i, i \succ j \} , \forall i \in I$. It shall also be based on the scores in the second criterion of any i is $m_i = \# \{ j \in I \,| j \neq i, j \succ i \}$, as follows: if $n_i = n_j$, then $\sigma_i < \sigma_j$ if $m_i < m_j$.*

Pseudo dominance rankings are less reliable than the complete dominance rankings but have the considerable advantage that they can always be produced on the basis of any given dominance relation. The pseudo dominance ranking of any institution can be interpreted as a structural measurement of its position in the (direct) adjusted dominance network $\vec{g}'_{\succ}$.

4 Comparing and ranking universities' research

In this section, we show how the general theory introduced above applies to the comparison of the scientific production of various institutions. This requires that some specifications be made concerning the measurement of scientific productions and of their impact, which will constitute a basis for computing quality in this context. A discussion of the appropriate assumptions for the value function is also in order before presenting the data and the results.

4.1 Scientific production

The index a now denotes an article in A the set of all articles. An impact measure $x_a \in \mathbb{R}^+$ is associated to each a, and the $1 \times n_i$ vector $s_i := (s_1^i, s_2^i, ..., s_{n_i}^i)$ describes the publication/impact production of university i, for all $i \in I$, the set of all universities considered. The question of how the impact can and should be measured is discussed in the following subsection. Since the average impact varies significantly across disciplines, it is appropriate to first define $g_i^k(x)$, the publication performance of i with impact x in discipline $k \in K$, the set of all scientific disciplines:

$$g_i^k(x) := \sum_{a=1,...,n_i} \{x_a = x\} \cdot p_a^i \cdot q_a^k. \tag{3}$$

The term $p_a^i \in [0, 1]$ accounts for the fact that in practice most articles are attributed to several universities (since its authors are often employed by different universities; either one author is employed by several institutions or different authors are employed by different institutions). In practice, it is impossible to know the precise affiliations of authors, and one can only count the number of times an institution is referred to in the article. An article a, referencing at least one address associated to institution i, provides institution i with a gross volume of academic production of:

$$p_a^i := \frac{\#\{i \in \Delta(a)\}}{\#\Delta(a)}. \tag{4}$$

The expression $\#\{.\}$ denotes the cardinal of the set into brackets. The term $\Delta(a)$ is the set of references to institutions as listed by the authors of a. It can mention several times the same institution and so $\#\{i \in \Delta(a)\}$ counts the number of times i is mentioned in the list of institutions of article a. The right hand side of the equation indicates the weight of institution i among the various institutions mentioned by the authors of article a. So for instance, if three authors co-author an article, if two of them mention institution i

as their affiliated institution and if the third author mentions another institution, the ratio will be equal to 2/3.

The term $q_a^k \in [0, 1]$ accounts for the fact that not all papers are associated to discipline k, and that those that are, are not necessarily exclusively associated to discipline k, and then the weight of discipline k in article a shall be computed:

$$q_a^k := \frac{1\{k \in d(j(a))\}}{\#d(j(a))}. \tag{5}$$

Typically, in the scientometric databases, the information on disciplines comes through the journals. The term $j(a) \subset A$ denotes the subset of all papers published in the same journal as a. The term $d(j)$ is the set of disciplines to which journal j is to be associated to. Thus, q_a^k serves as a filter for selecting the articles related to discipline k, through the association of the journal in which it was published to one or several disciplines, and it helps give weight to discipline k when the journal is related to several disciplines.

4.2 Impact

Three proxies of articles' impact are proposed here. First, it can be measured by counting, for each article, the number of citations received in a given time window after publication. It is computed as follows:

$$x_a := \#\{u | \, t_u \in w(a) \text{ and } a \in r(u)\}, \tag{6}$$

with t_u the year of publication of article u, and $w(a)$ the citation window of article a (we use three year citation windows in practice) and $r(u)$ the reference list of article u. This measure of impact is very attractive because it measures the impact of each article directly. Its shortcoming is that it is also noisy, since some articles do attract a considerable amount of citations, not only because of their real scientific contribution, but also because of the modes of citation, or because of their nature (review papers).

One may alternatively consider the impact of the journals such as its impact factor as an appropriate (though more indirect) measure for scientific impact. It is computed as the average number of citations received by the articles published in the journal:

$$x'_a := \frac{\sum_{h \in j(a)} \#\{u | \, t_u \in w(h) \text{ and } h \in r(u)\}}{\#j(a)}, \tag{7}$$

with $\#j(a)$ the number of articles published in the journal in which article a appeared, and at the numerator, the total number of citations received by these articles. This measure of impact helps to better evaluate one's capacity to publish in well-established journals with large readerships. Clearly, the universities that perform well -when impact is computed this way- have a high academic reputation in the largest communities of the discipline, as shown by their ability to publish in the most visible journals. This measure (as well as the former one) has the drawback of favouring the most prominent specialties or communities sub-disciplines).

The last measure is intended to correct for such a potential bias. The last measure of visibility is the relative impact factor, that is the journal's impact factor benchmarked by the average impact factor in the specialty. More formally, it is computed as follows:

$$x''_a := \frac{x'_a}{\frac{1}{\#\varphi(j(a))} \sum_{\alpha \in \varphi(j(a))} \langle x'_a \rangle_\alpha}, \tag{8}$$

with $\varphi(j)$ the set of specialties to which j is associated and $\langle \cdot \rangle_\alpha$ denoting the average within set α. Such a measure is particularly useful when one aims to account for the ability to publish in the best journals of given fields because it controls for the ability to choose the most visible fields. Such a measure also controls for the various citation practices of the various specialties of the same discipline (e.g. applied and fundamental mathematics have different citation practices).

4.3 Quality

The simplest way of dealing with quality in this context would be to assume that impact is the right measure of quality. There are, however, good reasons preventing us from making this assumption. The main one is that impact varies dramatically among disciplines simply because citation practices vary across disciplines. For instance the average size of reference lists in chemistry is greater than in mathematics and thus the average impact is higher. Therefore impact can not be a reliable measure of quality per se. We propose to measure articles' quality through their relative position in the distribution of articles (according to their impact) within their corresponding discipline. In concrete terms, the quality of a given paper will be x if its impact is at least as high as that of x percent of the articles published in the discipline. This will enable us to aggregate articles across disciplines for all quality levels. Of course this subtlety is not necessary when we limit ourselves to comparisons of universities in one discipline.

Let $\left\{\phi^k(\cdot), \forall s \in S\right\}$ be the density distribution in discipline k of all production according to an impact measure scaled by s and $\Phi^k(.)$ the associated cumulative distribution. Let us now also define the conditional distribution $\left\{f_i^k(x), \forall x \in [0,1]\right\}$, with $f_i^k(x)$ the production performance of i the impact of which is exactly equal or inferior to x percent of the world production ranked (the impact of which is equal or inferior to $s = \left(\Phi^k\right)^{-1}(x)$) in discipline k. Therefore, by this definition of f_i and by the definition of g_i (see Equation 3), we have: $f_i^k\left(\Phi^k(s)\right) = g_i^k(s), \forall s \in S$. We can now meaningfully compute the scientific production of each institution i for any level of quality $x = \Phi^k(s^k), \forall k \in K$, by summing over all disciplines as follows:

$$f_i(x) := \sum_{k \in K} f_i^k(x), \forall x \in [0,1]. \tag{9}$$

This gives the scientific production of institution i that has the same impact as (or greater than) x percent of all articles in their associated discipline. The interdisciplinary conditional distribution of institution i is then $\{f_i(x), \forall x \in [0,1]\}$.

4.4 Value

Let us now redefine slightly function $v(\cdot) : [0,1] \to \mathbb{R}$ as the valuation function which gives the "value" of any unit of scientific production as a function of its position x in the distribution of quality s in its associated discipline. Then, the value of the whole production performance of agent i in discipline k is simply:

$$V_i^k = \sum_{x \in [0,1]} v(x) f_i^k(x). \tag{10}$$

The value of the whole publication performance of institution i can be computed either directly or by aggregating the values over all fields:

$$\begin{aligned} V_i &:= \sum_{x \in [0,1]} v(x) f_i(x) \\ &= \sum_{x \in [0,1]} v(x) \sum_{k \in K} f_i^k(x) \\ &= \sum_k V_i^k. \end{aligned} \tag{11}$$

The establishment of dominance relations between universities is therefore a natural extension of the general theory presented in Section 2. If one focuses on disciplinary comparisons, the publications data are the associated $f_i^k(x)$ and the corresponding values the V_i^k. When one focuses on the interdisciplinary comparisons, the publications data are the $f_i(x)$ and the corresponding values are the V_i.

Let us turn to the various assumptions for function $v(\cdot)$. It seems more than reasonable to assume that one additional article or an article of a higher quality can never decrease the total value of any scientific production. This is equivalent to assuming that the value function of any article is positive and non-decreasing with its quality. Hypotheses concerning the second derivative of the value function are more debatable. However, it appears implicitly or explicitly in most of the interviews conducted with several rectors and presidents of universities that convexity is a relatively widely accepted hypothesis, once it has been clarified with them. University CEOs and their trustees usually attribute a more than proportional weight to productions in the higher segments of impact distribution whereas little-cited papers tend to be less than proportionally considered. This focus on excellence seems to be common to the research universities, while other universities may have a broader focus.

4.5 The Data

A set of the top US universities were selected on the basis of their rank in the Academic Ranking of World Universities (ARWU) produced by Shanghai Jiao Tong University. This ranking is well known to be "research oriented", a specificity which - though based on very different premises to ours - fits well with them. Our goal is to restrict our analysis to research universities, and so a number of universities representing about 30% of all Ph.D. granting universities in the US was selected, that is 112 universities.

The publications of these institutions[13] and the citations these publications received have been collected in the Thomson-Reuters-Web of Science (WoS) database.[14] Since the publication data are available only from 2003 forward, and the citations data are only available up to the year 2007, this analysis was carried out using a set of smoothed data (from 2003 thr. 2005), with a 3-year citation window for each of these publication years. Over the period of observation and for the citation-window selected, the scientific production of the 112 universities/institutions considered in this experiment amounts to $329,910$ articles published in the journals referenced in the WoS database, journals which received $2,316,576$ citations. The citation scores achieved by these papers are between 0 and $1,292$ and the impact factor of the associated journals varies from 0 to 28 (all within the three-year citation window).

As we have seen above,the assignment of the papers to disciplines is based on an association of journals to nine categories of disciplines (see Table 1). The first eight

[13] The lexical tokens which were used to collect publications have been nicely provided by Cheng and Zitt (2009).

[14] These data are imported and maintained by the Observatoire des Sciences et Techniques (OST) for national evaluation purposes and research and thus all computations (citations, impact factors...) are performed in house.

correspond to clear disciplinary lines of inquiry, whereas the ninth, labelled Multidisciplinary Sciences, groups together journals that have a truly interdisciplinary focus and some large multidisciplinary journals that publish articles that pertain to several disciplines. In the disciplinary based comparisons, excluding the papers published in such large journals would introduce a significant bias since it would eliminate a significant percentage of the best articles of several disciplines. Therefore, the articles published in the most influential of these multidisciplinary journals (namely Proceedings of the National Academy of Sciences USA, Science and Nature) have been reallocated to their parent discipline thanks to a lexicographic work.

As above-mentioned, the impact of universities' publications is considered in three different manners: through the direct citations received by the articles, through the direct impact of the journals in which the considered articles were published, and through the relative impact, that is the impact factor of the journal relative to the average impact factor of the specialty to which the journal belongs. This measure helps correct for the different citation practices across subject categories within the same discipline (e.g. between applied and fundamental mathematics). Lastly, the scientific production curves of each institution were linearized in twenty points positioned at equal intervals between zero and the maximum impact reached.

4.6 Results

The first result proposed concerns the extent to which the various dominance relations do allow us to compare universities. For this purpose, we now introduce the notion of rate of completeness of dominance relations over institutions set I, of cardinal n, defined as follows:

$$C_{I,\succ} = \frac{\#\left\{(i,j) \in I^2 \,|i > j, i \succ j \text{ or } j \succ i\right\}}{\#\left\{(i,j) \in I^2 \,|i > j\right\}} = \frac{\#\left\{(i,j) \in I^2 \,|i > j, i \succ j \text{ or } j \succ i\right\}}{n(n-1)/2}, \tag{12}$$

which simply indicates the percentage of pairs of (distinct) institutions for which one can establish (at least) one dominance relation of type $\succ$. Table 2 presents the rates of completeness in eight first large disciplines and for all disciplines, associated to dominance relations $\blacktriangleright$, $\triangleright$, and $\trianglerighteq$. The information on completeness of dominance relations is reported according to the three proxies used for impact. The results show that completeness varies across domains and depends on the type of dominance, the type of impact, and the discipline. Of course, a weak dominance relation achieves significantly higher rates than other types of dominance, regardless of the domain. The rate of completeness of a weak dominance is most of the time close to ninety percent. The rate of completeness is also slightly greater when the citations are considered, which was also expected since direct citations are more unevenly distributed than impact factors. Completeness is minimal for mathematics and maximal in medicine. All are strictly below the unity and therefore we cannot establish complete dominance rankings from the different types of dominance relations exposed here. However, completeness is often very high and it seems reasonable to produce a pseudo complete dominance ranking, as defined in Subsection 3.3, especially weak dominance.

Tables 3, 4 and 5 provide the top-50 of the pseudo dominance rankings associated to strong dominance, dominance, and weak dominance relations when considering all disciplines. Table 3 presents pseudo rankings based on the direct citations received by the articles, Table 4 on the direct impact factors of journals and Table 5 on the relative impact factors. Two columns, rank (σ_i) and #dom (n_i), are reported for each dominance relation: rank is the appropriate ranking and #dom the number of universities/institutions dominated by a given university/institution. When institutions have equal scores in this dominant criterion, the second criterion is used by default to rank institutions (the number of institutions which dominate it by increasing order - see Definition 6).

Though unreported Spearman rank correlations indicate that pseudo rankings built on the three dominance relations reveal a significant correlation, some institutions do however have very different ranks depending on the associated dominance relation. For instance, MIT is not in the top-50 group when ranking is based on strong dominance, whereas it is in the ninth position in the pseudo weak dominance ranking (see Table 3). Of course this result is to be interpreted taking into account the size of this institution. The weak dominance relation gives a chance to excellent but smaller institutions to remain in the top of the (pseudo) ranking. Interestingly some institutions have significantly different ranks when different measures of impact are used. For instance, Berkeley ranks sixth and fifth in dominance and weak dominance relations when impact is measured through direct citations, while it ranks third and second in both types of dominance relations, but using the impact factor as a measure of impact. This means that scholars in Berkeley do particularly well at publishing articles in the most important journals. When the relative impact factor is used to proxy impact, Berkeley moves down to fourth position in both rankings. This indicates that Berkeley scholars, are not only excellent at getting their papers published in the best journals within their given specialties (doing the "job right"), but also at selecting sub-fields that attract more attention (the "right job"). For each impact measure and each form of dominance considered (except in the case of strong dominance), Harvard dominates all other universities and ranks first.[15]

It is interesting to not limit the investigations to the pseudo-ranking results, and to picture their associated dominance networks, which highlight the architecture of dominance relations. Figure 1 presents the adjusted dominance network associated to weak dominance ($\vec{g}'_{\trianglerighteq}$) among the top institutions, measuring the impact with the number of direct citations. We observe that just below Harvard, the dominance structure is more sophisticated than expected. As a matter of fact no dominance relation can be found between Michigan Univ (at Ann Harbor), Seattle, UCLA and Stanford. Stanford is better ranked than the other three because it dominates Berkeley and MIT while the others do not. MIT does better than the University of Pennsylvania on the second criterion: it is immune to Seattle's, UCLA's and Berkeley's domination while the University of Pennsylvania is not.

5 Ranking academic departments according to their prestige

In this section we apply the extended stochastic dominance theory to the comparison of academic departments according to their prestige. The basic assumption we make here is that the prestige of the department rests upon the prestige of its present members: the prestige of past members and of the department itself are not taken into account. Thus, two questions arise: how shall (and can) individuals' prestige be defined and how does the aggregated prestige of individuals form the departments' prestige?

The scientific prestige of a scholar is the recognition by the community of its interest in his work - this is what R. K. Merton called credit.[16] Prizes, honorary lectures, invitations and more generally all distinctions based on peer-reviews may provide useful information on such a credit. However this information turns out to be heterogeneous and difficult to handle in a systematic and quantitative study. R. K. Merton himself argued that the accumulated academic credit can be approximated by direct citations. This idea was extended and formalized by scientometricians such as Garfield (1963) and Price (1965).

If citations were also used for the comparison of academic institutions' scientific productions in the previous section, this approach is different in several respects. First,

[15] All results, which can not be reported in the present paper, are avaible at http://carayol.u-bordeaux4.fr/ranking.html.

[16] Cf. his collected articles in Merton (1973).

it is no longer the flow of production that is taken into account, but rather the credit a scholar accumulates over his career, not only in his present institution but also in the ones he was previously employed in. Therefore we shall not limit ourselves to the papers produced in the current period but rather take into account all the papers ever published. Secondly, citations also should not be limited to a given window period after publication. Indeed, citations to old articles are also very informative on the importance of these papers in the literature and thus on the scientific prestige of the author.

Let us now consider the question of the aggregation of individuals' prestige constituting the departments' prestige. A department i is now described on the basis of the prestige of each of its members through vector $s_i = \left(s_a^i\right)_{a=1,\ldots n_i}$ where a denotes a scholar and n_i is the number of members of the department. Now $f_i(s)$ denotes the number of members in department i with prestige s for any possible level of prestige $s \geq 0$ and is computed as stated in equation (1). Let also the prestige of the department i be given by V_i as stated in equation (2). Again, clarifying the premises associated with the aggregation boils down to formulating assumptions on function $v(\cdot)$, thereby defining the class of functions among which unanimity of judgement shall be imposed to infer a dominance relation.

Hiring scholars of higher individual prestige and hiring more scholars with a given level of prestige should both have a positive influence on the prestige of the department. Since both size and individual prestige are positively valued ($v(\cdot)$ shall be positive and non-decreasing), clearly strong dominance and dominance relations are based on acceptable assumptions (since $v(\cdot)$ shall be positive for strong dominance and positive and non-decreasing for dominance). Assumptions on the second derivative (if any) are more debatable. However, anecdotal evidence suggests that the prestige of the "stars" hired by the department contributes more than proportionally to that of the department. Indeed, it is often mentioned that a key issue for a department is to hire at least one of these very influential scholars, the prestige and reputation of whom can serve as foundations for building internal research dynamics, raising significant external funding and attracting attention from the academic community. If this intuition were accepted, then the convexity assumption would also be retained and the most accurate extended stochastic dominance is weak dominance.

5.1 Data

The data were collected from the Ideas-RepEc[17] website in June 2010 using a computerized data collection procedure. We collected data on all registered members but the study is limited to those affiliated to at least one of the economics departments ranked among the top 5% in the world as listed in the Rep-Ec database itself (239 departments). We rely on the Rep-Ec selection of departments, based on the aggregation of all measurements provided by this service. It turns out that $10,465$ registered members are affiliated to these 239 departments (out of more than $25,500$ registered authors in the Rep-Ec database), that is more than 40% of all members. The average department is composed of 43.7 members, who authored a total of 695 papers and received $11,414$ citations. By paper here we mean an article published in a journal or in working papers series, chapters of books, books and software components. Of course, a paper may appear in different formats, and double counting is then corrected by an automated recognition of identical titles and possible decisions of the authors. Citations may be made of a working paper or the published version of the paper but are not attributed to both, and thus the double counting cannot occur. Citations are collected in the reference lists of these papers.[18]

The membership to a department is declarative and therefore there might be some difference between the real membership of a department and the Rep-Ec membership. That difference may be essentially due to the fact that some scholars decided not to declare their membership to Rep-Ec. Although we are aware of this, we are inclined to think that the difference is very limited, especially among the top departments of economics. The procedure being declarative, this implies that Rep-Ec includes as members Ph.D. students and all non-permanent members of the department.

The boundaries of the institutions are based on their own definitions and different levels of aggregation coexist. Some registered institutions are just aggregations of other institutions. We do not consider them, since we decided to aggregate our data at the lowest possible level. This means, for instance, that economists in a business school are not aggregated with the economists of the economics department of the same university (if they did not declare their affiliation to both departments) which are considered different entities. We also observe that a limited number of scholars are affiliated to several institutions. We have chosen to attribute each of these multi affiliated scholars

[17] See http://ideas.repec.org.

[18] It should be noted that the citations made in all reference lists are not yet fully taken into account and thus the citation data is clearly not as complete as that used for the ranking of universities in the previous section.

to each and every institution they belong to since we did not find a more suitable way of dividing authors across departments. The difficulty here comes from the fact that multi-affiliation corresponds to very different situations. For instance, some institutions, such as the NBER, CEPR and IZA, are not "real" departments and it would be difficult to argue that being affiliated to one of these institutions and to a university is similar to being affiliated to two different universities. Lastly, the profile of each department is represented by points positioned at the median of twenty equal size intervals between zero and the maximum level of prestige reached.

5.2 Results

The presentation of ranking of academic departments is limited to the first 50 departments (see Table 7) but complete results are also available.[19] It presents the scores obtained depending on the dominant criterion used (n_i, the number of institutions dominated by the institution concerned) and pseudo rankings (σ_i) of these institutions according to strong dominance, dominance and weak dominance relations. It is interesting to compare the ranking of European departments in the dominance and weak dominance rankings. For instance, fifteen European departments are in the top fifty institutions and seven are in the top twenty, when the ranking is based on dominance relations. These figures decrease to nine in the top fifty and three in the top twenty when ranking is based on weak dominance. This leads to the remark that European departments are well ranked when one focuses on quality, but when excellence is the focus, the best US departments perform better than their European counterparts.

However, for a precise analysis we believe that one must only take into consideration the pseudo-rankings based on weak dominance relations (if one believes in the associated assumptions on the implicit value function), because, as Table (6) shows, the weak dominance relation is the only one with an acceptable rate of completeness (.75) while completeness drops to .40 and even .08 in the case of dominance and strong dominance relations.

The three specific institutions NBER, CPER and IZA are in the best three positions in the pseudo ranking associated to weak dominance. This result is unsurprising and has little significance since these institutions are not economics departments in the classical sense. The Harvard economics department ranks third ex aequo with IZA. The Princeton economics department ranks fifth, followed by the economics departments of Princeton, Berkeley and Chicago. LSE and Oxford follow.

[19] Again at http://carayol.u-bordeaux4.fr/ranking.html.

Again, it is useful to examine the architecture of weak dominance relations among top departments using the adjusted weak dominance networks ($\vec{g}'_{\trianglerighteq}$) exposed in Figure 2. Interestingly, aside from the dominance relations between the very top institutions, there appears to be a parallel channel of dominance that goes directly from the Harvard department of economics and IZA to the World Bank and even to CESifo. The latter institution is such a large institution that only the top four institutions do dominate it weakly. However, it does not employ enough highly cited scholars to be able to dominate forty eight institutions. This is why it is ranked thirty seventh, as compared to the Princeton economics department, for example, which is ranked fourth while no dominance relations can be established between these two institutions. A similar comparison can be established between the MIT economics department and the Tinbergen Institute: while no dominance can be established between the two, the former ranks twelfth while the latter ranks thirty eighth. There is a marked opposition between different types of institutions with strong positions either because they employ a limited number of highly prestigious scholars or a large number of less prestigious scholars.

6 Conclusion

This article introduces a new theory for establishing dominance relations; it is an extension of the well-known stochastic dominance theory. We have applied this theory for comparing the scientific production of US research universities and for comparing the prestige of academic departments of economics.

Our results highlight that this theory provides an original solution for the treatment of the size effect in the comparisons of scientific institutions. Though our tool is not size-independent (simply because it is not a desired implicit assumption), it does however give small institutions that perform well in terms of quality, the opportunity to compete with larger institutions.

We also believe that this theory has a great application potential because in many situations, quality and quantity are relevant for making comparisons; not so much in order to produce new rankings (for which the social demand has been high in recent years), but because it helps to better understand and discuss their premises, which more often than not are implicit. Therefore, such comparisons may become truly useful to the users and to the evaluated institutions. In this article we have not been able to describe the two applications in much detail but have done so in more applied papers (building reference classes, focusing on specific parts of the quality distribution...). More precise information can be produced with this theory, which hopefully helps to provide tools that can be used for benchmarking universities.

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8 Appendix A.

8.1 Proof of Theorem 1

The *if and only if* statement shall be proven by demonstrating that the causality holds both ways.

a) We first demonstrate the left-to-right implication: $i \blacktriangleright j \rightarrow \forall x \in [0, \bar{s}[\,, f_i(x) \geq f_j(x)$. Let us consider that $i \blacktriangleright j$ holds and let us further assume that there exists an $x_0 \in [0, \bar{s}[$ such that $f_i(x_0) < f_j(x_0)$. Given the latter statement, one can always find a non negative function $v(.)$ such that $V_i < V_j$. For instance, if $v(.)$ is such that $v(x_0) > 0$ and $v(x) = 0$ otherwise, then obviously $f_i(x_0) < f_j(x_0)$ implies that $\sum_{s \in S, s \leq \bar{s}} v(s) f_i(s) < \sum_{s \in S, s \leq \bar{s}} v(s) f_j(s)$. We thus obtain a contradiction with the initial statement $i \blacktriangleright j$. Thus the inequality $f_i(x) \geq f_j(x)$ is always verified when i strongly dominates j.$\square$

b) The right-to-left implication, $\forall x \in [0, \bar{s}[\,, f_i(x) \geq f_j(x) \rightarrow i \blacktriangleright j$, is immediate. Indeed when $\forall x \in [0, \bar{s}[\,, f_i(x) \geq f_j(x)$, we can multiply both sides by any non negative function $v(.)$ and the inequality still holds for all $x \in [0, \bar{s}[$. We can also integrate both sides of the inequality and then we have $\sum_{s \in S, s \leq \bar{s}} v(s) f_i(s) \geq \sum_{s \in S, s \leq \bar{s}} v(s) f_j(s)$, that is i strongly dominates j.$\square$

8.2 Proofs of Theorem 2 and Theorem 3

We here only present the proof of part i) of Theorem 3 because the proofs of Theorem 2 and of part ii) of Theorem 3 follow very similar paths and because it is the most original of the three.

a) We begin by the left-to-right implication: $i \trianglerighteq j \rightarrow \forall x \in [0, \bar{s}[, \sum_{s \in S, x \leq s \leq \bar{s}} s\left[f_i(s) - f_j(s)\right] \geq 0$. We assume the weak dominance of i over j and the existence of an $x \in [0, \bar{s}[$ such that $\sum_{s \in S, x \leq s \leq \bar{s}} s\left[f_i(s) - f_j(s)\right] < 0$. If function $v(.)$ is such that $v(s) = s$ if $s \geq x$ and $v(s) = 0$ otherwise (an increasing weakly convex function), we can deduce that

$\sum_{s\in S,0\leq s\leq \bar{s}} v(s)\left[f_i\left(s\right)\right] < \sum_{s\in S,0\leq s\leq \bar{s}} v(s)\left[f_j\left(s\right)\right]$ since $v(s) = 0$ when $s < x$. This inequality contradicts the initial statement. Accordingly, if i weakly dominates j, the inequality $\sum_{s\in S,x\leq s\leq \bar{s}} s\left[f_i\left(s\right) - f_j\left(s\right)\right] \geq 0$ must be true for all $x \in [0, \bar{s}[$ when $i \rhd j$.

b) Consider now the right-to-left implication: $\forall x \in [0, \bar{s}[\,, \sum_{s\in S,x\leq s\leq \bar{s}} s\left[f_i\left(s\right) - f_j\left(s\right)\right] \geq 0 \rightarrow \quad i \trianglerighteq j$. We first assume that $\forall x \in [0, \bar{s}[\,, \sum_{s\in S,x\leq s\leq \bar{s}} s\left[f_i\left(s\right) - f_j\left(s\right)\right] \geq 0$. Let us further consider the two alternative situations. First, there may exist some positive $s_0 < \bar{s}$ such that $s_0 = \arg\min_x \forall s \in [x, \bar{s}[\,, \left(f_i\left(s\right) - f_j\left(s\right)\right) < 0$. Then we necessarily have $\sum_{s\in S,s_0\leq s\leq \bar{s}} s\left(f_i\left(s\right) - f_j\left(s\right)\right) < 0$ which contradicts the initial statement. If the reverse is true then there exists now some $s_0 \geq 0$ which is the smallest x such that $0 \leq x < \bar{s}$ and $\forall s \in [x, \bar{s}[\,, \left(f_i\left(s\right) - f_j\left(s\right)\right) \geq 0$. If s_0 equals 0, then multiplying both sides of the inequality by any positive functions $v(\cdot)$ and integrating obviously leads to the dominance of i over j. If $s_0 > 0$, then let's define s_1 the smallest value such that $0 \leq s_1 < s_0$, and $\forall s \in [s_1, s_0[\,, \left(f_i\left(s\right) - f_j\left(s\right)\right) < 0$. In other words, this means that i does better than j for some higher quality region (between s_0 and $\bar{s}$), while j does better in a lower quality zone (between s_1 and s_0). If $s_1 = 0$, then the initial statement leads to $\sum_{s\in S,s_1\leq s\leq s_0} s\left(f_i\left(s\right) - f_j\left(s\right)\right) \geq -\sum_{s\in S,s_0\leq s\leq \bar{s}} s\left(f_i\left(s\right) - f_j\left(s\right)\right)$. Since any positive non decreasing and weakly convex function $v(\cdot)$ would put a more than proportional weight to the higher segments of quality, then $\sum_{s\in S,s_1\leq s\leq s_0} v(s)\left(f_i\left(s\right) - f_j\left(s\right)\right) \geq -\sum_{s\in S,s_0\leq s\leq \bar{s}} v(s)\left(f_i\left(s\right) - f_j\left(s\right)\right)$. Here $s_1 = 0$, and thus the dominance of i over j is then obviously verified. If $s_1 > 0$, then let s_2 be the smallest value such that $0 \leq s_2 < s_1$ and $\forall s \in [s_2, s_1[\,, \left(f_i\left(s\right) - f_j\left(s\right)\right) \geq 0$. If $s_2 = 0$, then since $\left(f_i\left(s\right) - f_j\left(s\right)\right) \geq 0$ for all $s \in [s_2, s_1[$, the previous statement naturally extends to this situation. If $s_2 > 0$, then we can again define s_3 as the smallest value such that $0 \leq s_3 < s_2$ and $\forall s \in [s_3, s_2[\,, \left(f_i\left(s\right) - f_j\left(s\right)\right) < 0$. If $s_3 = 0$, the initial statement implies $\sum_{s\in S,s_3\leq s\leq s_2} s\left(f_i\left(s\right) - f_j\left(s\right)\right) \geq \sum_{s\in S,s_2\leq s\leq \bar{s}} s\left(f_i\left(s\right) - f_j\left(s\right)\right)$, and then $\sum_{s\in S,s_3\leq s\leq s_2} v(s)\left(f_i\left(s\right) - f_j\left(s\right)\right) \geq \sum_{s\in S,s_2\leq s\leq \bar{s}} v(s)\left(f_i\left(s\right) - f_j\left(s\right)\right)$ would be true since the function $v(\cdot)$ is positive non decreasing and weakly convex function. Since $s_3 = 0$ in this situation then i dominates j. Otherwise, this reasoning can be repeated recurrently down to some $s_n = 0$. Therefore $i \trianglerighteq j$ if $\sum_{s\in S,x\leq s\leq \bar{s}} sf_i\left(s\right) \geq \sum_{s\in S,x\leq s\leq \bar{s}} sf_j\left(s\right)$ for all $x \in [0, \bar{s}[.\square$

9 Appendix B. Tables and figures

k	Domain
1	Fundamental biology
2	Medicine
3	Applied biology/ecology
4	Chemistry
5	Physics
6	Science of the universe
7	Engineering sciences
8	Mathematics

Table 1: The domains.

	Citations			Journal IF			Rel JIF		
Dominance relation	$\blacktriangleright$	$\rhd$	$\trianglerighteq$	$\blacktriangleright$	$\rhd$	$\trianglerighteq$	$\blacktriangleright$	$\rhd$	$\trianglerighteq$
Fundamental biology	.65	.86	.89	.43	.82	.85	.50	.84	.87
Medicine	.79	.91	.93	.57	.89	.91	.66	.90	.91
Applied biology/ecology	.66	.87	.90	.43	.84	.86	.46	.83	.85
Chemistry	.62	.86	.89	.38	.85	.88	.36	.87	.89
Physics	.72	.90	.92	.48	.87	.89	.49	.88	.89
Science of the universe	.69	.87	.89	.45	.82	.84	.53	.87	.88
Engineering	.79	.88	.91	.51	.85	.87	.56	.87	.89
Mathematics	.61	.82	.85	.33	.74	.77	.37	.78	.80
All disciplines	.38	.88	.91	.57	.86	.89	.63	.88	.90

Table 2: The rate of completness of a series of dominance relations over set of 112 US higher Education and research institutions.

Figure 1: The adjusted dominance network among the top US research universities associated to weak dominance relation, when impact is measured with citations and at the interdisciplinary level.

Figure 2: The adjusted dominance network among top departments in economics associated to weak dominance relation.

	Dominance relations based on citations					
	$\blacktriangleright$		$\triangleright$		$\trianglerighteq$	
	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i
Harvard	10	65	1	111	1	111
Stanford	11	59	2	107	2	107
Seattle	1	82	3	105	3	105
UCLA	7	72	3	105	3	105
UM_Ann_Arbor	7	72	5	104	6	104
Berkeley	17	41	6	104	5	105
Johns_Hopkins	19	41	7	103	7	104
Pennsylvania	28	29	8	101	8	103
WI_Madison	3	78	9	97	11	97
MIT	55	14	10	95	9	102
Columbia	13	48	11	94	10	98
Cornell	5	76	12	94	15	95
Twin_Cities	4	77	12	94	13	95
UCSD	14	48	14	93	12	96
UCSF	48	17	15	91	13	95
Yale	15	46	16	89	16	91
Pittsburgh	51	16	17	89	18	89
Duke	12	57	18	89	17	90
Urbana_Champaign	17	41	19	88	19	88
Northwestern	37	23	20	84	21	87
WU_St_Louis	23	35	21	84	20	88
UNC	16	44	22	84	22	87
UC_Davis	2	80	23	83	25	83
PA_Univ_Park	22	36	24	83	26	83
Mayo_Coll_Med	87	4	25	83	24	84
Caltech	76	8	26	81	23	84
Florida	5	76	27	79	27	79
Columbus	9	66	28	79	28	79
Arizona	19	41	29	77	31	77
Austin	43	18	30	76	30	77
USC	46	18	31	75	32	75
Chicago	26	32	32	74	29	79
Texas_AM	21	40	33	68	36	68
Vanderbilt	35	24	34	65	34	71
UC_Irvine	39	21	35	65	35	71
TX_Anderson	78	7	36	62	33	74
Iowa	25	34	37	62	43	62
Purdue	29	29	37	62	43	62
MD_Coll_Park	44	18	39	61	46	62
Baylor_Coll_Med	77	8	40	61	39	64
NYU	32	26	40	61	39	64
Emory	33	25	42	60	37	68
Utah	26	32	43	60	47	62
Virginia	30	26	43	60	45	62
Georgia_Inst_Tech	65	12	45	59	47	62
Boston	31	26	46	59	42	63
Michigan	24	34	47	56	51	56
Princeton	70	10	48	55	38	65
UCSB	81	6	49	55	41	63
Iowa_State	38	22	50	55	53	55

Table 3: Top 50 pseudo ranking of 112 US higher Education and research institutions in all disciplines, build upon three dominance relations.

	Dominance relations based on journal IF					
	$\blacktriangleright$		$\triangleright$		$\trianglerighteq$	
	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i
Harvard	1	91	1	111	1	111
Berkeley	7	82	2	106	2	106
Stanford	11	79	2	106	2	106
Seattle	2	89	4	104	4	104
UCLA	5	87	5	103	5	103
UM_Ann_Arbor	2	89	6	102	6	102
Johns_Hopkins	6	83	7	101	7	102
Pennsylvania	13	77	8	100	8	102
MIT	20	62	9	97	9	101
WI_Madison	4	88	10	96	11	96
Cornell	9	80	11	94	12	95
Columbia	16	72	12	94	10	97
UCSD	19	64	13	93	12	95
Twin_Cities	9	80	14	91	15	92
UCSF	31	47	15	91	14	95
Yale	24	54	16	89	16	91
Urbana_Champaign	7	82	17	88	17	88
Duke	26	53	18	87	18	88
UC_Davis	12	78	19	86	19	87
Pittsburgh	17	70	20	86	22	86
Northwestern	32	47	21	84	19	87
WU_St_Louis	35	45	22	83	21	87
UNC	23	55	23	83	23	85
Caltech	51	27	24	81	24	84
Columbus	15	72	25	80	25	80
PA_Univ_Park	18	66	26	78	28	78
Austin	25	53	27	78	28	78
Mayo_Coll_Med	34	46	27	78	27	79
Arizona	21	59	29	77	30	77
Florida	14	75	30	76	31	76
Chicago	47	30	31	73	26	79
Vanderbilt	42	37	32	69	32	74
USC	28	52	33	67	35	67
Texas_AM	22	58	34	66	36	66
MD_Coll_Park	33	46	35	63	37	66
TX_Anderson	86	11	36	61	33	73
Baylor_Coll_Med	54	24	37	61	38	64
Boston	45	32	38	61	40	64
NYU	41	37	38	61	43	63
Purdue	27	52	38	61	45	61
Virginia	37	43	38	61	38	64
UC_Irvine	49	29	42	60	42	63
Iowa	30	48	43	59	47	59
Utah	39	40	44	59	46	61
Emory	46	31	45	58	44	62
Georgia_Inst_Tech	36	45	45	58	48	59
Princeton	61	19	47	56	34	67
UCSB	63	16	48	55	41	63
Michigan	29	49	49	55	53	55
Iowa_State	38	42	50	55	50	57

Table 4: Top 50 pseudo ranking of 112 US higher Education and research institutions in all disciplines, build upon three dominance relations.

	Dominance relations based on Rel JIF					
	$\blacktriangleright$		$\triangleright$		$\trianglerighteq$	
	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i
Harvard	1	100	1	111	1	111
Stanford	10	83	2	107	2	107
Seattle	4	95	3	106	3	106
UM_Ann_Arbor	1	100	3	106	3	106
Berkeley	5	89	5	106	5	106
UCLA	3	97	6	105	6	105
Johns_Hopkins	6	89	7	101	9	101
Pennsylvania	13	79	8	100	7	102
MIT	19	72	9	97	8	101
WI_Madison	7	88	10	96	10	96
Columbia	15	79	11	95	10	96
Cornell	9	85	12	95	12	96
UCSD	16	78	13	94	13	95
Twin_Cities	8	87	14	93	14	94
UCSF	30	52	15	92	15	93
Yale	23	61	16	90	16	92
Urbana_Champaign	12	81	17	90	17	90
UC_Davis	13	79	18	87	19	88
Duke	22	68	19	87	18	89
Pittsburgh	18	77	20	87	21	87
Northwestern	32	51	21	85	20	88
WU_St_Louis	28	56	22	84	22	86
UNC	26	57	23	84	23	86
Florida	11	81	24	83	24	83
Mayo_Coll_Med	34	50	25	82	26	83
Columbus	17	78	26	81	27	81
PA_Univ_Park	21	70	27	81	28	81
Caltech	48	33	28	80	25	83
Austin	27	56	29	80	29	80
USC	29	55	30	75	31	75
Texas_AM	20	70	31	73	32	73
Chicago	36	43	32	72	30	77
Arizona	25	57	33	70	34	70
Vanderbilt	51	32	34	69	33	73
Purdue	24	58	35	69	35	69
Georgia_Inst_Tech	38	42	36	66	36	67
MD_Coll_Park	35	44	37	64	38	65
UC_Irvine	45	36	37	64	36	67
Iowa	33	51	39	62	44	62
Utah	42	40	40	62	42	63
NYU	46	35	41	61	46	61
Virginia	37	43	42	61	41	63
Baylor_Coll_Med	53	30	43	60	47	60
Boston	43	37	43	60	42	63
Emory	50	32	45	60	45	62
Michigan	31	52	46	59	48	59
Princeton	59	22	47	57	40	63
Iowa_State	40	41	48	57	50	57
TX_Anderson	66	19	49	56	51	56
UCSB	71	17	50	55	39	63

Table 5: Top 50 pseudo ranking of 112 US higher Education and research institutions in all disciplines, build upon three dominance relations.

Dominance relation	$\blacktriangleright$	$\rhd$	$\unrhd$
Economics	.08	.40	.75

Table 6: The rate of completness of a series of dominance relations over set of 239 Economics Departments Worldwide.

	Dominance relations					
	$\blacktriangleright$		$\triangleright$		$\trianglerighteq$	
	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i	Rank σ_i	#Dom n_i
NBER	17	23	1	237	1	238
CEPR	43	5	2	236	2	237
DE_Harvard	57	3	8	190	3	235
IZA	1	229	3	235	3	235
DE_Princeton	58	3	19	168	5	232
DE_Berkeley	71	2	26	157	6	227
DE_Chicago	44	5	24	161	6	227
LSE	3	197	4	223	8	225
DE_Oxford	5	176	5	220	9	224
KS_Harvard	34	8	16	171	10	224
BSB_Chicago	44	5	19	168	11	223
DE_MIT	71	2	51	92	12	221
DE_NYU	36	8	22	166	13	221
IMF	4	186	6	215	14	217
DE_Columbia	31	9	17	171	15	217
DE_Stanford	36	8	23	162	16	216
BS_Harvard	51	4	40	116	17	215
World_Bank	2	200	7	214	18	214
IFS	21	17	14	172	19	212
DE_Boston	41	6	21	168	20	211
GSB_Stanford	73	2	58	84	21	210
GSB_Columbia	27	11	28	145	22	209
HIWRP_Stanford	93	1	93	32	23	208
DE_UCL	14	31	12	175	24	206
DE_UMichigan	30	9	12	175	24	206
DE_UCSD	49	4	34	125	24	206
DE_Northwestern	52	4	47	100	27	205
WSB_Pennsylvania	18	21	15	172	28	202
CFRE_Yale	82	2	69	61	29	201
DE_WI_Madison	78	2	60	82	30	200
SSB_NYU	22	15	30	137	31	199
WHSB_Berkeley	52	4	56	86	32	197
FRB	12	43	11	178	33	195
DE_UCLA	59	3	29	144	34	193
DE_Yale	62	3	38	120	35	193
FRB_Minneapolis	97	1	63	69	36	193
CESifo	11	44	9	190	37	190
Tinbergen_Instituut	6	136	10	185	38	186
DE_Brown	32	9	32	129	39	186
KGSM_Northwestern	50	4	37	123	40	185
IGIER_Bocconi	46	5	33	126	41	184
DE_WashingtonU	64	3	68	63	42	184
DE_Minnesota	97	1	71	55	43	184
DE_UPennsylvania	52	4	59	83	44	183
Brookings_Institution	69	3	65	66	45	183
DE_Maryland	78	2	49	95	46	182
WWSPIA_Princeton	108	1	123	15	47	179
TSE	9	64	25	159	48	179
CREATES_Aarhus	46	5	36	123	49	179
PSE	7	127	18	169	50	176

Table 7: Top 50 pseudo ranking economics departements, build upon three dominance relations.

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