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The Absence of Deprivation as a Measure of Social Well-Being An Empirical Investigation*

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**Mesurer le bien-être social à partir de l'absence de privation
Une investigation empirique**

Résumé

Le critère de Lorenz généralisé est largement utilisé pour effectuer des comparaisons de bien-être au sein et entre pays sur la base de leurs distributions de revenu. Des études expérimentales ont contesté cette manière de procéder en montrant que le principe des transferts, qui sous-tend le critère de Lorenz généralisé, ne rencontre pas dans le public le large consensus auquel les théoriciens auraient pu s'attendre. Nous proposons de remplacer le critère de Lorenz généralisé par le critère de non-privation introduit par S.R. Chakravarty (Keio Economic Studies 34 (1997), 17–32). Ce critère est moins exigeant que le critère de Lorenz généralisé car il s'appuie sur une version plus faible du principe des transferts et il est par conséquent plus à même d'être accepté par le public. Nous utilisons les distributions de revenus de 17 pays fournies par le Luxembourg Income Study pour évaluer empiriquement les capacités de discrimination des critères de Lorenz généralisé et de non-privation. Bien que le quasi-ordre de non-privation soit moins décisif que le critère de Lorenz généralisé, nous montrons qu'il engendre un classement des distributions pratiquement identique à celui qui résulte de l'application du critère de Lorenz généralisé.

Mots-clés : Transferts progressifs, Bien-être social, Inégalité, Privation

**The Absence of Deprivation as a Measure of Social Well-Being
An Empirical Investigation**

Abstract

The generalised Lorenz criterion is widely used for making welfare comparisons within and across countries on the basis of their income distributions. Experimental studies have challenged this way of proceeding by showing that the principle of transfers, which underlies the generalised Lorenz criterion, does not meet with widespread agreement among the public that theorists would have expected. We propose to substitute the non-deprivation quasi-ordering introduced by S.R. Chakravarty (Keio Economic Studies 34 (1997), 17–32) for the generalised Lorenz criterion. This criterion is less demanding than the generalised Lorenz criterion as it builds on a weaker version of the principle of transfers and it is therefore more likely to be accepted by the public. We use income data from the Luxembourg Income Study for 17 countries in order to contrast the generalised Lorenz and the non-deprivation criteria. Although the non-deprivation quasi-ordering is less decisive than the generalised Lorenz criterion, it is shown that the former approximates the latter surprisingly well.

Keywords: Progressive Transfers, Social Welfare, Inequality, Deprivation

JEL: D30, D63

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1. Introduction

There is a widespread agreement in the literature to appeal to the generalised Lorenz dominance criterion for making welfare comparisons across societies starting with their distributions of income (see Kolm (1969), Shorrocks (1983)). Much of the attractiveness of the generalised Lorenz criterion – beyond its simplicity and elegance – stems from its association with the *principle of transfers*, according to which any transfer from a richer individual to a poorer one that does not modify their respective positions on the income scale – a so-called progressive transfer – reduces inequality and increases welfare. However, notwithstanding its wide application in theoretical and empirical work, the approach based on the Lorenz curve is *neither* the only possibility *nor* immune to criticism.

On the one hand, the ability of the principle of transfers to capture the very idea of inequality has been challenged by a number of experimental studies, all of which find that a large proportion of respondents reject it (see, e.g., Amiel and Cowell (1999)). On the other hand, there is evidence that the social status of an individual – approximated for instance by her position in the social hierarchy – plays an important role in her appraisal of her well-being (see, e.g., Weiss and Fershtman (1998)). Attitudes such as envy or resentment have been argued to be important components of individual judgements that might be taken into account in the assessment of alternative situations. The notion of individual *deprivation* originating in Runciman (1966) accommodates such views by making the individual's own appraisal of a given social state depend on her situation compared with the situations of all the individuals who are treated more favourably than her.

Chateauneuf and Moyes (2006) have shown that, if the distributions have equal means, then the idea of deprivation can be captured by the *generalised Gini social welfare function* due to Yaari (1987) provided that suitable conditions be imposed. Furthermore, they have proposed a stronger version of the notion of a progressive transfer – the so-called *uniform on the right transfer* – having the property that overall deprivation decreases when one distribution is obtained from another by means of a finite sequence of such transformations, and conversely. Contrary to a progressive transfer, where the positions of the individuals involved play no role, a uniform on the right progressive transfer imposes a minimum amount of solidarity among the donors: if some income is taken from an individual, then the same amount has to be taken from every non poorer individual. Dispensing with the equal mean restriction, Magdalou and Moyes (2009) have shown that the *non-deprivation quasi-ordering* constitutes the analogue of the generalised Lorenz quasi-ordering when attention is paid to deprivation.¹

By definition the non-deprivation quasi-ordering is ethically less demanding than the standard generalised Lorenz quasi-ordering as it relies on weaker value judgments. As a consequence, if a distribution is ranked above another one by the non-deprivation quasi-ordering, then so it is by the generalised Lorenz quasi-ordering. Thus, the greater acceptability of the non-deprivation quasi-ordering is counterbalanced by the fact that it is likely to produce rankings less clearcut than those produced by the generalised Lorenz quasi-ordering. It is then natural to question how far away the ranking generated by the non-deprivation criterion will be from that implied by the application of the generalised Lorenz test in practice. The paper addresses this question empirically by contrasting the rankings of distributions one obtains

¹ It is fair to note that the non-deprivation quasi-ordering was initially proposed by Chakravarty (1997) who called it the *satisfaction quasi-ordering* (see Magdalou and Moyes (2009) for a discussion).

starting with the non-deprivation quasi-ordering and related criteria with those obtained when building on the generalised Lorenz quasi-ordering. To this aim we have chosen to measure well-being and inequality in 17 countries using income data from the Luxembourg Income Study (LIS) for years 1999–2000.

We present in Section 2 the different quasi-orderings we appeal to in the paper. We provide a short description of the database in Section 3 and then present the results of the pairwise comparisons. Section 4 concludes the paper. Finally, we give an overview of the statistical inference techniques used for implementing our different criteria in Appendix A.

2. Social Welfare and Inequality Criteria

We assimilate an *income distribution* with a random variable X with values in a closed and bounded interval $D \subseteq \mathbb{R}$. We indicate by $\mathcal{Y}(D)$ the set of income distributions and we denote as $F(z; X)$ the *cumulative distribution function* of $X \in \mathcal{Y}(D)$, where $z \in \mathbb{R}$. We denote by $F^{-1}(p; X)$ the *quantile function* of X defined by $F^{-1}(0; X) := \inf\{z \in \mathbb{R} \mid F(z; X) > 0\}$ and $F^{-1}(p; X) := \inf\{z \in \mathbb{R} \mid F(z; X) \geq p\}$, for all $p \in (0, 1]$. The *mean income* of $X \in \mathcal{Y}(D)$ is indicated by $\mu(X)$.

The first criterion we appeal to for making welfare comparisons only incorporates considerations for greater efficiency. Letting $RO(p; X) := F^{-1}(p; X)$, for all $p \in [0, 1]$, we have:

DEFINITION 2.1. Given two distributions $X, Y \in \mathcal{Y}(D)$, we say that X *rank-order dominates* Y , which we write $X \geq_{RO} Y$, if and only if $RO(p; X) \geq RO(p; Y)$, for all $p \in [0, 1]$.

The *non-deprivation quasi-ordering* builds on the idea according to which a person feels deprived if she realises that some other persons enjoy some item she does not have access to, but sees no reason why she is not entitled to get it (see Runciman (1966)). The *non-deprivation curve* $ND(p; X)$ of distribution $X \in \mathcal{Y}(D)$ indicates for every individual occupying rank p in the population the average difference between her income and the incomes of all individuals richer than her. Formally, we have $ND(0; X) := F^{-1}(0; X)$ and

$$ND(p; X) := \int_0^p (1 - q) dF^{-1}(q; X), \forall p \in (0, 1].$$

DEFINITION 2.2. Given two distributions $X, Y \in \mathcal{Y}(D)$, we say that X *non-deprivation dominates* Y , which we write $X \geq_{ND} Y$, if and only if $ND(p; X) \geq ND(p; Y)$, for all $p \in [0, 1]$.

Following Kolm (1969) and Shorrocks (1983), it is current practice to use the *generalised Lorenz curves* to perform welfare comparisons. The generalised Lorenz curve $GL(p; X)$ of distribution $X \in \mathcal{Y}(D)$, defined by

$$GL(p; X) := \int_0^p F^{-1}(q; X) dq, \forall p \in [0, 1],$$

indicates the cumulated income received by every fraction p of the population deflated by the population size. Then, we have:

DEFINITION 2.3. Given two distributions $X, Y \in \mathcal{Y}(D)$, we say that X *generalised Lorenz dominates* Y , which we write $X \geq_{GL} Y$, if and only if $GL(p; X) \geq GL(p; Y)$, for all $p \in [0, 1]$.

Magdalou and Moyes (2009) have shown that the three above quasi-orderings are nested in the sense that $\geq_{RO} \subseteq \geq_{ND} \subseteq \geq_{GL}$. Therefore, the non-deprivation quasi-ordering provides a ranking of the distributions under comparison that is more complete than the one resulting from the application of the rank-order quasi-ordering but less complete than the ranking implied by the generalised Lorenz quasi-ordering.

The previous approach can be extended to inequality measurement by appropriate normalisation of the distributions under comparisons (see, e.g., Moyes (1999)). Letting $D \subseteq \mathbb{R}_{++}$ and dividing every income by the mean, we derive the *relative Lorenz curve* and the *relative non-deprivation curve* of $X \in \mathcal{Y}(D)$ defined respectively by $RL(p; X) := GL(p; X/\mu(X))$ and $RND(p; X) := ND(p; X/\mu(X))$, for all $p \in [0, 1]$. If one subtracts the mean from every income, then one obtains the *absolute Lorenz curve* and the *absolute non-deprivation curve* of $X \in \mathcal{Y}(D)$ given respectively by $AL(p; X) := GL(p; X - \mu(X))$ and $AND(p; X) := ND(p; X - \mu(X))$, for all $p \in [0, 1]$. Then, the *relative Lorenz*, *relative non-deprivation*, *absolute Lorenz*, *absolute non-deprivation* quasi-orderings are defined by comparing the corresponding curves as we have done above for the generalised Lorenz and the non-deprivation quasi-orderings.

3. Empirical Comparisons of the Quasi-Orderings

The LIS database indicates for each household its disposable income (DHI) – that is its total income after taxation and transfer payments – in local currencies and its composition. We have converted incomes using the purchasing power parities (PPP) proposed by the OECD to make them comparable across countries. Furthermore, household incomes have adjusted by

Table 3.1: The countries under comparison

NO	COUNTRY	YEAR	DHI PER CAPITA	SAMPLE SIZE
1	Mexico	2000	5 788.32	10 072
2	Poland	1999	6 330.86	30 812
3	Hungary	1999	6 513.81	1 927
4	Greece	2000	13 598.82	3 873
5	Spain	2000	17 627.86	4 761
6	Finland	2000	18 152.60	10 419
7	Sweden	2000	18 356.47	14 471
8	Netherlands	1999	19 249.84	4 331
9	Germany	2000	20 123.77	10 982
10	United Kingdom	1999	20 733.02	24 830
11	Austria	2000	20 945.18	2 329
12	Belgium	2000	21 177.11	2 080
13	Canada	2000	23 583.00	28 902
14	Norway	2000	24 603.77	12 870
15	Switzerland	2000	25 825.90	3 627
16	USA	2000	29 028.81	49 294
17	Luxembourg	2000	29 504.36	2 415

means of the *square-root equivalence scale* in order to take family needs into account and the resulting equivalent incomes have been weighted by the number of persons in the household (see, e.g., Atkinson, Rainwater, and Smeeding (1995)). Table 3.1 gives the list of countries

we have retained and indicates for each of these the year when the data were collected, the corresponding adjusted DHIs per capita in \$US, and the number of households in the samples.

Because the distributions under comparisons are samples drawn from larger populations, we have implemented statistical inference using the *intersection-union* (IU) method advocated by Howes (1994) and Kaur, Rao, and Singh (1994), and the *union-intersection* (UI) method introduced by Bishop, Chakraborti, and Thistle (1989) (see Appendix A for more details). For each dominance criterion we have performed both tests at a 5% significance level. We only present here the tables giving the results of the pairwise comparisons obtained under the restrictive IU method for each criterion, and we refer the interested reader to Magdalou and Moyes (2008, Appendix B) which provides the corresponding tables for the UI method.

Table 3.2 gives the results of the comparisons of the distributions of adjusted household incomes by the rank order, the generalised Lorenz and the non-deprivation quasi-orderings. The 17 countries are listed in order of adjusted DHI per capita ranging from the lowest (Mexico) to the highest (Luxembourg) figures. The performance of the rank order criterion is surprisingly good and allows us to rank conclusively 65 out of 136 pairs of countries, hence a success rate of 47.79%. Application of generalised Lorenz dominance permits to reach definite

Table 3.2: Social welfare rankings of countries under the IU method

No	Country i	Country j															
		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	Mexico	-N	-N	-N	-R	-R	-R	-N	-R	-R	-R	-R	-R	-R	-R	-R	-R
2	Poland		-L	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R
3	Hungary			#	#	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R	-R
4	Greece				#	-R	-R	-N	-R	-R	-R	-R	-R	-R	-R	#	-R
5	Spain					-N	-N	-N	-N	-R	-N	-N	-R	-R	-R	#	-R
6	Finland						#	#	#	#	#	#	#	#	#	#	-R
7	Sweden							#	#	#	-L	#	#	-R	-R	#	-R
8	Netherlands								#	#	#	#	#	#	#	#	-R
9	Germany									#	#	#	#	-R	-R	#	-R
10	United Kingdom										#	#	#	-N	-R	#	-N
11	Austria											#	#	#	-R	#	-R
12	Belgium												#	#	-L	#	-N
13	Canada													-N	-R	#	-N
14	Norway														#	#	-N
15	Switzerland															#	-N
16	USA																-N
17	Luxembourg																

Legend “-R” if country i is rank-order dominated by country j .
 “-N” if country i is non-deprivation dominated by country j .
 “-L” if country i is generalised Lorenz dominated by country j .
 “#” if countries i and j are not comparable by the three social welfare quasi-orderings.

conclusions in 87 out of 136 cases, which gives a success rate of 63.97% and confirms that the ranking generated by the generalised Lorenz criterion is finer than the one implied by the rank order criterion. Contrary to what might have been anticipated, the substitution of the non-deprivation quasi-ordering for the generalised Lorenz one does not result in a significant decrease in the number of pairs of countries that can be ranked. As it can be checked in Table 3.2, the rankings of countries provided by the generalised Lorenz and the non-deprivation quasi-orderings are almost identical. The only difference concerns three pairs of countries that are ordered by the generalised Lorenz criterion while they are not comparable under the non-deprivation criterion: Poland and Hungary, Sweden and Austria, and Belgium and Switzerland.

The application of the standard Lorenz tests and of the non-deprivation based criteria to the comparisons of inequality results in more contrasted pictures. Table 3.3 indicates the results of the pairwise comparisons of countries on the basis of the relative Lorenz and of the relative non-deprivation quasi-orderings. Similarly, the rankings of countries generated by

Table 3.3: Relative inequality rankings of countries under the IU method

No	Country <i>i</i>	Country <i>j</i>															
		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	Mexico	-N	-N	-N	-N	-N	-N	-N	-N	-N	-N	#	-N	-N	-N	#	-N
2	Poland		#	#	#	#	#	-N	#	#	-N	#	#	#	#	-L	-N
3	Hungary			#	#	#	#	#	#	+N	#	#	#	#	#	+N	-N
4	Greece				#	#	#	-N	#	#	-L	#	#	#	#	+N	-N
5	Spain					#	#	-N	-L	#	-N	#	#	#	#	+L	-N
6	Finland						#	#	#	+N	#	+N	+N	#	+N	+L	#
7	Sweden							#	#	+N	#	#	+N	#	#	+L	#
8	Netherlands								#	+N	#	#	+N	#	#	+N	#
9	Germany										+N	#	#	+N	#	#	+N
10	United Kingdom											#	#	#	#	-N	#
11	Austria												#	+N	#	#	+N
12	Belgium													#	#	#	#
13	Canada														#	-L	+L
14	Norway															#	#
15	Switzerland																+L
16	USA																
17	Luxembourg																

Legend “+N” if country *i* relative non-deprivation dominates country *j*.
 “-N” if country *i* is relative non-deprivation dominated by country *j*.
 “+L” if country *i* relative Lorenz dominates country *j*.
 “-L” if country *i* is relative Lorenz dominated by country *j*.
 “#” if countries *i* and *j* are not comparable by the two relative inequality quasi-orderings.

Table 3.4: Absolute inequality rankings of countries under the IU method

No	Country <i>i</i>	Country <i>j</i>															
		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	Mexico	#	#	#	+N	+N	+L	#	+N	+N	#	+N	+N	+N	+N	+N	+N
2	Poland		#	+N	+N	+N	+N	+L	+N	+N	+N	+N	+N	+N	+N	+N	+N
3	Hungary			+N	+N	+N	+N	+N	+N	+N	+N	+N	+N	+N	+N	+N	+N
4	Greece				+N	#	#	#	+N	+N	#	+N	+N	+N	+N	+N	+N
5	Spain					#	#	#	#	+N	#	#	+N	+L	+N	+N	#
6	Finland						#	#	#	+N	#	+N	+N	+N	+N	+N	+L
7	Sweden							#	+L	+N	#	+N	+N	+N	+N	+N	+L
8	Netherlands								+N	+N	+L	+N	+N	+N	+N	+N	+N
9	Germany									+N	#	#	+N	+N	+N	+N	+L
10	United Kingdom										#	#	#	#	#	+L	#
11	Austria											#	+N	+N	+N	+N	+N
12	Belgium												#	#	#	#	#
13	Canada													#	#	+N	#
14	Norway														#	+L	#
15	Switzerland																+L
16	USA																
17	Luxembourg																

Legend “+N” if country *i* absolute non-deprivation dominates country *j*.
 “-N” if country *i* is absolute non-deprivation dominated by country *j*.
 “+L” if country *i* absolute Lorenz dominates country *j*.
 “-L” if country *i* is absolute Lorenz dominated by country *j*.
 “#” if countries *i* and *j* are not comparable by the two absolute inequality quasi-orderings.

the absolute Lorenz and by the absolute non-deprivation quasi-orderings are summarised in

Table 3.5: Success rates of the social welfare and inequality quasi-orderings

		RO	ND	GL	RND	RL	AND	AL
IU Test	Nb	65	84	87	46	56	82	93
	(%)	(47.79)	(61.76)	(63.97)	(33.82)	(41.18)	(60.29)	(68.38)
UI Test	Nb	84	101	104	86	101	107	117
	(%)	(61.76)	(74.26)	(76.47)	(63.24)	(74.26)	(78.68)	(86.03)

Table 3.4. A first remark is that, on the whole, the inequality quasi-orderings are far less discriminatory than the corresponding welfare quasi-orderings. This originates in part in the fact that the size effect due to differences in disposable per capita income, which explains much of the rankings of countries obtained on the basis of the welfare criteria, is eliminated through the normalisation procedures used in the definition of our inequality criteria. A second remark is that the inequality quasi-orderings rooted in the notion of non-deprivation are less powerful than those derived from the generalised Lorenz curve, something which is more in line with our expectations. This is particularly the case for relative inequality where the non-deprivation approach permits us to rank only 46 pairs of countries out of 136, while the relative Lorenz quasi-ordering is able to establish a conclusive verdict in 56 cases. As a consequence the success rate falls from 41.18% to 33.82% when the relative non-deprivation quasi-ordering is substituted for the relative Lorenz quasi-ordering. A similar situation obtains in the case of absolute inequality: application of the absolute Lorenz criterion allows to rank 93 pairs of countries (68.38% of the cases), while this figure falls only to 82 (60.29% of the cases) when one appeals to the absolute non-deprivation quasi-ordering.

Finally, we note that substituting the more liberal UI method for the IU one results in an appreciable gain in decisiveness, as it is shown in Table 3.5. However, this does not affect the general conclusions drawn from the application of the dominance criteria.

4. Conclusion

Contrary to what might have been expected, our empirical investigation suggests that the non-deprivation quasi-ordering performs rather well as compared with the generalised Lorenz quasi-ordering. Things are more contrasted as far as inequality comparisons are concerned, but still there the non-deprivation based inequality quasi-orderings do not perform too badly by comparison with the standard relative and absolute Lorenz criteria. Although it is certainly premature to generalise, this empirical study provides a response to a serious objection that could be made against the non-deprivation criteria, precisely their lower degrees of decisiveness as compared with the Lorenz criteria. In addition, the application of the non-deprivation quasi-ordering provides additional information about the nature of the equalising process that leads to the domination of one distribution by another.

A. Statistical Inference Tests

Let incomes be grouped into K *quantiles* with abscissa $p_k := k/K$, where $k \in \mathcal{K} := \{1, 2, \dots, K\}$ and $K = 100$. Following Beach, Chow, Formby, and Slotsve (1994), we estimate the *quantile mean* $\mu_k(X)$ by

$$\mu_k(\widehat{X}) := K \int_{p_{k-1}}^{p_k} F^{-1}(p; \widehat{X}) dp,$$

where \widehat{X} is the *sample distribution* corresponding to $X \in \mathcal{D}$. We compute the sample curve $J(\widehat{X}) := (J(p_1; \widehat{X}), J(p_2; \widehat{X}), \dots, J(p_K; \widehat{X}))$ for each $J \in \{RO, ND, GL, RND, RL, AND, AL\}$. We denote by $\gamma_k(X)$ the *cumulative mean* of rank-ordered incomes up to p_k and we estimate it by $\gamma_k(\widehat{X}) := GL(p_k; \widehat{X})/p_k$. The corresponding *cumulative variance* is indicated by $\lambda_k(X)$ and its sample estimator by $\lambda_k(\widehat{X}) := GL(p_k; (\widehat{X} - \mu_k(\widehat{X}))^2)/p_k$. Beach and Davidson (1983) have shown that the vector $GL(\widehat{X})$ is – under reasonable conditions – asymptotically normal with mean zero and covariance matrix $\mathbf{\Omega}(X) := [\omega_{ik}(X)]$, where

$$\begin{aligned} \omega_{ik}(X) := & p_i \left[\lambda_i(X)^2 + (1 - p_k) (\mu_i(X) - \gamma_i(X)) \right. \\ & (\mu_k(X) - \gamma_k(X)) (\mu_i(X) - \gamma_i(X)) + \\ & \left. (\gamma_k(X) - \gamma_i(X)) \right], \end{aligned}$$

for $1 \leq i \leq k \leq K$. For all the curves $J(\widehat{X})$, it can be shown that there exists a matrix $\mathbf{R}^J := [r_{ik}^J]$ independent of \widehat{X} and defined by

$$r_{ik}^J := \frac{\partial J(p_i; \widehat{X})}{\partial GL(p_k; \widehat{X})},$$

for all $i, k \in \mathcal{K}$. It follows from Rao (1965) that $J(\widehat{X})$ is asymptotically normal with mean zero and covariance matrix $\mathbf{\Theta}^J(X) := [\theta_{ik}^J(X)] = \mathbf{R}^J \mathbf{\Omega}(X) (\mathbf{R}^J)^T$. Thus, in order to compare two incomes distributions $X, Y \in \mathcal{D}$ by means of the quasi-ordering J , we can use for each abscissae p_k the standard normal test statistic:

$$T_k := \frac{J(p_k; \widehat{X}) - J(p_k; \widehat{Y})}{\left(\frac{\theta_{kk}^J(\widehat{X})}{n(\widehat{X})} + \frac{\theta_{kk}^J(\widehat{Y})}{n(\widehat{Y})} \right)^{1/2}},$$

where $n(\widehat{X})$ and $n(\widehat{Y})$ are the *sample sizes* of \widehat{X} and \widehat{Y} , respectively.

According to the IU method, non-dominance constitutes the null hypothesis:

$$H_0(X, Y) : \exists k \in \mathcal{K} \mid J(p_k; X) - J(p_k; Y) < 0 \text{ and}$$

$$H_1(X, Y) : J(p_k; X) - J(p_k; Y) \geq 0, \forall k \in \mathcal{K}.$$

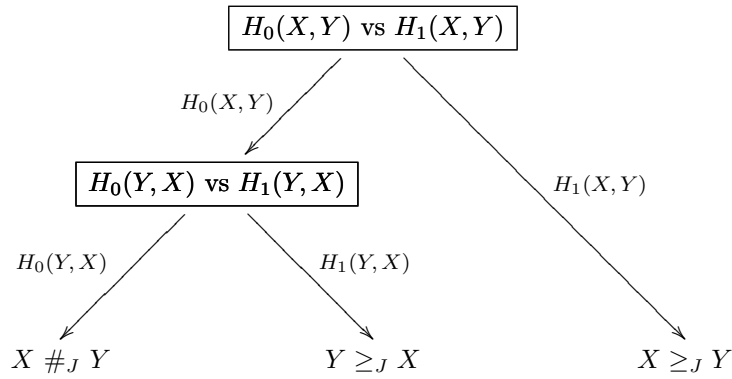
The curve $J(X)$ is significantly above the curve $J(Y)$ at abscissa p_k if and only if $Z_\alpha < T_k$, where Z_α is the critical value for a significance level of α derived from Student's t -distribution. We first test $H_0(X, Y)$ against $H_1(X, Y)$: *either* we accept $H_1(X, Y)$ in which case distribution X weakly dominates distribution Y , *or* we reject $H_1(X, Y)$ and we move to the second stage where $H_1(Y, X)$ is tested against $H_0(Y, X)$. Then, *either* we accept $H_1(Y, X)$ and distribution Y weakly dominates distribution X , *or* we reject $H_1(Y, X)$ and we conclude that distributions X and Y are non-comparable. The sequence of tests for the IU method is summarised in Figure A.1, where “ $X \#_J Y$ ” indicates that the distributions X and Y are non-comparable according to the J criterion. Application of the IU rule leads to the conclusion that:

$$X \geq_J Y \iff Z_\alpha < \min \{T_k\},$$

$$Y \geq_J X \iff \max \{T_k\} < -Z_\alpha,$$

X and Y are non-comparable otherwise,

Figure A.1: Test procedure under the IU method



We note that this method does not allow for equivalence: *either* one country weakly dominates the other one, *or* the two countries are non-comparable.

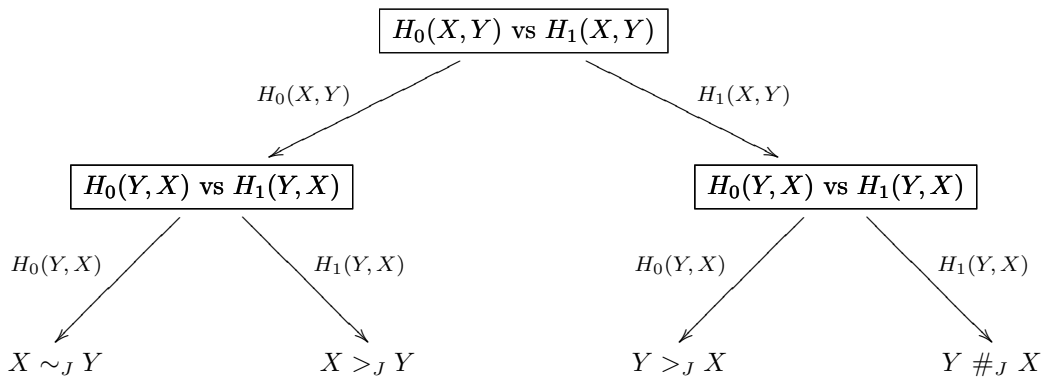
On the contrary, equivalence constitutes the null hypothesis in the UI method:

$$H_0(X, Y) : J(p_k; X) - J(p_k; Y) = 0, \forall k \in \mathcal{K} \text{ and}$$

$$H_1(X, Y) : \exists k \in \mathcal{K} \mid J(p_k; Y) - J(p_k; X) > 0 .$$

In a first step, we test $H_0(X, Y)$ against $H_1(X, Y)$ and there are two possibilities. If we accept $H_0(X, Y)$, then we test in a second stage $H_0(Y, X)$ against $H_1(Y, X)$: *either* we accept $H_0(Y, X)$, in which case we conclude that distributions X and Y are equivalent, *or* we accept $H_1(Y, X)$ and we conclude that distribution Y strongly dominates distribution X . If in the first step we accept $H_1(X, Y)$, then we test in a second stage $H_0(Y, X)$ against $H_1(Y, X)$: *either* we accept $H_0(Y, X)$, in which case we conclude that distribution X strongly dominates distribution Y , *or* we accept $H_1(Y, X)$ and we conclude that distributions X and Y are non-comparable. We have represented in Figure A.2 the test procedure for the UI method. Application of the UI rule allows us to conclude that:

Figure A.2: Test procedure under the UI method



$$\begin{aligned}
X >_J Y &\iff -C_\alpha < \min \{T_k\} \text{ and } C_\alpha < \max \{T_k\}, \\
Y >_J X &\iff \min \{T_k\} < -C_\alpha \text{ and } \max \{T_k\} < C_\alpha, \\
Y \sim_J X &\iff -C_\alpha < \min \{T_k\} \text{ and } \max \{T_k\} < C_\alpha, \\
X \text{ and } Y &\text{ are non-comparable otherwise,}
\end{aligned}$$

where C_α is the critical value for a significance level of α determined from the Student Maximum Modulus (SMM) distribution provided by Stoline and Ury (1979).

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