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**Dominance relations and ranking when quantity and quality
both matter: Applications to US universities and econ.
departments worldwide**

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**Relations de dominance et classement lorsque la quantité et la qualité importent:
Applications aux universités US et aux départements d'économie**

Résumé

Dans cet article, nous proposons une extension du concept de dominance stochastique à des comparaisons de productions composites dont la quantité et la qualité sont importantes. Cette théorie permet de requérir l'unanimité de jugement au sein de nouvelles classes de fonctions. En outre, nous introduisons et caractérisons axiomatiquement un nouvel indice, nommé Importance, permettant de classer des institutions sur des bases explicites en utilisant l'information contenue dans tout ensemble de comparaisons bilatérales unanimes (un tournoi incomplet). Cette théorie est appliquée au classement des universités de recherche US en prenant en considération à la fois le volume de publication et l'impact de chaque article. L'autre application proposée concerne la comparaison et le classement des départements académiques en économie lorsque l'on prend en compte à la fois la taille du département et le prestige de chacun de ses membres.

Mots-clés : Dominance Stochastique , Classements, Tournois, Axiomatique, Citations.

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Abstract

In this article, we propose to extend the concept of stochastic dominance, extensively used in decision theory and social choice, to the comparison of composite outcomes, both the quality and quantity of which do matter. Unanimity of judgment among new classes of functions is also studied. We introduce and characterize axiomatically a new index, called Importance, which allows us to rank institutions on clear grounds using the information contained in any set of available unanimous bilateral comparisons (an incomplete tournament). This theory is applied to the ranking of U.S. research universities taking into account both the volume of publications and their impact. We also compare and rank academic departments in economics taking both the size of the department and the prestige of its members into account.

Keywords: Stochastic Dominance, Ranking, Tournaments, Axiomatics, Citations.

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Dominance relations and ranking when
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Applications to US universities and
econ. departments worldwide¹

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1 Introduction

Ever since the seminal contributions of Quirk and Saposnick (1962), Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970), economists have extensively applied the notion of stochastic dominance to the theory of choice under uncertainty, in which one basically compares lotteries (or density distributions) in an unambiguous manner among given classes of utility functions. Another application, pointed out by Atkinson (1970), concerns the unanimous comparison of income distributions for all social welfare functions which are symmetric and increasing concave in all arguments (equivalent, in this context, to the anonymity condition and the Pareto principle). In these two contexts, the value judgments do not take size into account. In the theory of choice under uncertainty, when comparing lotteries, the sum of probabilities is clearly always equal to unity. Only matter the probability of occurrence of each possible state of the world and its associated returns which, in more general terms, we associate here with the notion of quality. In the context of income distribution comparisons, value judgments could, in principle, take quantity into account (in this case population size), but are designed not to do so; they only consider incomes and their distribution. Size clearly does not matter when comparing two income distributions within the same population (any distribution can then be obtained from any other through a finite set of transfers). When comparing the income distributions of two different countries with different population sizes, as stated by Dasgupta et al. (1973), the use of an average social welfare criterion allows any influence of population size *per se* to be excluded.

There are, however, numerous contexts in which the quantity and quality of each item both matter. For instance, schools care about the numbers of students they train and their future wages. Social clubs care about both the number of members and their social status. Museums value both the number of artistic items and their importance in the history of arts (assuming that such a quality can be unambiguously assessed). A typical example, discussed later in this article, is related to scientific production. The community of scientometricians (or, informetricians) has long been concerned

with the necessary quantification of the scientific production of various academic institutions and actors. Two main approaches have been developed for this purpose. The first one aims to measure the volume of publications or, basically, the number of articles published.¹ The second one, mostly concerned with the quality of these publications (since obviously not all papers are equal), mainly relies on the average number of citations^{2,3} that articles receive. However, a unique measurement of scientific production, taking into account both the volume of publication (quantity) and the quality of each article, is often needed. One basic way of computing such a measurement would simply be to sum up the number of citations that all articles receive, a solution which is far from satisfactory, since it implies that the quality of articles increases proportionally with the number of citations received.⁴ Interest in this issue was recently stimulated by the introduction of the h-index (Hirsh, 2005), precisely designed to simultaneously account for both quality and quantity in a specific manner,⁵ and which has led to much criticism and

¹Various corrections, however, need to be introduced. An applied literature in scientometrics discusses which list of journals should be retained, how to account for the length of articles, and how to (and whether we should) correct for co-authorship.

²Ellison (2002) considers two dimensions of article quality: the quality of ideas that may be proxied by citations, and other aspects of quality, such as exposition and completeness, which may be approximated by the submission acceptance time.

³Peer judgment is an obvious alternative (although more time-consuming) way of measuring quality. One may also consider journal's attributes as an appropriate (albeit more indirect) measurement of its scientific impact. For instance, journal's Impact Factor (approximately, the average number of citations received by the articles published in the journal within a fixed window period) is a widely used statistic.

⁴A measurement of both the quantity and quality of articles was initially introduced by Lindsay (1978). It is equal to the average number of citations multiplied by the square root of the total number of citations. This measurement has, however, no theoretical foundation.

⁵An author's h is the maximum number of articles written by her/him which have received at least h citations. This index has the specific characteristic that it neither takes into account citations received by articles with fewer than h citations, nor considers citations received by papers above the threshold of the first h citations. The citations that are not considered are said to fall outside the h -core. Hirsch (2005) argues that highly cited papers should not be taken into account in proportion to the number of citations received, because some papers attract an anomalous number of citations.

several improvements.⁶ To our knowledge, no academic article has yet attempted to derive comparisons and the ranking of scientific production from explicit value judgments that take into consideration both their quantity and impact. This is a question we can and do address, using the more general theory developed in this article.

The first contribution of this article consists in providing a rationale for bilaterally comparing the outcome of various institutions (or individuals) when both quality and quantity matter, while rendering the underlying associated value judgments completely explicit. This contribution can be interpreted as a (slight) generalization of the stochastic dominance theory, with the standard applications of this theory to choice under uncertainty and income distributions being particular cases in which only quality is taken into consideration. Although most of the theorems we introduce in this respect have analogs in the stochastic dominance literature, our problem is more general, and the mathematical proofs follow different lines and are, therefore, new. Moreover, several applications of the theory have prompted us to explore the consequences of assuming that the social value function is convex (rather than concave) in each of its arguments. This is an assumption which is quite exotic from the point of view of the theory of choice under uncertainty or the theory of income distribution.⁷

When a dominance relation can be established between two institutions, ordering the two institutions is quite obvious. However, in principle, a dominance relation does not necessarily exist between the two agents of each pair, and it is well known that this is likely to be the case when a large set of objects is to be compared. We are then left with a partial ordering only, which may be considered unsatisfactory when faced with a ranking problem. An alternative approach would be to identify a specific valuation of quality and quantity, satisfying several desired properties, so as to obtain a complete ordering.⁸ The second theoretical contribution of our pa-

⁶Many contributions have aimed to overcome such shortcomings (Egghe, 2006, Anderson et al., 2008, Woeginger, 2008).

⁷This issue has been examined in a recent study (Bazen and Moyes, 2012).

⁸This line of inquiry, which consists in picking the desired index and characterizing it

per consists in providing a middle ground between these two approaches. More precisely, we propose an axiomatization of the use of (potentially incomplete) bilateral dominance relations, assessed between institutions in a given set, to infer a ranking. This amounts to considering that the collection of unanimous bilateral comparisons is the only information that should be considered. Technically speaking, establishing a ranking in this context is very close to the issues raised by the tournament literature initiated by Wei (1952), Kendall (1955) or Daniels (1969), recently revisited in an axiomatic approach (Palacios-Huerta and Volij, 2004; Slutzky and Volij, 2006; Demange, 2013). We discuss several well-known indexes and also propose a new vectorial centrality measure, which we label *Importance*, and which is built upon Katz'(1953) centrality.⁹ We study whether these indexes respect several axioms we introduce and show that the Importance index is the only one that is consistent with all proposed axioms.

In line with the above example, we first apply this theory to the ranking of U.S. research universities.¹⁰ We assume that the social value of research by universities is additively separable in all its arguments, namely the quality of each article recorded. The quality of the articles can be assessed from their scientific impact (normalized per discipline), which can be computed using several procedures. We use here the number of direct citations received by the papers within a given period of time, and the impact of the journals in which the papers were published.¹¹ These two proxies reflect different dimensions of quality, whose pros and cons are discussed later.

It seems acceptable to assume that publishing a higher impact paper, or

axiomatically, was followed by Woeginger (2008) and Marchant (2009).

⁹See Jackson, 2010 for a review of such centrality indexes. An up-to-date and comprehensive review of ranking methods can be found in Langville and Meyer (2012).

¹⁰We should here mention Lubrano and Protopopescu (2004), the first to apply the notion of dominance to the academic sphere. However their concern is different from ours: they investigate the distribution of articles in the population of economists of various countries.

¹¹Basically, this is the same as the direct citations but averaged over the total number of papers published in the journal within the same period. As we will see below, this measurement can also be normalized at sub-field level.

publishing an additional article, can never decrease the total value of any scientific production. Assumptions about the second derivative are more debatable. However, it appears, implicitly or explicitly, that convexity is a widely accepted assumption in the academic sphere. Most university representatives consider it important for their institution to appear in papers that reach high scores in terms of impact, as this enables the institution to increase its visibility within a community characterized by very high scientific standards. Typically, convexity means, for instance, that the value of two articles of quality x is never higher than the value of one article of quality $2x$. One can also associate various points of view to a set of hypotheses, which we believe can guide the comparisons : *volume*, when the only assumption is that the value is positive; *quality*, when the value is positive and does not decrease with impact; and *excellence*, when the value is positive, non-decreasing, convex with impact, and null impact takes value zero. It is useful to compare the rankings we obtain with other existing rankings. It appears that 17 of the 20 best ranked US universities in the ARWU ranking (Shanghai) are also in our top 20 universities when we focus on excellence. Such a high correlation highlights the ability of our methodology to measure excellence, which is precisely the goal of the ARWU ranking (under the terminology “world class universities”), but in a much more parsimonious manner. We only use publication and citation data, whereas the ARWU ranking, like most universities rankings, relies on different sources of information to capture the underlying scientific hierarchy.¹² The other main advantage of our approach is that our rankings are built on clear premisses, unlike most other university rankings.

In this article, we propose a second application for our theory, namely comparing the prestige of academic departments. The academic prestige of a department may be measured using several variables; however, it would seem reasonable that it should be calculated using information about the prestige of each of its *current* members. Scott and Mitias (1996) call this

¹²Only a limited number of results is presented here due to space constraints. More detailed results (in particular at disciplinary level) are available on the companion website: <http://ncarayol.u-bordeaux4.fr/ranking.html>.

the “stock” approach, as opposed to the “flow” approach, which attributes the research to the institution in which the research was conducted. Lubrano et al. (2003) connect the former approach to a definition of the “human capital” of research institutions. This was also used by Dusantsky and Vernon (1998) in their ranking of U.S. economics departments, and by Combes and Linnemer (2003) in their ranking of European economics departments. Basically, all of these studies calculate the sum of individual performances to obtain that of their departments. However, several scholars acknowledge that this is to the disadvantage of small departments and, consequently, propose rankings based on per capita scores (e.g. Graves et al. 1982; Scott and Mitias, 1996; Dusantsky and Vernon, 1998), or even produce comparisons between individual economists (Medoff, 1989). However, using the sum or average of performances is not a convincing solution.

This is just another typical example of a situation in which both quantity (in this case, department size) and quality (each member’s prestige) matter, and there may be a certain consensus on how to evaluate each of them. The form of consensus chosen is delineated by the assumptions to be made about the way the prestige of each member affects the prestige of the department. For instance, assuming that the contribution of individual prestige to the prestige of the department is convex, this seems to capture the intuition that research departments need to hire a number of academic leaders that are likely to contribute more than proportionally (via their own prestige) to the prestige of the department. We develop this application by using the Rep-Ec dataset and by comparing the economics research departments positioned among the world’s top five percent. Quantity is measured through the number of (registered) members. The prestige of each member is here approximated by the number of citations that each member’s papers have received. Several scholars (e.g. Medoff, 1989; Beilock et al., 1986) argue that it is preferable to use citations rather than publications in journals (weighted for page adjusted size and/or journal quality). We also build upon a long tradition, initiated by sociologist R. K. Merton, and subsequently extended and formalized by scientometricians (e.g. Garfield, 1963), which acknowledges that the total number of citations received by an

individual is highly correlated to his prestige. We find that several government or business schools compete against top ranking prestigious economics departments (twelve in the top fifty). We also observe how differences in the design of US and European departments impact their position in the top ranking. European departments are usually larger but are characterized by a lower concentration of prestigious members, which results in lower ranks. Though their larger size often immunizes them from dominance by several smaller US departments, they dominate few or none of the latter, which keeps them low in the ranking, and thus do not reach the top positions. This illustrates how our theory deals with the quality/quantity issue.

The rest of the article is organized as follows. Our basic theory of extended dominance relations is developed in the following section. The third section presents how unilateral pairwise dominance relations can be turned into rankings through an axiomatic characterization. In the fourth section, we show how this theory can be used to compare the scientific production of research institutions and then apply it to US research universities. The fifth section is dedicated to the comparison and ranking of the world's top research departments in economics. The last section concludes.

2 The extended theory of dominance relations

2.1 Notations

Let us define a set $I \in \mathfrak{S}$ of n agents $i = 1, \dots, n$, which can denote either individuals or institutions. Each item produced by any of these agents is denoted by an index $a = 1, \dots, n_i$, with n_i the total number of items produced by agent i . Each item a is characterized by an associated quality $s_a \in S$, with $S (\subset \mathbb{R}^+)$ the bounded set of all possible values of quality ($s \in S \rightarrow 0 \leq s < \infty$). The outcomes of agent i are described by a $1 \times n_i$ vector $s_i := (s_1^i, s_2^i, \dots, s_a^i, \dots, s_{n_i}^i)$. Let us now define $f_i(s)$ the production performance of i with quality s :

$$f_i(s) := \sum_{a=1, \dots, n_i} 1_{\{s_a^i=s\}}, \quad (1)$$

with $1_{\{\cdot\}}$ the indicator function, which is equal to one if the condition in brackets is satisfied and zero otherwise. The distribution $F_i = \{f_i(s) \mid \forall s \in S\}$ describes the production of agent i , and $\forall i \in I, F_i \in F$ the set of all possible production distributions.

The valuation function $v(\cdot) : S \rightarrow \mathbb{R}$ gives the “value” of any unit item as a function of its quality. Assuming that the value of the whole production performance of agent i , V_i is the result of the sum of the value of each element, this writes as follows:

$$V_i = \sum_{s \in S} v(s) f_i(s). \quad (2)$$

2.2 Dominance relations

We now introduce three dominance relations: strong dominance, dominance and weak dominance. Each dominance relation requires unanimity within a given particular category of judgment which is given by a class of admissible $v(s)$ functions. Definitions 1 to 3 require that the total value of an institution’s production be superior to that of another institution for any function $v(s)$, within clearly defined classes, for it to be dominant. Theorems 1 to 3 establish the necessary and sufficient conditions for each dominance relation to hold.

Let us define the notion of strong dominance over the set of agents I . This dominance relation only requires that function $v(\cdot)$ be non-negative, i.e. no item will contribute negatively to the performance of any agent.

Definition 1 *The production of agent i strongly dominates that of agent j , noted $i \blacktriangleright j$, if, for any non-negative function $v(\cdot)$ over set S : $V_i \geq V_j$.*

Theorem 1 *$i \blacktriangleright j$ if and only if $\forall u \in S, f_i(u) - f_j(u) \geq 0$.*

Theorem 1 simply means that the necessary and sufficient condition for there to be a strong dominance of one agent over another is that it does not perform less for any possible level of quality. This condition is intuitive, since strong dominance requires unanimity of judgment for any non-negative

value function, which may arbitrarily increase the value of any positive level of quality.¹³

The notion of dominance requires unanimity among all non-negative and now also non-decreasing functions $v(\cdot)$, that is to say that articles of a higher quality should never have a lower value.

Definition 2 *The production of agent i dominates that of agent j , and is noted $i \triangleright j$, if, for any non-negative and non-decreasing function $v(\cdot)$ over set S : $V_i \geq V_j$.*

Theorem 2 *$i \triangleright j$ if and only if $\forall u \in S, \sum_{s \in S, s \geq u} (f_i(s) - f_j(s)) \geq 0$.*

Additional hypotheses can be introduced relative to the second derivative of the value function. Definition 3 introduces the notion of weak dominance which requires that $v(\cdot)$ be convex. Weak dominance also requires that the value of a null quality item should be zero.

Definition 3 *The production of agent i (convex) weakly dominates that of agent j , noted $i \trianglerighteq j$, if, for any non-negative, non-decreasing and weakly convex function $v(\cdot)$ over set S , such that $v(0) = 0$: $V_i \geq V_j$.*

We now have the following theorem.

Theorem 3 *$i \trianglerighteq j$ if and only if $\forall u \in S, \sum_{s \in S, s \geq u} s [f_i(s) - f_j(s)] \geq 0$.*

Each one of the three notions of dominance require the unanimity of the judgments associated with any value function belonging to a specific class. The results in the three axioms are important because they make it possible to compute the dominance relations, without having to further specify the functional forms of the various dominance relations. The notion of strong dominance requires only that the value function be non-negative, i.e. that no item should have a negative effect on the performance of any institution. This very weak condition implies that quality plays almost no role. The

¹³All the proofs are exposed in Appendix A.

notion of dominance requires the value function to be non-negative and non-decreasing, i.e. that items of a higher quality are better valued (within a given domain). This assumption is also likely to be considered acceptable when quality has to be considered. The notion of weak dominance requires, in addition to the above mentioned properties, that the value function also be weakly convex and that any null quality item should have the value zero. These additional assumptions imply that the value function gives proportional or more than proportional weight to the best quality items. Thus, weak dominance reflects a focus on excellence, that is on the highest quality items.

As the relevance of the various sets of assumptions is context-dependent, this will thus be discussed in Sections 4 and 5, which are dedicated to the proposed applications.

2.3 Some basic properties of dominance relations

We present here some simple properties of the dominance relations that will prove useful in the next section. Before doing so, we need to define a principle of comparison of dominance relations. A dominance relation \succsim is stronger than any other dominance relation \succsim' , noted $\succsim \rightarrow \succsim'$, if, $\forall I \in \mathfrak{S}, \forall i, j \in I$, $i \succsim j$ implies $i \succsim' j$. The symbols \succsim and \succsim' account for any one of the dominance relations introduced above ($\succsim, \succsim' \in \{\blacktriangleright, \triangleright, \trianglerighteq\}$). In other words, a dominance relation \succsim is said to be stronger than another if a dominance of the first type of any given agent over another implies a dominance relation of the second type between those two agents.

A first lemma establishes connections between dominance relations; a second lemma introduces several properties that all dominance relations fulfill. This states that the weaker the dominance relation, the greater the number of dominance relations it is possible to establish between the agents of any given set of agents I . The proofs are derived directly from the definitions of the different forms of dominance, which have conditions of increasing strength.

Lemma 4 $\blacktriangleright \rightarrow \triangleright$ and $\triangleright \rightarrow \trianglerighteq$.

The second lemma below focuses on some properties of the dominance relations. Part i) simply establishes that all of the dominance relations introduced are transitive. Part ii) states the basic reflexivity property derived from the definitions of the different forms of dominance in which inequalities are of the type “greater than or equal to” The last two parts state that bilateral dominance is obtained if, and only if, production performances are identical for all possible quality levels, excluding the zero case for weak dominance only.

Lemma 5 *The following statements hold:*

- i) *if $i \succcurlyeq j$ and $j \succcurlyeq h$, then $i \succcurlyeq h, \forall \succcurlyeq \in \{\blacktriangleright, \triangleright, \trianglerighteq\}$; (transitivity)*
- ii) *$i \succcurlyeq i, \forall i \in I, \forall \succcurlyeq \in \{\blacktriangleright, \triangleright, \trianglerighteq\}$; (reflexivity)*
- iii) *$i \succcurlyeq j$ and $j \succcurlyeq i$ if and only if $f_i(s) = f_j(s), \forall s \in S, \forall \succcurlyeq \in \{\blacktriangleright, \triangleright\}$.*
- iv) *$i \trianglerighteq j$ and $j \trianglerighteq i$ if and only if $f_i(s) = f_j(s), \forall s \in S \setminus \{0\}$.*

3 From bilateral dominance relations to rankings

The previous section showed how to construct partial ordering from any set of production distributions, in such a way that any bilateral comparison should be based in the unanimity among all judgments that are consistent with some well-defined conditions concerning the shape of the value function. Now, we will build a complete order on the basis of that information only. This amounts to considering that any bilateral, non-unanimous comparison should not be considered at all. There are many applications in which this information constitutes more of a noise than a truly relevant item of information.

For this purpose, let us first consider a dominance relation \succcurlyeq that could be any one of the three dominance relations examined above, or even any other form of dominance that would respect i) and ii) of Lemma 5. For any agent set $I \in \mathfrak{S}$, the set of all possible sets of agents, such that $\#I = n$, all the relevant information can be represented by the matrix $G \equiv (g_{ij})_{i,j=1,\dots,n}$ which is built as follows: $g_{ij} = 1$ if $i \succcurlyeq j$ and zero otherwise. We call such

a matrix the dominance matrix of set I associated with dominance relation \succ .

Let us make the further assumption that dominance relations are asymmetric in the following sense: $\forall I \in \mathfrak{S}, \forall i, j \in I, i \neq j$, if $g_{ij} = 1$ then $g_{ji} = 0$, *i.e.* there is no reciprocal dominance for any pair of distinct agents in I .¹⁴ A second simplifying assumption is that there are no separate leagues in I , in the sense that $\forall i, j \in I$, there is a finite sequence i_0, i_1, \dots, i_T with $i_0 = i$ and $i_T = j$ such that either $\prod_{t=1, \dots, T} g_{i_{t-1}i_t} = 0$ or $\prod_{t=1, \dots, T} g_{i_t i_{t-1}} = 0$. The set of all possible dominance matrices between the elements of set I (respecting *i* and *ii* of Lemma 5 and the two assumptions) of cardinal $n \geq 2$, is denoted Γ_n , and $\Gamma = \cup_{n \geq 2} \Gamma_n$ is the set of all possible dominance matrices.

As it is often useful to provide graphs associated with dominance relations, we introduce the notion of (directed) dominance network g_{\succ} that can be simply built from any dominance matrix G as follows: $ij \in g_{\succ}$, if and only if $g_{ij} = 1$.¹⁵ Since there is no separate league, these networks only have one connected component.

A ranking problem is a pair (G, I) with $G \in \Gamma_n$ and $I \in \mathfrak{S}$ such that $\#I = n$ for some $n \geq 2$. We denote by \mathfrak{R} the set of all such ranking problems. A ranking method is a function $\phi : \mathfrak{R} \rightarrow \mathbb{R}_+^N$, with N the set of all finite subsets of \mathbb{N} . That function returns a vector $1 \times n$ of non-negative scores for each set of agents $I \in \mathfrak{S}$ of cardinal n and associated dominance matrix $G \in \Gamma_n$. Thus, for any given agent set I , and any dominance matrix G associated with a binary relation \succ , we obtain a vector noted (with a slight abuse of notation) $\phi(G)$ of $n = \#I$ lines, of which the i th entry $\phi_i(G)$

¹⁴That is $\nexists i, j \in I, i \neq j$, such that both $i \succ j$ and $j \succ i$. This situation would only arise when i and j have exactly the same production, for all quality levels if $\succ \in \{\blacktriangleright, \triangleright\}$ and for all non-null quality levels if $\succ = \triangleright$ (as stated in *iii* and *iv* of Lemma 5). In most empirical applications, there is always a sufficiently detailed definition of the production item under consideration such that two agents can not be found with exactly the same production.

¹⁵In the graphical representations, for the sake of clarity, we remove self-dominance and redundant dominance relations, which are uninformative, since transitivity always holds. Such a network obtained from g_{\succ} is denoted by h_{\succ} and called adjusted dominance network.

is the evaluation of agent i in I with respect to \succsim . The set of evaluation functions over a given set of agents I is Φ_I , and Φ , is the set of all evaluation functions. We are not interested here in the cardinal valuations but only in ordinal comparisons; thus a ranking method will be considered as unique up to any weakly increasing transformation. Two ranking methods ϕ and ϕ' are said to be ordinally equivalent (that we write $\phi \doteq \phi'$) if and only if each agent's score in one ranking can be obtained from its score in the other ranking through a non-decreasing function: \exists a non-decreasing function $F()$, such that for all ranking problems (G, I) , and $\forall i \in I : \phi_i(G) = F(\phi'_i(G))$.

The structure of our problem is reminiscent of the tournament literature initiated by Wei (1952), Kendall (1955), David (1963) and Daniels (1969). However, the type of tournaments we have here has specificities that should be kept in mind for the remainder of this section. First, there is at most one comparison between two distinct agents: any matrix $G \in \Gamma$ is such that $g_{ij} + g_{ji} \in \{0, 1\}$. Secondly, since the dominance relations are reflexive, self comparisons always yield a one (the first diagonal entries $g_{ii} = 1$). Third, transitivity holds in any dominance matrix (see Lemma 5) and, therefore, the presence of inconsistencies that Wei (1952), Kendall (1955) and Daniels (1969) try to deal with is not the problem here. There is no cycle and, consequently, a complete order can be built avoiding any contradiction with the dominance relations. Instead, incompleteness, when present, is the problem that we are trying to solve so as to obtain a complete order. When two agents can not be compared (there is no dominance relation between them), we are willing to use the information on how others relate to both in order to infer some sound form of differentiation. This kind of argument will prompt us to break any form of irrelevance of independent alternatives axiom that Rubinstein proposed (in the context of complete tournaments) and which led him to the row sum ranking method. However, since G can correspond to a complete tournament, correspondance between the ranking methods and the row sum will be considered here.

In the following subsection, we introduce several axioms that any ranking method should respect. The following subsection discusses known ranking methods and also proposes a new one, which we characterize in the last

subsection.

3.1 Axioms

The first axiom we want any ranking method to satisfy is the standard anonymity axiom.

Axiom 6 [*Anonymity*] Let $\kappa : I \rightarrow I, \forall I \in \mathfrak{S}$ be a permutation function, and for any $G \in \Gamma$, let $G_\kappa = (g_{\kappa(i)\kappa(j)}) \in \Gamma$. A ranking method ϕ is anonymous if, for all ranking problems (G, I) in \mathfrak{R} , all $i \in I$ and any permutation function κ : $\phi_i(G) = \phi_{\kappa(i)}(G_\kappa)$.

The following dominance consistency axiom is simple but important, in the sense that it posits that the ranking of any two agents should be strictly consistent with their direct dominance relation (when it exists).

Axiom 7 [*Dominance consistency*] A ranking method ϕ satisfies Dominance consistency if, for all ranking problems (G, I) in \mathfrak{R} , and $\forall i, j \in I$: if $g_{ij} = 1 - g_{ji} = 1$ then $\phi_i(G) > \phi_j(G)$.

Up to this point, the proposed axioms are based on the initial partial order directly obtained with the dominance relation used.

A symmetry axiom is also in order, since there is no reason why upward dominance relations should be treated differently from downward relations. This idea was first expressed by Ramanujacharyulu (1964). According to him, the score an agent has in a network should be the inverse of what that agent would have if all dominance relations were reversed, that is when the transpose of the dominance matrix is used.

Axiom 8 [*Symmetry*] A ranking method ϕ satisfies Symmetry, if, for all ranking problems (G, I) in \mathfrak{R} , and $\forall i \in I$: $\phi_i(G) \phi_i(G^T) = 1$.

It is obvious that Dominance consistency and Symmetry imply anonymity¹⁶, since the conditions in both axioms need to be satisfied for any agent and any G in Γ .

¹⁶Meaning here that, taken separately, each axiom implies the Anonymity axiom.

We now introduce the condition that a ranking method should correspond to the row sum ranking in the subset of complete tournaments in Γ . Let $\Gamma^1 \subset \Gamma$ be the subset of all dominance matrices such that $g_{ij} + g_{ji} = 1$, and Γ^2 the complement of Γ^1 on Γ . The set Γ^1 is the subset of all complete tournaments in Γ , and Γ^2 the subset of incomplete tournaments. The row sum is defined as follows: $r_i(G) = \sum_{j \in I} g_{ij}$.

Axiom 9 [*Row sum correspondence*] *A ranking method ϕ satisfies the condition of Row sum correspondence if, $\forall G \in \Gamma_1, \forall I \in \mathfrak{S}$, and $\forall i, j \in I$: $\phi_i(G) \geq \phi_j(G)$ if and only if $r_i(G) \geq r_j(G)$*

The Symmetry axiom has led us to consider both upward dominance and downward dominance, and we therefore also introduce the column sum given by $c_i(G) = \sum_{j \in I} g_{ji}$. Given that we have $c_i(G) = r_i(G^T)$, it is easy to show that symmetry and row sum correspondence imply column sum correspondence, that is $\forall G \in \Gamma_1, \forall I \in \mathfrak{S}$, and $\forall i, j \in I$: $\phi_i(G) \geq \phi_j(G)$ if and only if $c_i(G) \leq c_j(G)$.

We now investigate the idea that indirect dominance should be considered positively if it is an outgoing dominance relation. The next axiom builds directly on the ideas developed by David (1963) who proposed that the row sum of the defeated agents should account for the value of each win. We propose that one more dominance relation of any of the agents agent i beats should be good for i . Let us denote G_{uv} a matrix with all zero entries except for the entry of the u^{th} line and the v^{th} , column which equals unity.

Axiom 10 [*Indirect dominance consistency*] *A ranking method ϕ respects the condition of Indirect dominance consistency if, for all ranking problems (G, I) in \mathfrak{R} , and for all $G' \in \Gamma$ such that $G' = G + G_{uv}$: $g_{iu} = 1$ implies $\phi_i(G') > \phi_i(G)$.*

Similarly, one more dominance relation over an agent that beats i , should be bad for i because i is then defeated by a less performing agent. However it is not necessary to state this in a new axiom since the Symmetry axiom will naturally bring in this property, provided Indirect dominance consistency is

verified. The following very simple examples should clarify most aspects of what this axiom is intended to capture.

Example 11 Consider the following dominance network $g = \{ik, ku, ju, iu\}$, whose adjusted dominance network is $h = \{ik, ku, ju\}$, depicted in Figure 1. A ranking should reflect a preference for the position of i , followed by the position of j , then k and finally u . Although i does not dominate j , its position is preferable since it dominates k , while j does not. Moreover, k is not dominated by j ; the latter's position is, however, preferable since it is not dominated by i .

Example 12 Let us now consider two dominance networks: $g^1 = \{ik, ij, ku, iu, ju\}$ and $g^2 = \{ij, jk, ik, ku, iu, ju\}$ whose adjusted dominance networks are depicted in Figure 2: $h^1 = \{ik, ij, ku, ju\}$ and $h^2 = \{ij, jk, ku\}$. Agent i 's position in g^2 is preferable to i 's position in g^1 because in g^2 , agent j (which i dominates in both cases) has an increased importance. Symmetrically, u 's position in g^1 is preferable to u 's position in g^2 because it is dominated by a more dependent agent k .

Indirect dominance consistency amounts to consider not only the direct dominance relations but also the indirect dominance relations of length equal to two. As suggested by Wei (1952) and Kendall (1955), there is no reason to restrict ourselves to indirect dominances of lengths two, and we should consider dominance paths of all lengths. Moreover, in its simplest form, indirect dominance relations may be considered as votes of equal weight. Each dominance path could be considered as a vote and, thus, the scores should be proportional to the number of outgoing dominance paths to others. The next axiom builds on this idea.

Axiom 13 [Indirect dominance homogeneity] A ranking method ϕ respects the condition of indirect dominance consistency, if, for all ranking problems (G, I) in \mathfrak{R} , and $\forall i \in I : \phi_i(G) \doteq \psi_i(G_i^2) \cdot \sigma_i(G_i^1)$ with $\sigma_i(G_i^1) = \sum_{j \in I} \sum_{k=0, \dots, \infty} h_{ij}^{(k)}$.

The term $h_{ij}^{(k)}$ denotes the i^{th} line and j^{th} column of matrix $(G - I)^{(k)}$, which is the k^{th} power of matrix $(G - I)$.¹⁷ It is well known from graph theory that the i^{th} line and j^{th} column entry of the k^{th} exponent of the adjacency matrix of some digraph gives the number of paths of length k from i to j . The matrix G_i^1 is built from G , by replacing by zero all entries that concern, in column, agents that i does not dominate: $G_i^1 \equiv \left(g_{ju} \cdot 1_{\{g_{ij}=1\}} \right)_{j,u \in I}$. The matrix $G_i^2 \equiv G - G_i^1$, captures the remaining information. It is clear that $h_{ij}^{(k)}$ only uses the non null information contained in G_i^1 : it is also the i^{th} line and j^{th} column of matrix $(G_i^1 - I)^{(k)}$.

The axiom of Indirect dominance homogeneity requires the score of agent i to be proportional to the number of paths originating from i , not excluding that it could also be affected by some other function which would not use the information contained in G_i^1 .

3.2 Ranking methods

In this section we introduce several ranking methods and examine how they relate to the different axioms introduced above. Due to space constraints, only a certain number of the existing methods can be examined. The first one, proposed by David (1962), gives each agent a score equal to the sum of all outgoing paths of length one and two, minus the sum of all incoming paths of the same lengths. This can be written as follows:

$$da_i(G) = \sum_{j \in I} g_{ij} \cdot (r_j(G) - c_j(G)), \quad (3)$$

where $r_j(G)$ is the row sum and $c_j(G)$ the column sum of agent j . Since g_{ii} equals unity, the row and column sums of the considered agent are also counted.

The first ranking method intended to weight each win by the score of the outranked agent was developed by Wei (1952) and Kendall (1955), and

¹⁷Matrix $(G - I)$ is considered here instead of matrix G , because it makes no sense to include the others' self-dominance as votes for the considered agent. One's self-dominance is however included since, by convention, $H^{(0)} = I$.

is accordingly called the Wei-Kendall method. The most common way of writing this score is as follows:

$$wk_i(G) = \lambda \sum_{j \in I \setminus i} g_{ij} \cdot wk_j(G). \quad (4)$$

This states that the score of each agent is proportional to the sum of the scores of all the agents it dominates. Moon and Pullman (1970) and Daniels (1969) have proposed two improvements to this scoring function that are better known as the invariant method and the fair bets method. The invariant method, which is the core of the page rank algorithm used by Google for ranking web pages (Brin and Page, 1998), is given by the following equation:

$$im_i(G) = \sum_{j \in I \setminus i} \frac{g_{ij}}{\sum_{k \in I \setminus i} g_{jk}^T} \cdot im_j(G), \quad (5)$$

According to this method, the score of each agent is proportional to the sum of the scores of all the agents that it dominates, the contribution of each agent being divided by the number of agents that dominate it. The fair bets method is very similar to the invariant method, but it relies on a different normalization of the contribution of each agent j , to the score of the focal agent i : instead of dividing it by the number of agents that dominate j , it considers the number of agents that dominate i . Thus $fb_i(G)$ will simply be obtained from the right side of Equation 5, where g_{jk}^T and $im_j(G)$ is replaced by g_{ik}^T and $fb_j(G)$ respectively.

We now propose an evaluation function which, like the last three ones, has a fixed point inspiration, and which can be traced back to Katz (1953):

$$\alpha_i(G) = \varepsilon \sum_{j \in I \setminus i} g_{ij} \cdot \alpha_j(G) + \delta, \quad (6)$$

with δ and ε two strictly positive parameters. This function increases proportionally with the sum of the score of the agents that the considered agent dominates, plus some exogenous parameter. In fact, this measure is very similar to the “real” page rank, that is when one also considers the perturbations of the matrices introduced by Brin and Page (1998) who transform

them into irreducible ones.¹⁸ In matrix form this becomes (after some trivial recombination):

$$\alpha(G) = \delta(I - \varepsilon(G - I))^{-1} \mathbf{1} \quad (7)$$

where $\alpha(G)$ is the influence vector, I the identity matrix and $\mathbf{1}$ a column vector of one.

In the spirit of Ramanujacharyulu (1964), we would like to capture not only the dominance structure “below” agents but also the structure “over” agents. In other words, we also intend to account for the inability to free oneself from the dominance of agents that are themselves dominated. In fact, this is exactly what we would obtain with an index $\alpha'(G) \equiv \alpha(G^T)$. To take into account both upward and downward dimensions, we can construct a synthetic measure called *importance* that combines the two. This is given, in matrix form, by:

$$\gamma(G) = \lambda(G) \text{diag}(1/\alpha(G^T)) \alpha(G), \quad (8)$$

with $\text{diag}(1/\alpha(G^T))$ the diagonal matrix, the i 'th entry of its diagonal being equal to $1/\alpha_i(G^T)$, and with $\lambda(G)$ a normalization factor.¹⁹ In a general problem, ε cannot be arbitrarily large because, at some point, the system may diverge. However, here transitivity always holds and we have assumed asymmetry ($\forall i, j \in I$, if $g_{ij} = 1$ then $g_{ji} = 0$), and thus there is no cycle in $(G - I)$ and in $(G^T - I)$. Therefore $(I - \varepsilon(G - I))$ and $(I - \varepsilon(G^T - I))$ are invertible and a unique solution is always found for any $G \in \Gamma$.

3.3 Characterization

We now investigate how the proposed indexes relate to the different axioms introduced previously. Table 1 synthesizes all the information. What may surprise the reader is the very weak performance of the Wei-Kendall, the

¹⁸It can easily be shown that, after some normalization, adding perturbations amounts to adding some exogenous value to each agent. On this point, one may refer to Newman (2010).

¹⁹If the entries of $\gamma(G)$ need to sum up the unity, then the normalization parameter must be defined as follows: $\lambda(G) = (\text{diag}(1/\alpha(G^T)) \alpha(G) \mathbf{1}^T)^{-1}$

invariant and the fair bets methods. However, a little reflection leads to the conclusion that these measures are not relevant to the class of matrices considered here. In particular, they do not satisfy the axiom of dominance. That is because, by transitivity, all dominance paths necessarily end at one agent who dominates no other and, therefore, all scores are null. Consequently these indexes always yield a vector of zeros and are not even able to differentiate two agents when one dominates the other.²⁰ In comparison, David's index is considerably more robust since it satisfies four of the six axioms.

The only index that satisfies all the axioms is importance. In fact, it turns out that only two axioms (Symmetry and Indirect dominance homogeneity) are necessary to characterize importance as stated in the following theorem.

Theorem 14 *A ranking method ϕ satisfies the axioms of Symmetry and Indirect dominance homogeneity if and only if it is ordinally equivalent to the Importance evaluation function γ with $\varepsilon = 1$.*

Setting ε to unity ensures that each dominance path counts the same (see equations 7 and 8). With this theorem in hand, we can now further investigate the ranking problem from an empirical point of view.

4 Comparing and ranking research universities

In this section, the general theory introduced above is applied to the comparison of the scientific production of various institutions. This requires clarifications on the computation of scientific production and its impact, which will constitute a basis for computing quality in this context. The appropriate assumptions for the value function are subsequently discussed. The data are then presented and, lastly, the results are introduced.

²⁰This kind of indexes is more adapted to ranking scientific journals, for instance (see Palacios-Huerta and Volij, 2004; Demange, 2013).

4.1 Fractional counts of scientific production

In this paper we use the publication counting method known in the scientometric literature as fractional count, which splits each paper between institutions and disciplines, given that bibliometric data have some imperfections.

Let index a now specifically denote an article in A , the set of all articles, and let $p_a^i \in [0, 1]$ account for the fact that, in practice, most articles are attributed to several universities and should, therefore, be divided between them (since their authors are often employed by different universities: either one author is employed by several institutions or several authors are employed by different institutions). In practice, it is impossible to know the precise affiliations of authors, therefore one can only count the number of times an institution is referred to in the article. An article a , referencing at least one address associated with institution i , provides institution i with a gross volume of academic production of:

$$p_a^i := \frac{\#\{i \in \Delta(a)\}}{\#\Delta(a)}. \quad (9)$$

Expression $\#\{.\}$ denotes the cardinal of the set defined in brackets. Term $\Delta(a)$ is the set of references to institutions as listed by the authors of a . The same institution can be mentioned several times and so $\#\{i \in \Delta(a)\}$ defines the number of times i is mentioned in the list of institutions of article a . The right-hand side of the equation indicates the weight of institution i among the various institutions mentioned by the authors of article a . For instance, if three authors co-author an article, and two of them mention i as their affiliated institution and the third mentions another institution, then the ratio will be equal to $2/3$.

Let us also introduce the term $q_a^k \in [0, 1]$, which is intended to account for the fact that not all papers are associated with discipline k , while those that are, are not necessarily exclusively associated with discipline k ; therefore, the weight of discipline k in article a is computed as follows:

$$q_a^k := \frac{1_{\{k \in d(j(a))\}}}{\#d(j(a))}. \quad (10)$$

Typically, in scientometric databases, the information on disciplines comes from journals. The term $j(a) \subset A$ denotes the subset of all papers published in the same journal as a . The term $d(j)$ is the set of disciplines with which journal j is associated. Thus, q_a^k serves as a filter for selecting the articles related to discipline k through the association of the journal in which it was published with one or several disciplines, and it helps give weight to discipline k when the journal is related to several disciplines.

The share of paper a that goes to institution i in discipline k is thus simply given by $p_a^i \cdot q_a^k$.

4.2 Impact and quality

Three proxies for an article's quality are proposed here. First, it can be measured by counting the number of citations received by each article in a given time window after publication. This is computed as follows:

$$x_a := \# \{u \mid t_u \in w(a) \text{ and } a \in r(u)\}, \quad (11)$$

with t_u the year of publication of article u , and $w(a)$ the citation window of article a (we use three-year citation windows in practice) and $r(u)$ the reference list of article u . This measure is very attractive because it is a direct measure. Its shortcoming is that it is also noisy, since some articles attract a considerable amount of citations, not only because of their real scientific contribution but, also, because of the modes of citation, or because of their nature (review papers).

One may alternatively consider the impact factor of the journals as an appropriate (though more indirect) measure of scientific impact. This is computed as the average number of citations received by the articles published in the journal:

$$x'_a := \frac{\sum_{h \in j(a)} \# \{u \mid t_u \in w(h) \text{ and } h \in r(u)\}}{\#j(a)}, \quad (12)$$

with $\#j(a)$ the number of articles published in the journal in which article a is published, at the numerator, the total number of citations received by these

articles. This measure of impact helps to better evaluate an author’s capacity to publish in well-established journals with large readerships. Clearly, when impact is computed this way, universities that perform well have a high academic reputation in the largest communities of the discipline, as shown by their ability to publish in the most visible journals. This measure (like the former) has the drawback of favoring the most prominent specialties or sub-discipline communities.

The last measure, which is intended to correct for such potential bias, is the relative impact factor, that is the journal’s impact factor bench-marked by the average impact factor in the specialty. More formally, this is computed as follows:

$$x''_a := \frac{x'_a}{\frac{1}{\#\varphi(j(a))} \sum_{\alpha \in \varphi(j(a))} \langle x'_a \rangle_\alpha}, \quad (13)$$

with $\varphi(j)$ the set of specialties with which j is associated, and $\langle \cdot \rangle_\alpha$ denoting the arithmetic mean within set α . Such a measure is particularly useful when one aims to account for the ability to publish in the best journals of given fields because it controls the ability to choose the most visible fields. The relative impact factor also controls the various citation practices of the various specialties of the same discipline (e.g. applied and fundamental mathematics have different citation practices).

The simplest way of dealing with quality in this context would be to select one of the three impact measures presented above and to assume that it is the appropriate measure of quality. There are, however, good reasons not to make this assumption when one intends to describe comparisons across disciplines.²¹ The main one is that impact varies dramatically among disciplines, simply because citation practices vary across disciplines. For instance, the average size of reference lists in chemistry is greater than in mathematics and thus the average impact is higher. Thus, impact can not be a reliable measure of quality. We therefore normalize impact in each discipline through its relative position in the distribution of articles (according to its impact)

²¹This subtlety is not necessary when we limit ourselves to comparisons of universities in a single discipline.

within its corresponding discipline: the quality of a given article a in discipline k is equal to the maximum integer s , such that its impact x_a is at least as high as that of s percent of the articles published in discipline k . In other words, its quality is equal to the probability that a randomly drawn article in discipline k would have a lower (or equal) impact. The quality of paper a in discipline k is thus $s_a^k = \lfloor 100 \cdot \Pr(X^k \leq x_a) \rfloor$, with $\lfloor x \rfloor$ meaning the highest integer smaller than x .

4.3 Quantity and quality

Let us now redefine the conditional distribution of institution i in discipline k : $\{f_i^k(s) \mid \forall s \in S\}$, where $S = \{1, \dots, 100\}$, and which is computed as follows:

$$f_i^k(s) := \sum_{a=1, \dots, n_i} 1_{\{s_a^k=s\}} \cdot p_a^i \cdot q_a^k. \quad (14)$$

This can be aggregated across disciplines for any quality levels s : $f_i(s) := \sum_{k \in K} f_i^k(s)$.

4.4 Value

Let us now slightly redefine function $v(\cdot) : S \rightarrow \mathbb{R}$ as the valuation function which calculates the “value” of any unit of scientific production as a function of its position in the distribution of quality in its associated discipline. Then, the value of the whole production performance of agent i in discipline k is simply:

$$V_i^k = \sum_{s \in S} v(s) f_i^k(s). \quad (15)$$

The value of the whole publication performance of institution i can be computed either directly, or by aggregating the values over all fields:

$$V_i := \sum_{s \in S} v(s) f_i(s) = \sum_k V_i^k.$$

The establishment of dominance relations between universities is, therefore, a natural extension of the general theory presented in Section 2. If one focuses on comparisons by discipline, the publication data are the associated $f_i^k(s)$,

and the corresponding values are the V_i^k . When one focuses on comparisons across disciplines, the publication data are the $f_i(s)$, and the corresponding values are the V_i .

Let us now turn to the various assumptions for function $v(\cdot)$. It seems more than reasonable to assume that one additional article, or an article of higher quality, can never decrease the total value of any scientific production. This is equivalent to assuming that the value function of any article is positive and non-decreasing with its quality. Hypotheses concerning the second derivative of the value function are more debatable. However, in most of the interviews conducted with several rectors and presidents of universities, convexity appears (implicitly or explicitly) to be a relatively widely accepted hypothesis, once this has been clarified with them. University CEOs and their trustees usually attribute more than proportional weight to production in the higher segments of impact distribution, whereas little-cited papers tend to be less than proportionally considered. This focus on excellence seems to be common to all research universities, while other types of universities may have a broader focus.

4.5 Data

A set of the top US universities was selected on the basis of their rank in the Academic Ranking of World Universities (ARWU) produced by Shanghai Jiao Tong University. This ranking is well known to be “research oriented”, a specificity which, though based on very different premisses to ours, fits well with them. As our goal was to restrict our analysis to research universities, the best-ranked universities, representing about 30% of all Ph.D. granting universities in the US, was selected, i.e. a total of 112 universities.

Publications by these institutions²² and the citations they have received have been collected in the Thomson-Reuters-Web of Science (WoS) database.²³

²²The lexical tokens used to collect publications have been kindly provided by Cheng and Zitt (2009).

²³These data are imported and maintained by the Observatoire des Sciences et Techniques (OST) for national evaluation purposes and research and thus all computations (citations, impact factors, etc.) are performed in-house.

Since the publication data are available only from 2003 onwards and the citations data are only available up to the year 2007, this analysis was carried out using a set of smoothed data (from 2003 to 2005), with a 3-year citation window for each of these publication years. Over the period of observation and for the citation window selected, the scientific production of the 112 universities/institutions considered in this experiment amounted to 329,910 articles published in the journals referenced in the WoS database, journals which received 2,316,576 citations. The citation scores achieved by these papers were between 0 and 1,292, and the impact factor of the associated journals varied from 0 to 27.6 (all within the three-year citation window).

The assignment of the papers to disciplines was based on the association of the journals with nine discipline categories (see Table 2). The first eight correspond to clear disciplinary lines of inquiry, whereas the ninth, labeled Multidisciplinary Sciences, groups together journals that have a truly multidisciplinary focus, as well as some large multidisciplinary journals that publish articles pertaining to several disciplines. In the disciplinary based comparisons, excluding papers published in such large journals would introduce a significant bias since it would eliminate a significant percentage of the best articles across several disciplines. Therefore, the articles published in the most influential of these multidisciplinary journals (namely Proceedings of the National Academy of Sciences USA and Science and Nature) were reallocated to their parent discipline lexicographically.

As mentioned above, the impact of publications by universities was considered in three different ways: through the direct citations the articles received, the direct impact of the journals in which the considered articles were published and the relative impact, that is the impact factor of the journal relatively to the average impact factor of the specialty. This measure helped correct for the different citation practices across subject categories within the same discipline (e.g. between applied and fundamental mathematics). Lastly, the scientific production curves of each institution were linearized in fifty points, positioned at equal intervals between zero and the maximum

impact reached.²⁴

4.6 Results

The first result proposed concerns the extent to which the various dominance relations allow us to compare universities. For this purpose, we shall now introduce the notion of rate of completeness of dominance over set I , of cardinal n , defined as follows:

$$C_{I, \succ} = \frac{\# \{(i, j) \in I^2 \mid i > j, i \succ j \text{ or } j \succ i\}}{n(n-1)/2}, \quad (16)$$

which is simply equal to the percentage of pairs of (distinct) institutions for which one can establish at least one dominance relation of type \succ . Table 3 presents the rates of completeness for the eight disciplines and for all disciplines, associated with dominance relations \blacktriangleright , \triangleright , and \trianglerighteq . The information on dominance completeness is reported for each one of the three proxies used for impact. The results show that completeness varies across domains and depends on the type of dominance, the type of impact and the discipline. A weak dominance relation achieves significantly higher rates than other types of dominance, regardless of the domain. The completeness rate of weak dominance is generally close to ninety percent. The completeness rate is slightly higher when the citations are considered, which was expected since direct citations are more unevenly distributed than impact factors. Completeness is minimal for mathematics and maximal in medicine. All are strictly below unity and, therefore, we cannot establish a dominance ranking from the different types of dominance relations discussed here, without relying upon some scoring function. However, the rate of completeness is often very high, and it seems reasonable to produce a dominance ranking, as defined in Section 3, especially the one based on weak dominance.

The top 50 rankings of strong dominance, dominance and weak dominance relations are provided in Tables 4, 5, and 6 respectively (considering

²⁴Different numbers of points were tried (twenty, one hundred...). Results changed only marginally, provided that a sufficiently fine grain of quality is taken into consideration (a sufficient number of points).

all disciplines and using the importance scoring function γ). Table 4 presents rankings based on the direct citations received by the articles, Table 5 on the direct impact factors of journals, and Table 6 on the relative impact factors. Two columns, rank and importance (γ_i), are reported for each dominance relation. As expected, the hierarchy is more pronounced in the weak dominance relation as compared to dominance, and more pronounced with dominance as compared to strong dominance. Indeed, as the dominance relation is weakened, more dominance relations can be inferred and, thus, the dominance network becomes more hierarchical.²⁵

Though unreported Spearman rank correlations indicate that rankings built on the three dominance relations reveal a significant correlation. Some institutions, however, have very different rankings, depending on the associated dominance relation. For instance, MIT is forty-eighth when ranking is based on strong dominance, whereas it is in ninth position in the weak dominance ranking (see Table 4). This result should be interpreted by bearing in mind the size of the institution. The weak dominance relation provides an opportunity for excellent but smaller institutions to remain at the top of the ranking. Interestingly, certain institutions have significantly different rankings when different proxies of impact are used. For instance, Berkeley ranks sixth and fifth respectively in dominance and weak dominance relations when impact is measured through direct citations, while it ranks second in both types of dominance relations using the direct impact factor. This means that scholars in Berkeley do particularly well at publishing articles in the most important journals. When the relative impact factor is used to proxy impact, Berkeley moves down to fifth position in both rankings. This indicates that Berkeley scholars are excellent not only at having their papers published in the best journals within their given specialties (doing the “job right”), but also at selecting sub-fields that attract more attention (the “right job”). For each impact measure and each form of dominance considered (except in the case of strong dominance), Harvard dominates all other

²⁵Although it is not apparent in the tables reported in the present paper, which present only the top fifty institutions according to the dominance relation, this is also verified at the bottom of the ranking.

universities and ranks first.

It is interesting not to confine the investigation to the ranking results, but also to picture their associated (adjusted) dominance networks, which highlight the architecture of dominance relations. Figure 3 presents the adjusted dominance network associated with weak dominance (h_{\triangleright}) among the top institutions, proxying impact with the number of direct citations. We observe that, just below Harvard, the dominance structure is more sophisticated than expected. In fact, no dominance relation can be found between Michigan Univ. at Ann Arbor, Seattle, UCLA and Stanford. Stanford is, however, more important and thus better ranked than the other three institutions, because it dominates Berkeley and MIT while the others do not. Berkeley is less important than Seattle and UCLA because it is dominated by Stanford, while the other two are not.²⁶ Although Michigan Univ at Ann Arbor is not dominated by Stanford, it is, however, as important as Berkeley because it does not dominate John Hopkins, whereas Berkeley does. As a matter of fact, the dominance of Stanford reduces the dependence of Berkeley by exactly the same factor as its dominance over John Hopkins increases its influence (factor two).

The relation between the ranking we obtain when focusing on excellence and the ARWU (Shanghai) ranking should be highlighted. As Table 7 shows, 17 of the 20 best ranked US universities in ARWU are also in our top 20 universities. Such a high correlation tends to highlight the ability of our methodology to capture excellence, in particular when compared to the ARWU (Shanghai) ranking which is much less parsimonious, and combines different sources of information to capture scientific hierarchy. The only two exceptions,²⁷ Princeton and Chicago, have very good positions in ARWU, thanks mainly to their very high scores in the indexes which count the number of Nobel price winners and Fields medalists they employ and

²⁶This difference in importance is not apparent in Table 7 since the dominance by Stanford reduces very slightly the score of Berkeley, because Stanford itself is only dominated by Harvard.

²⁷The third one, Washington University in St Louis, has very similar positions in both rankings: 20 in ARWU and 21 in our ranking based on weak dominance.

have trained.

5 Ranking of academic departments according to their prestige

In this section, we apply the extended stochastic dominance theory to the comparison of academic departments according to their prestige. The basic assumption we make here is that the prestige of the department rests upon the prestige of its present members: the prestige of past members and of the department itself are not taken into account. Thus, two questions arise: how should an individual's prestige be defined, and how does the aggregated prestige of individuals form the prestige of the department?

The scientific prestige of a scholar is the recognition by the community of its interest in her/his work - what R. K. Merton calls credit.²⁸ Prizes, honorary lectures, invitations and, more generally, all distinctions based on peer-reviews may provide useful information on such credit. However, this information turns out to be heterogeneous and difficult to handle in a systematic and quantitative study. R. K. Merton himself argues that the accumulated academic credit can be approximated by direct citations. This idea has been extended and formalized by scientometricians such as Garfield (1963) and Price (1965). Stigler and Friedland (1975) also argue that "to some degree, citations are influence".

While citations were also used to compare the scientific production of academic institutions in the previous section, the approach followed in this section is different in several respects. First, it is no longer the flow of production that is taken into account, but rather the credit a scholar accumulates over her/his career, not only in her/his present institution but also in those in which s/he was previously employed. Therefore, we shall not limit ourselves to the papers produced in the current period but will take into account all of the papers ever published. Secondly, citations should also not be limited to a given window period after publication. Citations of old articles are also

²⁸Cf. his collected articles in Merton (1973).

very informative about the importance of these papers in the literature and thus about the scientific prestige of the author.

Let us now consider the question of the aggregation of the prestige of individuals constituting the department's prestige. A department i is now described on the basis of the prestige of each of its members through vector $s_i = (s_a^i)_{a=1, \dots, n_i}$, where a denotes a scholar and n_i is the number of members of the department. Now $f_i(s)$ denotes the number of members in department i with prestige s for any possible level of prestige $s \geq 0$, and is computed as stated in Equation 1. Let the prestige of the department i be given by V_i as stated in Equation 2. Again, clarifying the premisses associated with the aggregation amounts to formulating assumptions on function $v(\cdot)$, thereby defining the class of functions among which unanimity of judgment should be imposed to infer a dominance relation.

Hiring scholars of higher individual prestige, and hiring more scholars with a given level of prestige, should both exert a positive influence on the prestige of the department. Since both size and individual prestige are positively valued ($v(\cdot)$ should be positive and non-decreasing), strong dominance and dominance relations are based on acceptable assumptions (since $v(\cdot)$ is positive for strong dominance, and positive and non-decreasing for dominance). Assumptions on the second derivative (if any) are again more debatable. However, anecdotal evidence suggests that the prestige of the “stars” contributes more than proportionally to that of their department. Indeed, it is often mentioned that a key issue for a department is to hire at least one of these very influential scholars, whose prestige and reputation can serve as foundations for building internal research dynamics, raising significant external funding and attracting attention from the academic community. If this intuition is accepted, then the convexity assumption should also be retained, and the most accurate extended stochastic dominance is weak dominance.

5.1 Data

The data was collected from the Ideas-RepEc²⁹ website in June 2010 using a computerized data collection procedure. We collected data on all registered members; the study, however, was limited to those affiliated to at least one of the economics departments ranked among the top 5% in the world, as listed in the Rep-Ec database itself (239 departments). We relied on the Rep-Ec selection of departments, based on the aggregation of all measurements provided by this service. It turned out that 10,465 registered members were affiliated to these 239 departments (out of over 25,500 registered authors in the Rep-Ec database), or more than 40% of all members. The average department was composed of 43.7 members who had written a total of 695 papers and received 11,414 citations. By paper we mean an article published in a journal or in a series of working papers, a chapter of a book, a book or a software component. A paper might appear in different formats, and double counting was then corrected by an automated recognition of identical titles and possible decisions of the authors. Citations may have been made to a working paper or to the published version of the paper but they were not attributed to both, thus avoiding double counting. The citations were collected from the reference lists of these papers.³⁰

As the membership of a department is declarative, there might be differences between the real membership of a department and Rep-Ec membership. These differences are due to scholars having decided not to declare their membership to Rep-Ec. Although we are aware of this, we are inclined to think that the difference is very limited, especially among the top departments of economics. Note also that, since the procedure is declarative, Rep-Ec membership includes Ph.D. students and other non-permanent members of the departments.

The boundaries of the institutions are based on their own definitions, and different levels of aggregation coexist. Some registered institutions are just

²⁹See <http://ideas.repec.org>.

³⁰It should be noted that the citations made in all reference lists have not yet been fully taken into account, and thus the citation data is clearly not as complete as that used for the ranking of universities in the previous section.

aggregations of other institutions. We do not take these into account, since we decided to aggregate our data at the lowest possible level. This meant, for instance, that economists in a business school were not aggregated with the economists of the economics department of the same university (if they did not declare their affiliation to both departments). The two departments were considered as two different entities. We also observed that a limited number of scholars are affiliated to several institutions. Like Lubrano et al. (2003), we chose to allocate each of these multi-affiliated scholars to all of the institutions to which they belong, since we did not find a more suitable way of dividing authors across departments. The difficulty here comes from the fact that multi-affiliation corresponds to very different situations. For instance, some institutions, such as the NBER, CEPR and IZA, are not “real” departments, therefore it would be difficult to argue that being affiliated to one of these institutions and to a university is similar to being affiliated to two different universities. Lastly, the profile of each department is represented by points positioned at the median of twenty equal size intervals between zero and the maximum level of prestige reached.

5.2 Results

The presentation of the ranking of academic departments is limited to the first 50 departments; the complete results are, however, available on the companion website. Table 9 presents the importance scores obtained (γ_i) and the associated rankings of these institutions according to strong dominance, dominance and weak dominance relations.

We were surprised by the good ranking of many departments that are not economics departments but government or business schools (twelve in the top fifty). It is also interesting to compare the ranking of European departments in the dominance and weak dominance rankings. For instance, sixteen European departments³¹ are in the top fifty institutions, and five are in the top twenty when the ranking is based on dominance relations. These figures decrease to nine in the top fifty and to three in the top twenty when

³¹Excluding CPER and IZA.

the ranking is based on weak dominance relations. This leads to the remark that European departments are well ranked when one focuses on quality but, when excellence is the focus, the best US departments perform better than their European counterparts (excluding LSE and Oxford, which both rank in the top ten, in terms of volume, quality and excellence).

However, for a fully reliable analysis we believe that one must only take into consideration rankings based on weak dominance relations (if one believes in the associated assumptions about the implicit value function) because, as Table 8 shows, the weak dominance relation is the only one with an acceptable rate of completeness (.75), while completeness drops to .40 and even to .08 in the case of dominance and strong dominance relations.

Three specific institutions - NBER, CPER and IZA - are in the best three positions in the ranking associated with weak dominance. This result is not surprising and has little significance, since these institutions are not economics departments in the classical sense. The Harvard economics department ranks third *ex aequo* with IZA. The Princeton economics department ranks fifth, followed by the economics departments of Berkeley and Chicago and then followed by LSE and Oxford.

Again, it is useful to examine the architecture of weak dominance relations among top departments using the adjusted weak dominance networks (h_{\geq}) exposed in Figure 7. Interestingly, aside from the dominance relations between the very top institutions, there appears to be a parallel channel of dominance that goes directly from the Harvard department of economics and IZA to the World Bank, and even to CESifo. The latter institution is so large that only the top four institutions dominate it weakly. However, it does not employ enough highly cited scholars to be able to weakly dominate forty eight quite influential institutions. This is why it is ranked thirty-seventh, as compared to the Princeton economics department, for example, which is ranked fifth, while no dominance relations can be established between these two institutions. A similar comparison can be established with the MIT economics department, though the latter is dominated by four more institutions (the economics departments of Princeton, Chicago and Berkeley, and

KS Harvard). This is because it is much less prejudicial to be dominated by such a list of four quite independent institutions than it is favorable to dominate a series of influential institutions like the economics departments of MIT and Stanford or Harvard business school), something the MIT does but that CESifo does not. Similar statements could be made for the Tinbergen Institute, which ranks thirty-eighth. There is here a marked opposition between different types of institutions with strong positions, either because they employ a limited number of highly prestigious scholars or a large number of less prestigious scholars.

6 Conclusion

This article introduces a new theory for ranking institutions when both quantity and quality matter: it extends the well-known stochastic dominance theory and proposes a new ranking method based on the unanimous comparisons that is characterized axiomatically. We have applied this theory to compare and rank the scientific production of US research universities and the prestige of academic departments in the field of economics. We should emphasize, in conclusion, that this theory provides an original solution for the size problems that most rankings face. Although our tool is not size-independent (simply because it is not a desired implicit assumption), it does however give those smaller institutions that perform well in terms of quality the opportunity to compete with larger institutions, in particular when excellence is the focus of the underlying dominance relations.

We also believe that this theory has a great application potential because, in many situations, quality and quantity are relevant for making comparisons, not so much in order to produce new rankings (for which the social demand has been high in recent years), but because it helps to better understand and discuss their premisses which, more often than not, are implicit. Therefore, such comparisons may become truly useful to the users and to the evaluated institutions. We have not been able to describe the two applications in great detail in this article, but we have done so in more applied papers (building reference classes, focusing on specific parts of the

quality distribution, etc.). More precise information can be produced with this theory which, we trust, will help to provide tools that can be used for benchmarking universities (cf. Carayol et al., 2012, 2013).

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Appendix A.

Before proceeding with the proofs related to Section 2 of the paper, let us precise the strict upper bound of all quality levels reached by items produced by any agent in a given set $I \in \mathfrak{S}$:

$$\bar{s} = \min s \text{ such that } : \bar{s} > \max_{i \in I} \max_{j=1, \dots, n_i} s_j^i,$$

that is, \bar{s} is the lowest quality, no item produced by any agent in set I reached.

Proof of Theorem 1

The *if and only if* statement shall be proven by demonstrating that the causality holds both ways.

i) We first demonstrate the left-to-right implication: $i \blacktriangleright j \rightarrow \forall x \in [0, \bar{s}[, f_i(x) \geq f_j(x)$. Let us consider that $i \blacktriangleright j$ holds and let us further assume that there exists an $x_0 \in [0, \bar{s}[$ such that $f_i(x_0) < f_j(x_0)$. Given the latter statement, one can always find a non-negative function $v(\cdot)$ such that $V_i < V_j$. For instance, if $v(\cdot)$ is such that $v(x_0) > 0$ and $v(x) = 0$ otherwise, then obviously $f_i(x_0) < f_j(x_0)$ implies that $\sum_{s \in \mathcal{S}, s < \bar{s}} v(s) f_i(s) < \sum_{s \in \mathcal{S}, s < \bar{s}} v(s) f_j(s)$. We thus obtain a contradiction with the initial statement $i \blacktriangleright j$. Thus the inequality $f_i(x) \geq f_j(x)$ is always verified when i strongly dominates j . \square

ii) The right-to-left implication, $\forall x \in [0, \bar{s}[, f_i(x) \geq f_j(x) \rightarrow i \blacktriangleright j$, is immediate. When $\forall x \in [0, \bar{s}[, f_i(x) \geq f_j(x)$, we can multiply both sides by any non-negative function $v(\cdot)$ and the inequality still holds for all $x \in [0, \bar{s}[$. We can also integrate both sides of the inequality and then we have $\sum_{s \in \mathcal{S}, s < \bar{s}} v(s) f_i(s) \geq \sum_{s \in \mathcal{S}, s < \bar{s}} v(s) f_j(s)$, that is i strongly dominates j . \square

Proofs of Theorem 2 and Theorem 3

We here only present the proofs of Theorem 3 because it is the most original and because the proofs of Theorem 2 follow very similar paths to that of Theorem 3.

i) We begin by the left-to-right implication:

$$i \triangleright j \rightarrow \forall x \in [0, \bar{s}[, \sum_{s \in S, x \leq s < \bar{s}} s \times [f_i(s) - f_j(s)] \geq 0.$$

We assume the weak dominance of i over j and the existence of an $x \in [0, \bar{s}[$ such that $\sum_{s \in S, x \leq s < \bar{s}} s [f_i(s) - f_j(s)] < 0$. If function $v(\cdot)$ is such that $v(s) = s$ if $s \geq x$, and $v(s) = 0$ otherwise (an increasing weakly convex function), we can deduce that $\sum_{s \in S, 0 \leq s < \bar{s}} v(s) [f_i(s)] < \sum_{s \in S, 0 \leq s < \bar{s}} v(s) [f_j(s)]$, since $v(s) = 0$ when $s < x$. This inequality contradicts the initial statement. Accordingly, if i weakly dominates j , then the following inequality $\sum_{s \in S, x \leq s < \bar{s}} s [f_i(s) - f_j(s)] \geq 0$ must be true for all $x \in [0, \bar{s}[$.

ii) Consider now the right-to-left implication:

$$\forall x \in [0, \bar{s}[, \sum_{s \in S, x \leq s < \bar{s}} s [f_i(s) - f_j(s)] \geq 0 \rightarrow i \triangleright j.$$

We first assume that $\forall x \in [0, \bar{s}[, \sum_{s \in S, x \leq s < \bar{s}} s [f_i(s) - f_j(s)] \geq 0$. Let us further examine the two alternative situations. First, consider the possibility that there may exist some positive $s_0 < \bar{s}$ such that s_0 is the smallest x such that $\forall s \in [x, \bar{s}[, (f_i(s) - f_j(s)) < 0$. Then, we necessarily have $\sum_{s \in S, s_0 \leq s < \bar{s}} s (f_i(s) - f_j(s)) < 0$, which contradicts the initial statement. This is thus impossible, and the reverse is necessarily true, that is there now exists some $s_0 \geq 0$, which is the smallest x , such that $\forall s \in [x, \bar{s}[, (f_i(s) - f_j(s)) \geq 0$. If s_0 equals 0, then, for any possible value of s , multiplying both sides of the inequality by $v(s)$, with $v(\cdot)$ any positive non-decreasing and weakly convex functions, and summing over all possible values of s obviously leads to the weak dominance of i over j . If $s_0 > 0$, then let us define s_1 the smallest value such that $s_1 < s_0$ and $\forall s \in [s_1, s_0[, (f_i(s) - f_j(s)) < 0$. In other words, this means that i does better than j for some higher quality region (between s_0 and \bar{s}), while j does better in a lower quality zone (between s_1 and s_0). If $s_1 = 0$, then the initial assumption implies that $\sum_{s \in S, s_1 \leq s < s_0} s (f_i(s) - f_j(s)) \geq -\sum_{s \in S, s_0 \leq s < \bar{s}} s (f_i(s) - f_j(s))$. Since any positive non-decreasing and weakly convex function $v(\cdot)$ would

put a more than proportional weight on the higher segments of quality, then this implies the following inequality: $\sum_{s \in S, s_1 \leq s < s_0} v(s) (f_i(s) - f_j(s)) \geq -\sum_{s \in S, s_0 \leq s < \bar{s}} v(s) (f_i(s) - f_j(s))$.

If $s_1 > 0$, then there is an s_2 which is the smallest value such that $s_2 < s_1$ and $\forall s \in [s_2, s_1[$, $(f_i(s) - f_j(s)) < 0$. If $s_2 = 0$, then since $(f_i(s) - f_j(s)) \geq 0$ for all $s \in [s_2, s_1[$, the previous statement naturally extends to this situation. If $s_2 > 0$, then we can again define s_3 as the smallest value such that $0 \leq s_3 < s_2$ and $\forall s \in [s_3, s_2[$, $(f_i(s) - f_j(s)) < 0$. If $s_3 = 0$, the initial statement implies $\sum_{s \in S, s_3 \leq s < s_2} s (f_i(s) - f_j(s)) \geq -\sum_{s \in S, s_2 \leq s < \bar{s}} s (f_i(s) - f_j(s))$, and then the following inequality applies: $\sum_{s \in S, s_3 \leq s < s_2} v(s) (f_i(s) - f_j(s)) \geq -\sum_{s \in S, s_2 \leq s < \bar{s}} v(s) (f_i(s) - f_j(s))$ since the function $v(\cdot)$ is a positive non-decreasing and weakly convex function. Since $s_3 = 0$ in this situation, then i dominates weakly j . Otherwise, this reasoning can be repeated recurrently down to some $s_n = 0$. Therefore $i \succeq j$ if $\sum_{s \in S, x \leq s < \bar{s}} s f_i(s) \geq \sum_{s \in S, x \leq s < \bar{s}} s f_j(s)$ for all $x \in [0, \bar{s}[$. \square

Proofs of Theorem 14

Applying the recursivity of the right-hand side of Equation 6, one obtains:

$$\alpha_i(G) = \varepsilon \sum_{j \in I \setminus i} g_{ij} \times \left(\varepsilon \sum_{k \in I \setminus j} g_{jk} \left(\varepsilon \sum_{y \in I \setminus k} g_{ky} (\dots) + \delta \right) + \delta \right) + \delta.$$

This expression reduces to:

$$\begin{aligned} \alpha_i(G) &= \delta \sum_{k=1, \dots, n-1} \varepsilon^k \sum_{j \in I} h_{ij}^{(k)} + \delta, \\ &= \delta \left(\sum_{k=1, \dots, n-1} \varepsilon^k \sum_{j \in I} h_{ij}^{(k)} + 1 \right), \\ &= \delta \left(\sum_{k=0, \dots, n-1} \varepsilon^k \sum_{j \in I} h_{ij}^{(k)} \right), \end{aligned}$$

where $h_{ij}^{(k)}$ is the i 'th line and j 'th column entry of the k^{th} power of matrix $G - I$. It can be shown easily that $h_{ij}^{(k)}$ is equal to the number of paths

of length k that start from i and end at j . We can stop the sum at $k = n - 1$ (recalling that $n = \#I$) because there is no cycle in the network associated with $G - I$ and, thus, the longest possible path is composed of $n - 1$ intermediary links.

Let us now write $h_{ij}^{T(k)}$ i 'th line and j 'th column entry of matrix $(G^T - I)^{(k)}$. Similar computations can be introduced for $\alpha_i(G^T)$ which leads to:

$$\alpha_i(G^T) = \delta \left(\sum_{k=0, \dots, n-1} \varepsilon^k \sum_{j \in I} h_{ij}^{T(k)} \right).$$

Therefore, if we let $\varepsilon = 1$, one obtains:

$$\gamma_i(G) = \alpha_i(G) / \alpha_i(G^T) = \frac{\sum_{k=1, \dots, n-1} \sum_{j \in I} h_{ij}^{(k)}}{\sum_{k=1, \dots, n-1} \sum_{j \in I} h_{ij}^{T(k)}}.$$

Since $h_{ij}^{T(k)}$, is also equal to the i 'th line and j 'th column entry of matrix $((G_i^2)^T - I)^{(k)}$, $\gamma_i(G)$ is of the form:

$$\gamma_i(G) = \psi_i(G_i^2) \cdot \sigma_i(G_i^1),$$

with $\sigma_i(G_i^1) = \sum_{j \in I} \sum_{k=0, \dots, \infty} h_{ij}^{(k)}$ and $\psi_i(G_i^2) = 1 / \sum_{k=1, \dots, n-1} \sum_{j \in I} h_{ij}^{T(k)}$. Therefore, the importance ranking method γ with $\varepsilon = 1$ respects the Indirect dominance homogeneity axiom.

Since $\gamma_i(G^T) = \alpha_i(G^T) / \alpha_i(G)$, then obviously $\gamma_i(G) \gamma_i(G^T) = 1$ and γ thus respects the Symmetry axiom.

We have proved that the importance ranking method respects the three axioms. To show the converse, we just need to show that there is always at least one ordinaly different ranking method which would respect any one of the two axioms. If Indirect dominance homogeneity is not respected, this is obvious. If symmetry is not respected, we can find a function $\psi_i(G_i^2)$ that would generate a different order than the importance method. This completes the proof. \square

Appendix B. Tables and figures

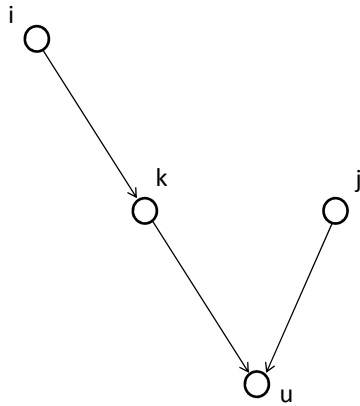


Figure 1: The adjusted dominance network (without self-dominance and dominance relations that could be inferred by transitivity) of Example 1.

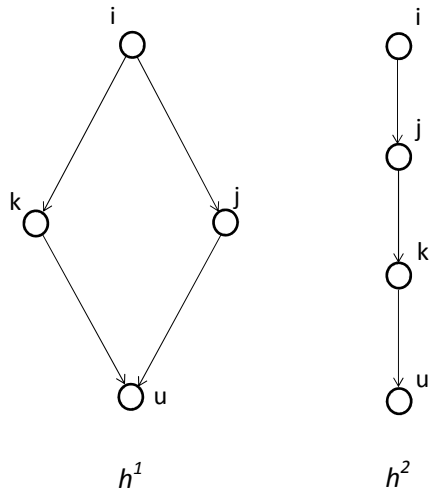


Figure 2: The adjusted dominance network (without self-dominance and dominance relations that could be inferred by transitivity) of Example 2.

Table 1: Ranking methods and axioms.

	A	DC	S	RSC	IDC	IDH
$r_i, 1/c_i$	√	√	-	√	-	-
da_i	√	√	-	√	√	-
wk_i, im_i, fb_i	√	-	-	-	-	-
α_i, α'_i	√	√	-	√	-	-
γ	√	√	√	√	√	√

Table 2: The domains.

<i>k</i>	Domain
1	Fundamental Biology
2	Medicine
3	Applied Biology/Ecology
4	Chemistry
5	Physics
6	Science of the Universe
7	Engineering Sciences
8	Mathematics

Table 3: The rate of completeness of a series of dominance relations in the set of 112 US higher education and research institutions.

Dominance relation	Citations			Journal IF			Rel. JIF		
	▶	▷	⊃	▶	▷	⊃	▶	▷	⊃
1 Fundamental Biology	.65	.86	.89	.43	.82	.85	.50	.84	.87
2 Medicine	.79	.91	.93	.57	.89	.91	.66	.90	.91
3 Applied Biology/Ecology	.66	.87	.90	.43	.84	.86	.46	.83	.85
4 Chemistry	.62	.86	.89	.38	.85	.88	.36	.87	.89
5 Physics	.72	.90	.92	.48	.87	.89	.49	.88	.89
6 Science of the Universe	.69	.87	.89	.45	.82	.84	.53	.87	.88
7 Engineering Sciences	.79	.88	.91	.51	.85	.87	.56	.87	.89
8 Mathematics	.61	.82	.85	.33	.74	.77	.37	.78	.80
All disciplines	.38	.88	.91	.57	.86	.89	.63	.88	.90

Table 4: Top 50 rankings of 112 US higher education and research institutions in all disciplines, built upon the three dominance relations.

Dominance relations based on citations						
	▶		▷		⊇	
	Rank	Importance	Rank	Importance	Rank	Importance
		γ		γ		γ
Harvard	8	.06631262979540	1	.71403332360848	1	.71836372786341
Stanford	10	.02946234989562	2	.09932232139276	2	.11430916521577
Seattle	1	.14607546280401	3	.06193494994972	3	.05222541340007
UCLA	2	.11863883565147	3	.06193494994972	3	.05222541340007
UM Ann Arbor	3	.11105727038510	5	.03450211911929	5	.02611270670003
Berkeley	12	.01455243470577	6	.01725105955964	5	.02611270670003
Johns Hopkins	18	.00503451681735	7	.00685820770761	8	.00435211778334
MIT	30	.00056601076645	8	.00144262616187	7	.00492916920781
Pennsylvania	24	.00135048182872	9	.00124469322889	9	.00100433487308
WI Madison	5	.09525365398508	10	.00100989832698	10	.00012300057440
Columbia	17	.00582395288634	11	.00011316591353	11	.00009989854470
UCSD	20	.00432327960425	12	.00010123029731	12	.00007614321272
Cornell	9	.06196328390585	13	.00008068630453	15	.00001300006071
Twin Cities	4	.10965713848916	13	.00008068630453	13	.00001974083293
UCSF	31	.00043940309501	15	.00005895520920	14	.00001903580318
Yale	21	.00298496204200	16	.00001355907399	16	.00001152145170
Duke	15	.00793408074371	17	.00000402398325	17	.00000090111913
Pittsburgh	43	.00007873082090	18	.00000401841889	18	.00000085457125
Urbana Champaign	11	.02408524761437	19	.00000356398402	19	.00000042728563
Florida	6	.07693277917638	20	.00000275443305	24	.00000003045377
UC Davis	7	.06829366747796	21	.00000129696886	23	.00000008107903
Caltech	42	.00008440511429	22	.00000074931259	20	.00000035621429
Northwestern	37	.00013554468354	23	.00000039002103	22	.00000009011191
WU St Louis	27	.00099548384801	24	.00000028394169	21	.00000012675860
UNC	22	.00189415006491	25	.00000023141248	25	.00000002759629
PA Univ Park	13	.01394173887882	26	.00000010583426	27	.00000000286111
Columbus	14	.01359418840448	27	.00000010201604	28	.00000000161522
Mayo Coll Med	49	.00005213257059	28	.00000003394804	26	.00000000369955
Arizona	19	.00471427388370	29	.00000000423716	31	.00000000019383
Texas AM	16	.00594311304770	30	.00000000269509	34	.00000000002055
Austin	34	.00026385464301	31	.00000000263323	30	.00000000026412
Chicago	35	.00015817092847	32	.00000000259915	29	.00000000041706
USC	50	.00000000000000	33	.00000000000000	33	.00000000000000

Table 5: Top 50 rankings of 112 US higher education and research institutions in all disciplines, built upon the three dominance relations.

Dominance relations based on journal IF						
	▶		▷		▹	
	Rank	Importance	Rank	Importance	Rank	Importance
		γ	σ	γ		γ
Harvard	1	.15215141770872	1	.70663630851583	1	.70785539021056
Berkeley	8	.05001972575715	2	.07405438570612	2	.07724760315529
Stanford	15	.01800625623904	2	.07405438570612	2	.07724760315529
Seattle	2	.11587131127794	4	.06301572552101	4	.06109244281970
UCLA	4	.09682832408176	5	.04370547893341	5	.04072829521314
UM Ann Arbor	3	.10878711347720	6	.02443379268513	6	.02036414760657
Johns Hopkins	6	.05862341904374	7	.00482756164690	8	.00509103690164
Pennsylvania	11	.04033939715204	8	.00385433724966	9	.00407282952131
MIT	16	.00829071790331	9	.00365164638534	7	.00534817419473
WI Madison	5	.07981185249237	10	.00085392434959	11	.00022058430493
Columbia	20	.00401985582296	11	.00028275745746	10	.00032109311347
Cornell	9	.04551600999163	12	.00022103215754	12	.00012151479855
UCSD	21	.00256365960836	13	.00019317863458	12	.00012151479855
Twin Cities	10	.04543933102272	14	.00008426206643	16	.00003738327333
UCSF	24	.00072662451487	15	.00004966463933	14	.00006075739928
Yale	25	.00067960368943	16	.00003044381549	15	.00003903175818
Caltech	40	.00004851118441	17	.00001395350485	17	.00001843962523
Urbana Champaign	7	.05519007908907	18	.00001281388256	18	.00000301489882
UC Davis	12	.03825811085317	19	.00000732987446	22	.00000134408668
Duke	23	.00108982499538	20	.00000536928237	19	.00000233228022
Pittsburgh	18	.00439417438137	21	.00000449250370	23	.00000126177978
Florida	13	.03139925532841	22	.00000241002121	27	.00000004115345
Northwestern	34	.00019100703539	23	.00000190379732	20	.00000203838405
WU St Louis	33	.00021196969141	24	.00000146635181	21	.00000179563556
UNC	28	.00051255737771	25	.00000073990229	24	.00000017866033
Columbus	14	.02598947582827	26	.00000033653802	26	.00000004837422
Mayo Coll Med	27	.00055240155419	27	.00000008514301	28	.00000002270696
PA Univ Park	19	.00404625008028	28	.00000006862328	30	.00000000790384
Chicago	44	.00001670884708	29	.00000006750265	25	.00000004946138
Austin	22	.00127380980998	30	.00000004337783	29	.00000001121607
Arizona	26	.00061408378329	31	.00000002128575	31	.00000000542942
Texas AM	17	.00662725374113	32	.00000000501392	35	.00000000023006
U. of Illinois	43	.00001500000000	33	.00000000000000	34	.00000000000000

Table 6: Top 50 rankings of 112 US higher education and research institutions in all disciplines, built upon the three dominance relations.

Dominance relations based on Rel JIF						
	▶		▷		⊇	
	Rank	Importance	Rank	Importance	Rank	Importance
		γ		γ		γ
Harvard	2	.20408809859583	1	.72404164564122	1	.72519364548870
Stanford	10	.01414619569790	2	.08834998690578	2	.09361712482322
Seattle	4	.14785768705618	3	.07412833960362	3	.07014502923916
UM Ann Arbor	1	.23760247864481	3	.07412833960362	3	.07014502923916
Berkeley	6	.03739360270965	5	.02208749672644	5	.02340428120580
UCLA	3	.18239274336202	6	.01235472326727	6	.01169083820653
MIT	17	.00361153997497	7	.00177770591277	7	.00293401194801
Johns Hopkins	9	.01985150031137	8	.00116756916290	9	.00101169747750
Pennsylvania	13	.00536516987294	9	.00115119733193	8	.00122551239930
WI Madison	7	.03079983097067	10	.00027539693741	11	.00010690746090
Columbia	15	.00383076411486	11	.00024681357274	10	.00030061647190
Cornell	12	.00733171269299	12	.00008634170215	12	.00006207529988
UCSD	16	.00362807971816	13	.00007674648880	13	.00005528079004
Twin Cities	5	.04940994375925	14	.00006652136540	14	.00004730532001
UCSF	25	.00025748136384	15	.00002544121993	15	.00002732885737
Yale	27	.00016747826285	16	.00001615178025	16	.00002416342840
Urbana Champaign	14	.00467474440921	17	.00000573281620	17	.00000291313799
Florida	8	.02634679528996	18	.00000545727699	22	.00000029122829
UC Davis	18	.00337768033865	19	.00000287856401	18	.00000196850285
Duke	22	.00044227749977	20	.00000238158843	19	.00000174655348
Pittsburgh	19	.00258817963698	21	.00000101596377	21	.00000040254010
Northwestern	32	.00006301211899	22	.00000090014994	20	.00000130576094
WU St Louis	31	.00007890231287	23	.00000067416480	23	.00000025608699
UNC	30	.00008196078121	24	.00000030296475	24	.00000014188603
Columbus	20	.00256269295670	25	.00000007319746	26	.00000001829511
PA Univ Park	21	.00126532662476	26	.00000005671535	28	.00000000856451
Caltech	36	.00001725480507	27	.00000004807556	25	.00000007613099
Mayo Coll Med	23	.00039920266109	28	.00000004102432	27	.00000001627230
Austin	26	.00025446378764	29	.00000000952510	29	.00000000303228
Texas AM	11	.00940988064384	30	.00000000465052	31	.0000000049854
Chicago	39	.00001015598266	31	.00000000265710	30	.00000000297022
USC	33	.00004989489193	32	.00000000158251	32	.00000000038023
Arizona	24	.00000440875573	33	.00000000030537	34	.00000000011151

Table 7: The weak dominance rankings of the top 20 US universities according to the ARWU Shanghai ranking published in 2005.

Dominance relation	\succeq		ARWU
	Cit	IF	2005
Harvard	1	1	1
Stanford	2	2	2
Berkeley	5	2	3
MIT	7	7	4
Caltech	20	20	5
Columbia	11	10	6
Princeton	36	36	7
Chicago	29	25	8
Yale	16	15	9
Cornell	15	12	10
UCSD	12	12	11
UCLA	3	5	12
Pennsylvania	9	9	13
WI Madison	10	11	14
Seattle	3	4	15
UCSF	14	14	16
Johns Hopkins	8	8	17
UM Ann Arbor	5	6	18
Urbana Champaign	19	18	19
WU St Louis	21	21	20

Table 8: The rate of completeness of a series of dominance relations in the set of 239 top departments in economics worldwide.

Dominance relation	▶	▷	⊇
Economics	.08	.40	.75

Table 9: Top 50 rankings of departments in economics worldwide, built upon the three dominance relations.

	Dominance relations					
	▶		▷		▹	
	Rank	Importance γ	Rank	Importance γ	Rank	Importance γ
NBER	9	.00764986026972	1	.57652326294970	1	.74457915061459
CEPR	17	.00145711624185	3	.14413081573743	2	.18614478765365
ED Harvard	28	.00091069765116	4	.01457026057300	3	.03102413127561
IZA	1	.38850361798352	2	.22998058918284	3	.03102413127561
ED Princeton	39	.00045534882558	10	.00131117381507	5	.00515936545026
ED Berkeley	42	.00036427906046	13	.00050167005087	6	.00067948572879
ED Chicago	31	.00072855812093	17	.00025303323474	6	.00067948572879
LSE	3	.14899013572926	5	.00887287875106	8	.00027061276939
ED Oxford	5	.05272939400198	6	.00834343036890	9	.00024421109639
KS Harvard	25	.00100176741627	12	.00061369783110	10	.00007254577207
BSB Chicago	31	.00072855812093	11	.00108475473997	11	.00004217343176
ED NYU	42	.00036427906046	15	.00028055962810	12	.00002353816022
World Bank	2	.15700427505944	8	.00482287401917	13	.00002245275786
ED MIT	42	.00036427906046	30	.00000725819402	14	.00001518617015
IMF	4	.12130492713410	7	.00507827287883	15	.00001320083650
ED Columbia	35	.00054641859069	14	.00028240223656	16	.00000221031526
ED Stanford	40	.00043713487256	20	.00013889416869	17	.00000111039281
BS Harvard	52	.00025499534232	32	.00000533373505	18	.00000102244731
GSB Stanford	62	.00014571162419	35	.00000235455317	19	.00000028708202
IFS	19	.00133568988836	18	.00022443154217	20	.00000027341006
HIWRP Stanford	88	.00005203986578	61	.00000012213043	21	.00000023229999
CESifo	10	.00573739520229	9	.00191174991935	22	.00000019343349
ED Boston	48	.00032785115442	19	.00018190621604	23	.00000010977563
ED UMichigan	20	.00127497671162	21	.00012726006934	24	.00000002900853
ED UCSD	35	.00054641859069	29	.00000730087440	25	.00000002274240
GSB Columbia	29	.00086516276860	31	.00000554937938	26	.00000002000586
ED UCL	12	.00285351935896	22	.00012142809697	27	.00000001887611
ED Northwestern	57	.00018213953023	38	.00000169797332	28	.00000000391853
Tinbergen Instituut	7	.02094604597661	16	.00026146380705	29	.00000000201212
WSB Pennsylvania	27	.00096273751694	23	.00009320013515	30	.00000000184942
ED UCLA	49	.00030356588372	26	.00004265336513	31	.00000000140101
ED WI Madison	68	.00010407973156	54	.00000023113844	32	.00000000097914
CEPR NYU	53	.00000100070518	57	.00000010010000	33	.00000000070000

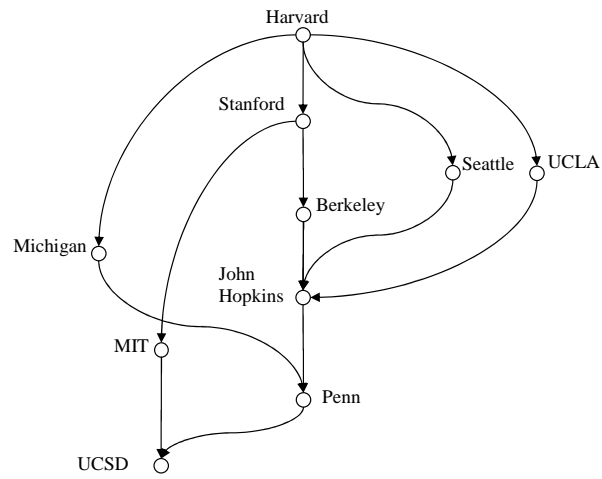


Figure 3: The adjusted dominance network (without self-dominance and dominance relations that could be inferred by transitivity) among the top US research universities associated with weak dominance, when impact is measured with citations and for all disciplines.

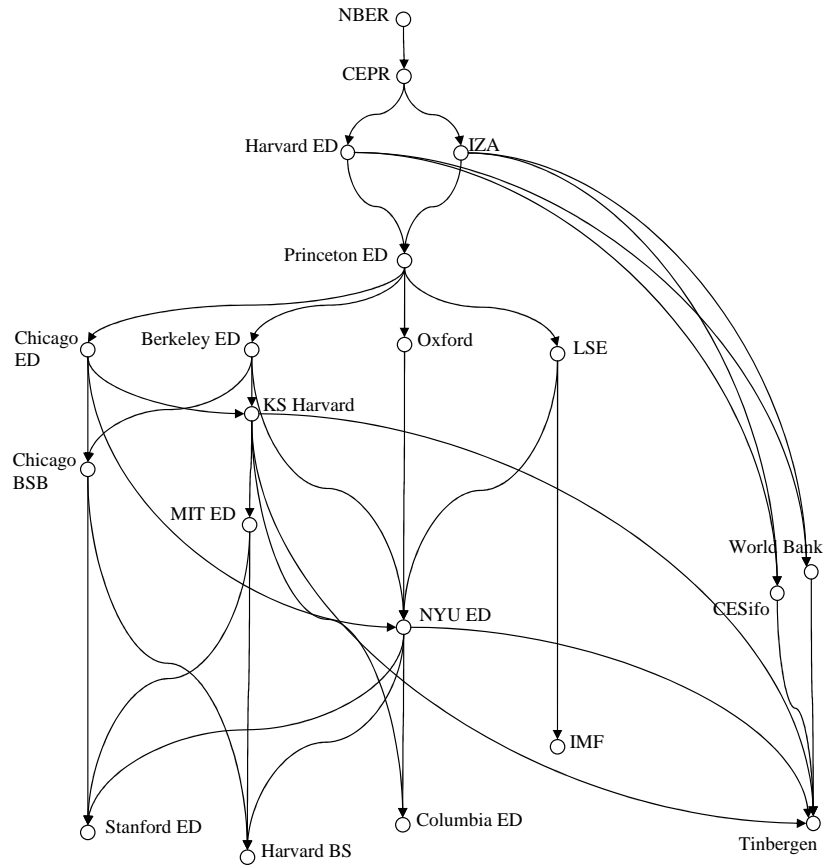


Figure 4: The adjusted dominance network (without self-dominance and dominance relations that could be inferred by transitivity) among top departments in economics associated with weak dominance .

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