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Groundwater management in food security context

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Gestion d'un aquifère en présence d'une contrainte de sécurité alimentaire

Résumé

Cet article analyse comment un instrument de marché du type permit transférable permet une gestion soutenable d'un aquifère destiné à satisfaire les besoins en eau d'une agriculture irriguée. Un modèle dynamique hydro-économique est proposé dans lequel une agence de l'eau cherche à garantir une contrainte de production agricole en présence d'exploitants myopes. Le noyau de viabilité qui détermine les trajectoires possibles de prélèvements en fonction de l'état de la ressource et des contraintes fixées est analytiquement calculé. Des simulations numériques reposant sur la calibration proposée par Gisser and Sanchez (1980) illustre les principaux résultats de l'article.

Mots clés: Aquifère, Agriculture, Irrigation, sécurité alimentaire, quotas individuels, Soutenabilité, Systèmes dynamiques, Viabilité.

Groundwater management in food security context

Abstract

This article studies the sustainability of market-based instrument such as tradable permits for the management of a renewable aquifer used in agriculture production. Based on a dynamic hydro-economic model, a water agency aims at satisfying a food security constraint within a tradable permit scheme in the presence of myopic heterogeneous agents. We identify analytically the viability kernel that defines the states of the resource yielding inter-temporal feasible paths able to satisfy the set of constraints over time. We then illustrate the results with numerical simulations based on the data from Gisser and Sanchez (1980).

Keywords: Groundwater, Agriculture, Irrigation, Food security, Individual permits, Sustainability, Dynamic model, Viability kernel.

JEL: 15, Q25, C61

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1. Introduction

With 20% of over-exploited aquifers all around the word (WWAP, 2015), groundwater resources are under extreme pressure (Wada *et al.*, 2010). With drinking water, water demand for agriculture remains the main pressure on aquifers, and this pressure continues to grow with the population increase. Indeed, it has been estimated that agriculture will need to produce more than 60% of food by 2050 as compared to the current situation (FAO, 2014), and will then demand more and more water for irrigation and crop production. To face this potential water crisis, water agencies are mobilized in order to ensure sustainable management of renewable aquifers by limiting the volume of water usable for irrigation. However such a limitation impacts indirectly the agricultural production. The question of sustainable management of renewable aquifers is thus strongly connected to the objective of food security.

The question of sustainable management of renewable water has been investigated in the literature. Notably, the use of transferable permits has been proposed as a promising way to replenish an aquifer (Provencher 1993), or to manage efficiently a groundwater aquifer for irrigated agriculture (Latinopoulos and Sartzetakis, 2015). The process behind this idea is that transferability ensures that water is used by farmers with the highest effectivenesses. As farmers differ in their productivity, economic efficiency implies that an efficient farmer will produce more than a less efficient farmer with the same water volume. In other words, efficiency implies that the total water extraction provides the maximum amount of food production. The crucial role of transferability has also been pointed out by Knapp *et al* (2003): they showed that transfers between agriculture and urban sectors and/or within a region as well as between regions reaches an efficient management of aquifers. Finally, the success of implementation of transferable quotas in fisheries (Branch, 2008; Chu, 2009; Pereau *et al*, 2012) calls for the use of individual permits in aquifers since similarities between groundwater and biological renewable resources have been highlighted (Roumasset and Wada, 2012).

This paper aims at addressing the management of water as renewable and limiting resource based on transferable quotas. We develop a dynamic hydro-economic model based on the seminal model of Gisser and Sanchez (1980). The state of art of this literature, including management issues and game theoretical models, has been addressed in Rubio and Casino (2001), Koundouri (2004), Booker *et al* (2012), Madani and Dinar (2013), Tomini (2014) and De Frutos Cachorro *et al* (2014). Similarly to Latinopoulos and

Sartzetakis (2015), we explicitly represent a water management system based on transferable permits among farmers. We extend this model by adding a food security constraint in the design of the water agency policy. This implies that the water agency has to adopt a multi-criteria management approach to balance economic efficiency, agricultural production and the water resource.

The analysis of our hydro-economic model relies on the weak invariance (Aubin, 1990) or viable control method (Clarke *et al*, 1995). This approach focuses on identifying inter-temporal feasible paths within a set of desirable objectives or constraints (Béné *et al*, 2001). This framework has already been applied to renewable resources management and especially to the regulation of fisheries (Martinet *et al*, 2007; Doyen and Pereau, 2012) but its application in groundwater management is completely new.

The paper is structured as follows. Section 2 is devoted to the description of the dynamic hydro-economic model and the objectives of the water agency. Section 3 characterizes the feasible resource states and water policies under several constraints. An application illustrates the main results in Section 4. The last section concludes.

2. The hydro-economic model

2.1. The resource dynamics

An aquifer is described by its state variable (ie the height of water) $H(t) \in [0; S_L]$ at time t where S_L stands for the height of the ground surface. At $H(t) = 0$, the aquifer is empty, at $H(t) = S_L$ the aquifer is full. The height of water increases with constant recharge $R > 0$ and decreases because of extraction $W(t)$ dedicated to agriculture by n farmers with $W(t) = \sum_{i=1}^{n} w_i(t)$. We assume that a proportion μ of the water used for irrigation comes back to the aquifer where $0 < \mu < 1$ stands for the non-absorption coefficient. Then total extraction is $(1-\mu)\,W$.

Based on Gisser and Sanchez (1980), the dynamics of the resource is

$$\begin{aligned} H(t+1) &= H(t) + \frac{R}{AS} - \frac{(1-\mu)}{AS} W(t), \\ H(0) &= H_0, \end{aligned} \tag{1}$$

with A stands for the area of the aquifer and S the storage coefficient.

Eq (1) can be rewritten as

$$H(t+1) = H(t) + \frac{1-\mu}{AS} \left(W_R - W(t) \right), \tag{2}$$

where $W_R = \frac{R}{1-\mu}$ stands for the level of extraction which maintains constant the table water $(H(t+1) = H(t))$. If the extraction is too high $(W(t) > W_R)$, the net recharge is lower than the extraction and the height of the aquifer decreases.

2.2. The water permit market

A set of n heterogeneous farmers use water denoted by w_i from the aquifer to irrigate their crops. The individual profit of farmer i is given by

$$\pi_i(t) = p_y y_i(t) - c(t)w_i(t) - m(t)\left(w_i(t) - w_i^-(t)\right). \quad (3)$$

The first term of eq (3) refers to the total income with p_y the price of the agricultural product (farmers are supposed to be price takers on the product market) and $y_i(t)$ the individual production which is assumed to be a quadratic form of the water use $w_i(t)$ as follows

$$y_i(t) = a_i w_i(t) - \frac{b_i}{2} w_i^2(t) \quad (4)$$

where $a_i > 0$ and $b_i > 0$ are technical parameters. Marginal productivity is positive and decreasing. Individual production reaches a maximum for $\overline{w_i} = a_i/b_i$ yielding $\overline{y_i} = a_i^2/2b_i = a_i\overline{w_i}/2$. It implies that individual water extraction is bounded as follows: $w_i \in [0, \overline{w_i}]$. We deduce that the maximum amount of water consumption is $\overline{W} = \sum_{i=1}^n \overline{w_i}$ and the maximum amount of production is then $\overline{Y} = \sum_{i=1}^n \overline{y_i}$. Hence farmers can be ranked according their efficiency $\overline{y}_1 > \overline{y}_2 > ... > \overline{y}_n$. Farmer n is the least efficient while farmer 1 is the most productive.

The second term of eq (3) refers to the extraction cost. The unitary cost $c(t)$ is given by

$$\begin{aligned} c(t) &= c_1(S_L - H(t)), \\ &= c_0 - c_1 H(t). \end{aligned} \quad (5)$$

where $c_0 = c_1 S_L$ stands for a fixed cost and c_1 is the marginal pumping cost. The unitary cost is the same at each point of the aquifer. It increases with the diminution of the water table. When the height of the water table is at its maximum, the unitary cost is nul (Rubio and Casino, 2001).

The third term of eq (3) refers to the transferable permit market whose the unitary price is $m(t)$. It is assumed that the water extraction is managed by a water agency which allocates transferable water permits to the n farmers at the beginning of each period t. After receiving their free of charge water entitlements $w_i^-(t)$ at each period, farmers decide whether to buy or sell water permits to other farmers, based on their annual water uses $w_i(t)$. It is assumed that water permits are not transferable through time, implying that banking or borrowing of water permits is forbidden.

The total water supply (ie the global amount of water allocated by the water agency) is equal to

$$W(t) = \sum_{i=1}^{n} w_i^-(t). \tag{6}$$

The total water quota demand depends on the optimal individual quotas, which emerge from the maximisation of individual profits

$$\max_{w_i} \pi_i(t) \tag{7}$$

First order conditions give the optimal individual water demand

$$w_i^*(t) = \left(\frac{a_i}{b_i} - \frac{c_0}{p_y b_i} - \frac{m(t)}{p_y b_i}\right) + \frac{c_1}{p_y b_i} H(t). \tag{8}$$

We deduce the aggregate water demand[1]

$$W^*(t) = \sum_{i=1}^{n} w_i^*(t) = \overline{W} - \beta\frac{c_0}{p_y} - \beta\frac{m(t)}{p_y} + \beta\frac{c_1}{p_y} H(t), \tag{9}$$

with

$$\overline{W} = \sum_{i=1}^{n} \overline{w_i}; \beta = \sum_{i=1}^{n} \frac{1}{b_i}. \tag{10}$$

The clearing market condition on the water market implies equality between water supply and demand

$$W(t) = \overline{W} - \beta\frac{c_0}{p_y} - \beta\frac{m(t)^*}{p_y} + \beta\frac{c_1}{p_y} H(t). \tag{11}$$

[1] Simple manipulations give the irrigation water demand function of Gisser and Sanchez (1980) $W = g - kp_w$ with $g = \overline{W}, k = \beta/p_y$ and p_w the price of water.

We deduce the equilibrium water price $m^*(t)$

$$m^*(t) = \frac{p_y}{\beta}(\overline{W} - W(t)) - c_0 + c_1 H(t). \tag{12}$$

This analytical expression of $m^*(t)$ confirms the economic intuitions with $\frac{\partial m^*}{\partial W} < 0$ and $\frac{\partial m^*}{\partial H} > 0$. An increase in the water supply faced to an unchanged demand implies a decrease in the water price. When the height of the water table is high, water extraction increases and thus the demand of permits, pushing up the water price.

3. The water agency constraints

We consider a management strategy of the water agency in a food security context. In other words, the total production generated by the n farmers has to fit with the food security goal. Since water is a limiting resource for farmers, the objective of the water agency can be in conflict with the agricultural production objectives. The section shows how the quota market and the food security constraints implies conditions on the water extraction and the water resource.

3.1. The water permit price constraint

If a positive permit demand exists, then the price of the water permit $m^*(t)$ is positive

$$0 \leq m^*(t). \tag{13}$$

This positivity condition on $m^*(t)$ yields a state-control constraint

$$W(t) \leq \overline{W} - \frac{\beta c_0}{p_y} + \frac{c_1 \beta}{p_y} H(t). \tag{14}$$

By denoting the above function by W_M, it implies

$$W(t) \leq W_M(H(t)) \tag{15}$$

The tradable water permit constraint entails a higher limit on the value of the total water extraction $W(t)$. This superior bound is an increasing function of the state variable $H(t)$. This bound depends on the economic parameters of farmers and on the price of agricultural product.

3.2. The food security constraint

To deal with agriculture agency objectives of food security, the aggregate production of the agricultural sector has to satisfy a minimum threshold

$$Y_{lim} \leq Y^*(t), \tag{16}$$

with the aggregated production $Y^*(t) = \sum_{i=1}^{n} y_i^*(t)$

By substituting m^* (eq 12) within w_i^* (eq 8), we obtain

$$w_i^*(t) = \frac{1}{b_i}\left(a_i - \left(\frac{\overline{W} - W(t)}{\beta}\right)\right), \tag{17}$$

and thus the optimal individual production becomes

$$y_i^*(t) = \frac{1}{2b_i}\left(a_i^2 - \left(\frac{\overline{W} - W(t)}{\beta}\right)^2\right). \tag{18}$$

Summing the individual productions ($y_i^*(t)$) yields the aggregated production

$$Y^*(t) = \overline{Y} - \frac{1}{2\beta}\left(\overline{W} - W(t)\right)^2, \tag{19}$$

with $\overline{Y} = \sum_{i=1}^{n} \overline{y_i}$.

The aggregated production constraint $Y_{lim} \leq Y^*(t)$ implies thus

$$Y_{lim} \leq \overline{Y} - \frac{1}{2\beta}\left(\overline{W} - W(t)\right)^2 \tag{20}$$

bounding the water supply $W(t)$ by an inferior limit W_{FS}

$$W_{FS} \leq W(t), \tag{21}$$

where W_{FS} is constant and independent from the state variable

$$W_{FS} = \overline{W} - \sqrt{2\beta\left(\overline{Y} - Y_{\text{lim}}\right)}. \tag{22}$$

Not surprisingly, the existence of W_{FS} impose that the objective of production Y_{lim} cannot exceed than its maximum value $\overline{Y}$. Moreover, the positivity of the food security constraint (ie $W_{FS} \geq 0$) yields a minimum threshold: $Y_{\text{lim}} \geq Y_{\text{lim}}^{min}$ with $Y_{\text{lim}}^{min} = \overline{Y} - \left(\overline{W}^2/2\beta\right)$.

3.3. The resource constraint

The existence of the water permit price constraint $W(t) \leq W_M(H(t))$ and of the food security constraint $W_{FS} \leq W(t)$ limits the level of the water table. Combining eq (15) and eq (21) gives

$$W_{FS} \leq W \leq W_M. \tag{23}$$

This yields a critical threshold on the water table

$$H_{\text{lim}} \leq H(t), \tag{24}$$

where H_{lim} is such that

$$H_{lim}(Y_{\text{lim}}) = S_L - \frac{p_y}{c_1 \beta}\sqrt{2\beta\left(\overline{Y} - Y_{\text{lim}}\right)},$$

with $S_L = c_0/c_1$. Substitute the value of Y_{lim}^{min} that ensures that the food security is binding $W_{FS} \geq 0$ implies a condition on the amount of resource threshold. It gives

$$H_{lim}^{min}(Y_{\text{lim}}^{min}) = \frac{1}{c_1}\left(c_0 - \frac{p_y \overline{W}}{\beta}\right).$$

The value of H_{lim}^{min} is positive under the condition

$$c_0 > \frac{p_y \overline{W}}{\beta}. \tag{25}$$

Eq (25) states that the marginal extraction cost of the last unit of water (c_0) is higher than the maximum value of marginal product (Rubio and Casino (2001))[2]. A violation of the condition (25) means that the food security constraint associated to the minimum extraction is not a binding constraint for the resource. On contrary, when $Y_{\text{lim}} \geq \overline{Y} - \frac{\beta}{2}\left(\frac{c_0}{p_y}\right)^2 > Y_{\text{lim}}^{min}$, a positivity constraint on H_{lim} holds. The next section shows how condition (25) impacts the set of quota supply of the water agency.

[2]The authors (pp 1123) derive a similar condition $c_0 \geq g/k$ that eliminates the possibility of a corner solution in which $H \leq 0$.

3.4. The maximum food security objective

Satisfying simultaneously the food security constraint ($W_{FS} \leq W(t)$) and the equilibrium constraint ($W(t) \leq W_R$) implies

$$W_{FS} \leq W_R. \tag{26}$$

This condition means that if the food security is too demanding, the water extraction requiring for the demanding production is higher than the recharge of the aquifer. The water table decreases towards zero, and it is not possible to define any sustainable water extraction. This allows to define a maximum threshold Y_{lim}^{max} able to sustain a non-empty groundwater. This value is computed with the limit case, where the equilibrium water extraction W_R and the food security constraint W_{FS} are overlapped ($W_{FS} = W_R$). We deduce then

$$Y_{lim}^{max} = \overline{Y} - \frac{(\overline{W} - W_R)^2}{2\beta}. \tag{27}$$

We can note that Y_{lim}^{max} is below the maximum amount of production $\overline{Y}$ for a nul extraction cost. When the food security objective Y_{lim} reaches its maximum Y_{lim}^{max}, the groundwater height as well as the allowed extraction remain constant. Moreover, the consistency of Y_{lim}^{max} (ie $Y_{lim}^{max} \geq Y_{lim}^{min}$) implies a upper bound on the water height such that

$$H_{lim}^{max}(Y_{lim}^{max}) = \frac{1}{c_1}\left(c_0 - \frac{p_y(\overline{W} - W_R)}{\beta}\right).$$

4. Results

Taking into account the described hydro-economic model, we consider that the water agency implements a quota policy in a dynamic context which satisfies all the constraints. We characterize the sustainability of the system based on the concept of viability kernel. The viability kernel is the set of initial height of water for which exists at least one regime of quotas satisfying the constraints along time. This section aims at identifying the viability kernel and the associated viable quotas.

4.1. Viability kernel

The dynamics of the aquifer given by eq (1) is taken into account in combination with

1. the water permit price constraint (15): $W(t) \leq W_M(H(t))$,
2. the food security constraint (21): $W_{FS} \leq W(t)$,
3. the resource constraint (24): $H(t) \geq H_{lim}$.

In a finite horizon context, the viability kernel can be formally defined as the set of initial situations H_0 such as it exists water extraction $W(t)$ and resources $H(t)$, satisfying the previous constraints, for any time between $t = 0, 1, ...T$.

We obtain the following proposition[3]

Proposition 1. *Assuming that $W_{FS} \leq W_R$ and $Y_{lim}^{min} \leq Y_{lim} \leq Y_{lim}^{max}$, we obtain*

- *If $H(0) < H_{lim}$ then no viability occurs $Viab = \emptyset$*
- *If $H(0) \geq H_{lim}$ the viability kernel is $Viab = [H_{\text{lim}}, S_L]$*

Proposition (1) shows that the viability of the quota management strategies depends on the initial amount of available water in the aquifer as compared with the minimum resource threshold H_{lim} emerging from the amount of water extraction W_{FS} needed to satisfy the food security constraints given by Y_{lim}.

We observe that two non viable cases can occur. The first situation emerges when the initial height of the water table H_0 is smaller than the tipping resource state H_{lim} and consequently does not belong to the viability kernel. The second case occurs when the objective of food production is too demanding. In this case, the water extraction W_{FS} exceeds the water volume extraction W_R which maintains constant the table water.

Figure (1) shows the viability kernel in the water table vs. water extraction space (H, W). The equilibrium extraction level is represented by the horizontal straight line W_R. When water extraction is above W_R, the recharge of the aquifer cannot compensate the extraction, generating then a decrease of the water table. On contrary, the volume of the aquifer increases for lower extractions ($W(t) < W_R$). The food security constraint is also represented by the horizontal straight line W_{FS}. The water permit constraint

[3]The proof is given in Appendix.

is represented by the increasing linear function W_M. The intercept with the Y-axis depends on the values of the parameters given by condition (25). The intersection of the two constraints W_M and W_R gives the critical stock H_{lim}. The viability domain corresponds to the area which lies above the food security constraint and below the water permit constraint W_M. In this area, the viability domain allows increasing or decreasing water dynamics depending on whether the system is above or below the sustainable water extraction W_R.

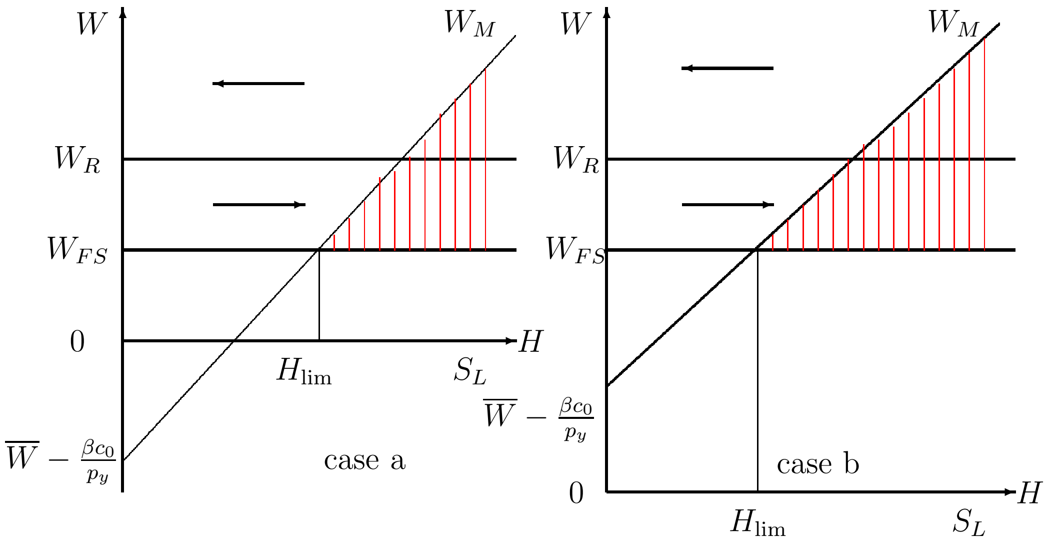


Figure 1: Viability domain when $c_0 > \frac{p_y \overline{W}}{\beta}$ (case a) and when $c_0 < \frac{p_y \overline{W}}{\beta}$ (case b).

4.2. Viable water quotas

We derive the viable water quotas from proposition (1). The viable controls have to maintain the water table of the aquifer within the viability kernel using the dynamic programming structure depicted in Doyen and De-Lara (2010). In other words, the viable quotas $W(t)$ have to comply with the additional intertemporal condition $H_{\text{lim}} \leq H(t+1)$. Using eq (1), we obtain

$$W(t) \leq W_R + \frac{AS}{(1-\mu)} \left(H(t) - H_{\text{lim}}\right). \tag{28}$$

Denoting by W_D the above function, the dynamic context of the resource threshold yields

$$W(t) \leq W_D(H(t)). \tag{29}$$

This dynamic constraint leads to a superior bound for the water extraction $W(t)$. This superior limit is an affine and increasing function of the state variable $H(t)$.

The comparison between the slopes of $W_D(H(t))$ and $W_M(H(t))$ shows[4] that the dynamic viability constraint $W_D(H(t))$ is binding and reduces the viability domain under the condition on the marginal pumping cost:

$$c_1 > \left(\frac{AS}{1-\mu}\right)\left(\frac{p_y}{\beta}\right). \tag{30}$$

Based on the extraction cost function (eq 5), a high value of c_1 means a low extraction cost. This creates incentives for farmers to increase their water consumption to get higher payoffs. It adds a dynamic constraint of the water quota setting for the water agency. It means that the viability domain is reduced and the room for manoeuvre to manage the aquifer is also reduced. When condition (30) is not satisfied, $W_D(H(t))$ is not active and the viable quotas will solely depend on $W_M(H(t))$ and W_{FS}.

We are then able to specify the viable quotas W associated to the viable water tables.

Proposition 2. *Considering that $W_{FS} \leq W_R$, the viable quotas associated to $Viab = [H_{\lim}, S_L]$ are*

$$W^{Viab} = [W_{FS}, min(W_M, W_D)].$$

Figure 2 displays the viable quota policies when the viability kernel is not empty and when condition (30) holds in the level state-control (H, W) space. The dynamic constraint W_D is represented by an increasing linear function with a negative intercept with the Y-axis for $H_{\lim} > \frac{(1-\mu)W_R}{AS}$. This configuration also satisfies condition (25). Figure 3 shows a case in which the quota dynamic constraint in not active. Based on numerical examples, the next section will show how such both configurations are possible.

[4]See appendix.

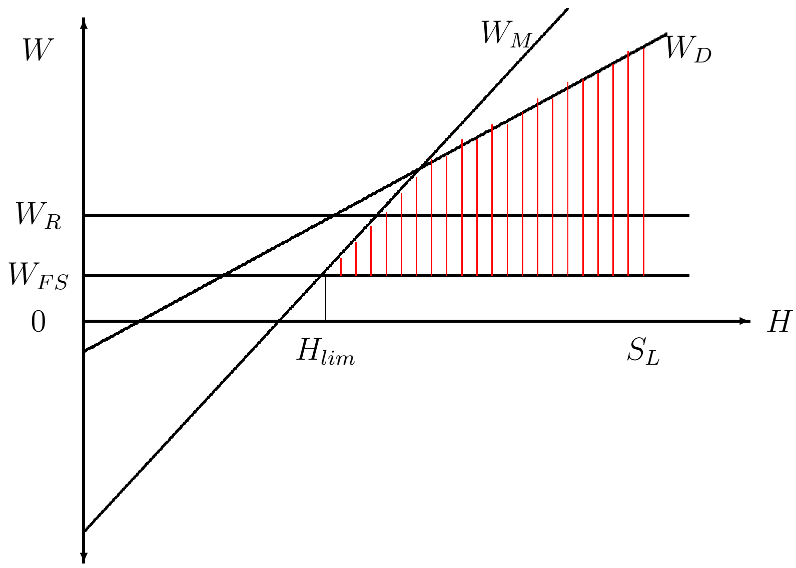


Figure 2: Viable water quotas when the quota dynamic constraint is active under the condition $c_1 > \left(\frac{AS}{1-\mu}\right)\left(\frac{p_y}{\beta}\right)$.

5. Numerical illustration

The numerical example is based on the case study of Gisser and Sanchez (1980) adapted to deal with heterogeneous farmers.

Parameters	Description	units	value
μ	Return flow coefficient	Unitless	0.27
R	Natural recharge	ac ft/yr	173000
AS	Aquifer area times storativity	ac ft/yr	1500
c_0	fixed cost	\$/ac ft	125
c_1	Pumping costs	\$/ac ft per foot of lift	0.035
H_0	Initial water table elevation	feet above sea level	3400
a	production coefficient	\$/ac ft	96.218676
b	squared production coefficient	\$/ac ft	0.0204562
n	number of farmers	Unitless	100
p_y	crop price	\$/ac ft	1.5

Gisser and Sanchez (1980) specified an aggregate linear water demand $W = g - kp_w$ with $g, k > 0$ (measured in ac ft/yr) and p_w the water price

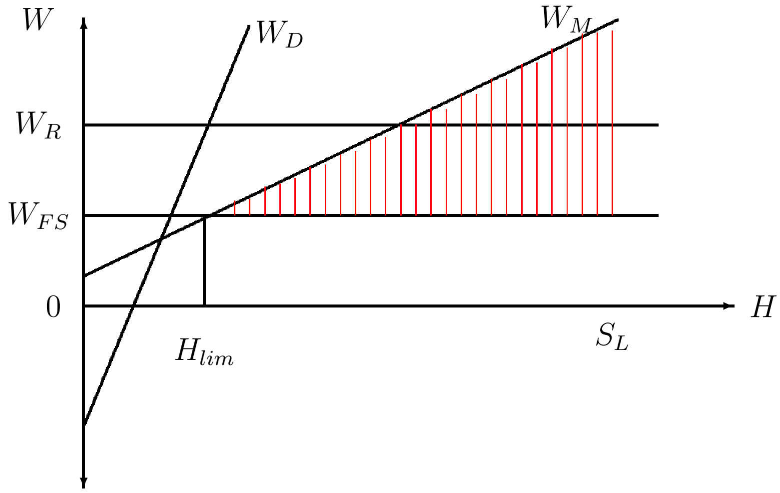


Figure 3: Viable water quotas when the quota dynamic constraint is not active under the condition $c_1 < \left(\frac{AS}{1-\mu}\right)\left(\frac{p_y}{\beta}\right)$.

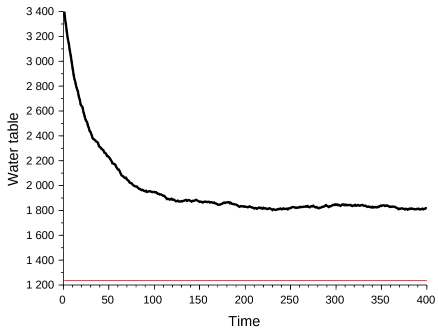
(in \$/ac ft). By identification with our model, and in particular eq (9) with $m(t) = 0$, the intercept of the demand-for-water function is $g = \overline{W}$ and the slope of the demand-for-water function $k = \frac{\beta}{p_y}$ with $\overline{W}$ and β given by eq (10).

Using the values of a, b, n and p_y gives the value of Gisser and Sanchez: $g = 470365$ and $k = 3259$. We have to note that H_0 is closed to the height of the ground surface $S_L = c_0/c_1$. Heterogeneity between the farmers is introduced through the values of b_i as a uniform random variable over the interval $[b*(1-\delta), b*(1+\delta)]$ with a dispersion rate $\delta = 10\%$. Compared to Gisser and Sanchez's case study, we choose a lower value of AS to reduce the simulation horizon.

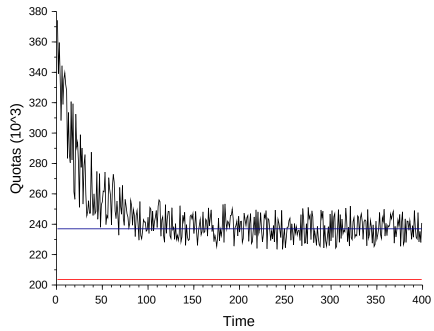
Based on these numerical specifications, it turns out that both conditions (25) and (30) are not satisfied. This configuration refers to case 3 where the dynamic constraint is not active. In other words, since $W_R = 236986$ ac ft/yr is higher than $W_{FS} = 203617$ ac ft/yr, the water agency can implement at each period a quota policy that belongs to the viability kernel.

The food security constraint corresponds to the production $Y_{lim} = 15352336$ (in lbs) and a threshold resource $H_{lim} = 1233$ feet which is higher than $\frac{(1-\mu)W_R}{AS}$. Figure 4 displays an associated viable trajectory for the water table

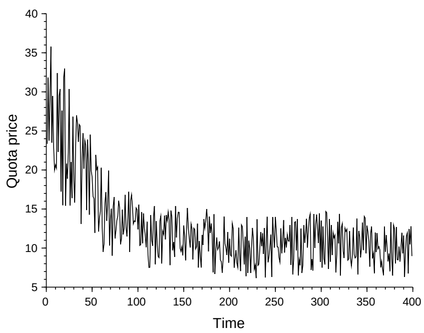
$H(t)$, the quotas $W(t)$, the price quota $m(t)$ and the mean individual profit $\sum_n \frac{\pi_i(t)}{n}$.



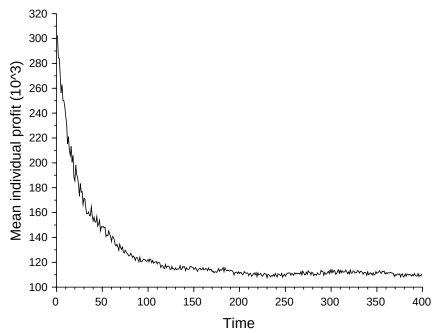
(a) Water table $H(t)$



(b) Quotas $W(t)$



(c) Price quota $m(t)$



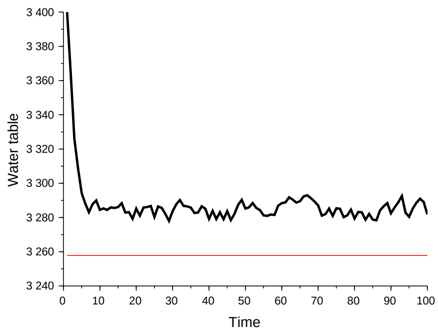
(d) Mean profit $\sum_n \frac{\pi_i(t)}{n}$

Figure 4: Trajectories of the water table $H(t)$, the quotas $W(t)$, the price quota $m(t)$ and the mean individual profit $\sum_n \frac{\pi_i(t)}{n}$. In figure (a), the red line stands for the $H_{lim} = 1233$ f. In figure (b), the red line stands for the food security constraint $W_{FS} = 203617$ ac ft/yr and the blue line represents $W_R = 236986$ ac ft/yr.

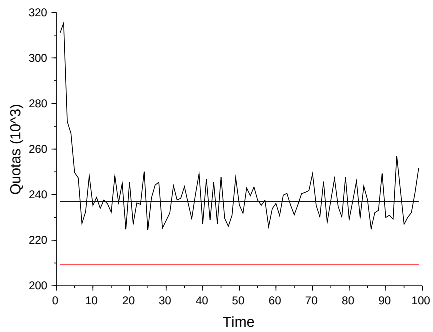
We provide a second simulation illustration, in which all the constraints are binding as explained in case 2. For that, we consider some new values for the parameters such that (25) and (30) are satisfied. It leads us to set $c_0 = 1250$ and $c_1 = 0.35$ such that S_L remains constant.

We also consider $b = 0.011$ which modifies the intercept and the slope of the demand-for-water function as follows $g = 874715$ and $k = 6060$. For these new values, the food security constraint corresponds to the production $Y_{lim} = 17742213$ lbs while its maximum value is $Y_{lim}^{max} = 19717481$ lbs.

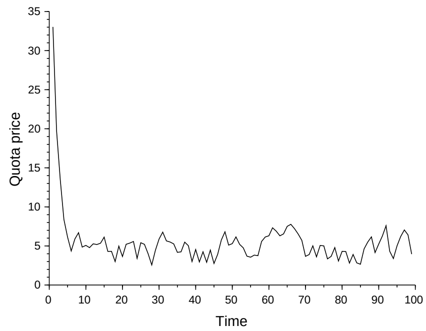
Increasing c_1 and decreasing b means that the pressure on the resource is reinforced due to a larger demand and the smaller cost of extraction. Both effects imply a higher constraint on the resource threshold value $H_{lim} = 3257$ f. Hence for a same food constraint, the resource threshold is 60% higher than in the previous case. Figure 5 displays an associated viable trajectory for the water table $H(t)$, the quotas $W(t)$, the price quota $m(t)$ and the mean individual profit $\sum_n \frac{\pi_i(t)}{n}$.



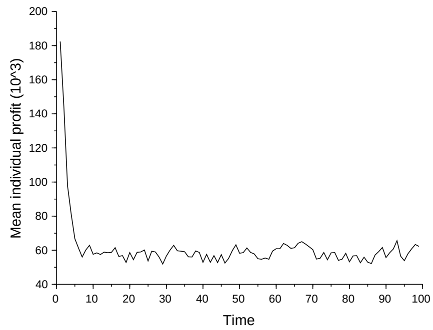
(a) Water table $H(t)$



(b) Quotas $W(t)$



(c) Price quota $m(t)$



(d) Mean profit $\sum_n \frac{\pi_i(t)}{n}$

Figure 5: Trajectories of the water table $H(t)$, the quotas $W(t)$, the price quota $m(t)$ and the mean individual profit $\sum_n \frac{\pi_i(t)}{n}$. In figure (a), the red line stands for the $H_{lim} = 3257$ f. In figure (b), the red line stands for the food security constraint $W_{FS} = 203617$ ac ft/yr and the blue line represents $W_R = 236986$ ac ft/yr.

6. Conclusion

This paper examines the problem of groundwater management in irrigated agriculture. A water agency is assumed to allocate a total amount of water to farmers using tradable permits. Our framework emphasizes how the water agency deals with the constraint of food security defined as a objective of a minimum amount of agricultural production for the whole agricultural sector. In a dynamic hydro-economic model, we determine the feasibility conditions under which the water agency ensures the joint sustainability of the resource and the agricultural activity.

Our results show that the food security constraint entails a threshold on the water resource. When the food security constraint is too demanding with respect to the net recharge, or when the initial level of the water table is below the threshold value, the over-exploitation of the aquifer leads to its depletion. Our results also show the conditions under which the water agency can select different amount of water quota among a viable set of regulation policies. The implementation of a tradable water permits ensures the economic efficiency in the use of the resource and gives flexibility to the water agency in the design of its policies. Numerical examples based on the data of Gisser and Sanchez (1980) show illustrations of the theoretical results of the paper.

Future extensions could be considered. A first one consists in introducing an individual constraint on farmers in terms of warranted payoffs. By dealing with an aggregate food security objective together with individual constraints for heterogenous farmers, the water agency will face equity and acceptability issues when setting the quota supply and the initial allocation of the water permits (Ballestero *et al*, 2002). A second extension relies on the introduction of stochasticity on the natural recharge rate of the aquifer. De Frutos Cachorro *et al* (2014) show that such an uncertainty can create incentives for the water agency to allow more extraction in the long run than in the short run. It suggests the use of robust viability theory to address dynamical control problems under constraints with uncertainty (Doyen and De Lara, 2010; Regnier and De Lara, 2015).

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9. Appendix

9.1. Proof viability kernel

Proof: Consider the dynamics

$$H(t+1) = H(t) + \frac{1-\mu}{AS}\left(W_R - W(t)\right),$$

with $W_R = \frac{R}{1-\mu}$.

We first show that $W_{FS} \leq W_R$ implies $Viab = [H_{\lim}, +\infty[$. Assume that $H_0 \geq H_{\lim}$, choose $W = W_{FS}$ then

$$H + \frac{1-\mu}{AS}(W_R - W) = H + \frac{1-\mu}{AS}(W_R - W_{FS}) \geq H \geq H_{\lim}.$$

Hence $[H_{\lim}, +\infty[$ is viable and $Viab = [H_{\lim}, +\infty[$.

Now if $W_{FS} > W_R$, we show by forward induction that

$$\begin{aligned} H(1) &= H(0) - \left(\frac{1-\mu}{AS}\right)(W_{FS} - W_R)), \\ H(2) &= H(0) - 2\left(\frac{1-\mu}{AS}\right)(W_{FS} - W_R)), \\ H(t) &= H(0) - t\left(\frac{1-\mu}{AS}\right)(W_{FS} - W_R)). \end{aligned}$$

Hence $\exists t^*$ such that $H(t^*) < H_{\lim}$, it implies that $Viab = \emptyset$.

9.2. Dynamic constraint W_D

The constraint on the state variable $H_{lim} \leq H(t+1)$ implies

$$H_{lim} \leq H(t) + \frac{R}{AS} - \frac{(1-\mu)}{AS}W(t), \tag{31}$$

$$\iff W(t) \leq W_R - \frac{AS}{(1-\mu)}H_{lim} + \frac{AS}{(1-\mu)}H(t). \tag{32}$$

By denoting $W_D = W_R - \frac{AS}{(1-\mu)}H_{lim} + \frac{AS}{(1-\mu)}H(t)$ it gives $W(t) \leq W_D(H(t))$ We look at the conditions depending on the sign of $W_M - W_D$ under which this dynamic constraint is binding and reduces the viability kernel. By definition, $W_D(H_{lim}) = W_R$ and since W is bounded by W_R, it implies that for $H = H_{lim}$

$$W_M(H_{lim}) < W_D(H_{lim}). \tag{33}$$

It yields

$$\overline{W} - W_R < \frac{\beta}{p_y}(c_0 - c_1 H_{lim}). \tag{34}$$

The expression of $W_M - W_D$ is given by

$$W_M - W_D = \overline{W} - W_R - \frac{\beta c_0}{p_y} + \frac{c_1\beta}{p_y}H - \frac{AS}{1-\mu}(H - H_{lim}). \tag{35}$$

Using (34), it gives

$$W_M - W_D < \frac{\beta}{p_y}(c_0 - c_1 H_{lim}) - \frac{\beta c_0}{p_y} + \frac{c_1 \beta}{p_y} H - \frac{AS}{1-\mu}(H - H_{lim}),$$
$$W_M - W_D < \frac{c_1 \beta}{p_y}(H - H_{lim}) - \frac{AS}{1-\mu}(H - H_{lim}). \quad (36)$$

When $H > H_{lim}$ the condition ensuring $W_M - W_D > 0$ is

$$c_1 > \left(\frac{AS}{1-\mu}\right)\left(\frac{p_y}{\beta}\right), \quad (37)$$

and corresponds to eq (30) in the text.

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