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Optimal taxation with intermittent generation

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Taxation optimale avec une production intermittente

Résumé

Cet article analyse le développement du secteur intermittent utilisant les énergies renouvelables pour produire de l'électricité, sous la concurrence du secteur historique utilisant des technologies conventionnelles. Le dommage à l'environnement est pris en compte dans l'analyse de l'interaction entre ces deux secteurs. Ceci permet de considérer les coûts sociaux de la production d'électricité dans l'analyse. On montre qu'il est socialement avantageux de maintenir certaines capacités conventionnelles en réserve. Le papier traite aussi la question de l'efficacité de la taxe environnementale dans l'internalisation du dommage à l'environnement. Puisque le dommage à l'environnement dépend de la disponibilité des sources renouvelable, le taux de taxe qui décentralise l'état optimal, doit aussi varier. Ceci ne semble pas réalisable sur le plan pratique. Faute de mieux, nous décrivons un système de marché capable d'implémenter l'équilibre 'second-best', et le taux de taxe qui le décentralise. La coexistence entre un prix de détail et une taxe, tous deux constants et la variation des sources renouvelables, favorise l'investissement dans le secteur renouvelable.

Mots-clés : Électricité, Intermittence, taxe, énergie renouvelable, pollution.

Optimal Taxation With Intermittent Generation

Abstract

The paper analyses the development of the intermittent technologies to produce electricity, facing the competition of the incumbent sector, using conventional technologies. In our analysis of the interaction between these two sectors, we consider the environmental damage caused by the electricity production from fossil fuel. This allowed us to represent the social cost of electricity production. We show that it is socially favorable to keep some conventional capacities in reserve. We then investigate the efficiency of environmental taxes in the internalization of the environmental damage. The paper shows that there is not a rate tax capable of implementing the first-best equilibrium. Effectively, this requires a variable tax rate, which seems unrealistic in practice. We also determine the constrained second-best equilibrium and the tax rate that decentralizes it. Interestingly, we find that the interaction between a retail price and tax, both constant and the intermittency of renewable energy, yield to two phenomena that, on average, promote the investment in intermittent capacities.

Keywords: Electricity, Intermittency, Tax, Renewable Energy, Pollution.

JEL: D24, D61, Q41, Q42, Q48

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http://ideas.repec.org/p/grt/wpegrt/2016-26.html .

1 Introduction

Electricity production from fossil fuel is one of the main cause of global warming due to greenhouse gas emissions. This sector has therefore attracted considerable attention in the debate on climate change mitigation. Several environmental policy instruments have been created to support renewable energies and to increase their share in the energy mix. Therefore, the increasing interest given to green the electricity production, has heightened the need to analyze the efficiency of implemented tools to achieve environmental goals.

The paper has mainly twofold aims. First, it analyses the development of intermittent renewable sector (wind, solar), given the competition of the incumbent sector using conventional technologies (coal, oil, natural gas). In this context, the environmental damage caused by the electricity production of the conventional polluting sector, is taken into account in the analysis. Second, the efficiency of the environmental tax in decentralization the environmental marginal damage, is investigated.

By treating these two issues, the present article matches two theoretical literatures. The first is dealing with the optimal and decentralized energy mix with intermittent sources. The second is that which focus on public policies aiming at greening electricity production.

The theoretical literature analyzing the penetration of the intermittent generation technologies and its effects on the electricity market, is still in its early stage, and most papers are em-

pirical and country specific. This includes, Bathurst et al (2002), Benitez et al. (2008), Bocard (2008), Green and Vasilakos (2010), Kennedy (2005), Menanteau et al. (2003) and Neuhoff et al. (2006, 2007). Sinden (2007) and Gross et al (2006), study the UK context, Holttinen (2005) , analyses the Nordic market, Sensfuss et al. (2007), study Germany while Sáenz de Miera et al. (2008), study Spain and they both estimate the impact of supporting renewable generators on wholesale market prices. We take a different approach by developing a microeconomics stylized model.

Most of papers that addressed the mix between renewable and non-renewable sources of energy in a theoretical framework, consider a deterministic supply of renewable inputs. See for example, Ambec and Doucet (2003), Crampes and Moreaux (2001) and also Garcia et al. (2001). However, we take a difference here by focusing on input variability. In this context, the closest papers to our framework, are Ambec and Crampes (2012) and Rouillon (2015)¹.

Ambec and Crampes (2012) analyze the optimal provision of electricity with intermittent sources of energy and discuss its implementation under perfect competition. First difference with the present paper, is that they postulate constant returns to scale technologies with capacity constraint, for both sectors. In contrast, we suppose an increasing marginal cost and no capacity constraints for the conventional sector. This hypothesis is more appropriate to reflect the initial situation of the electricity market, before the penetration of intermittent operators². Second, contrary to them, we can determine a constrained second-best equilibrium, while they fail in implementing it. This possibility can be attributed to our assumption of the existence of a wholesale spot market, where conventional and intermittent operators can exchange their electricity production at a price that reflects the climatic conditions. In contrast, Ambec and Crampes (2012), implicitly assume that no wholesale spot market exists.

¹ I wish to thank Mr. Sébastien Rouillon for his comments, which helped me to improve this paper

² See Rouillon (2015).

Rouillon (2015)³ analyzes the development of the intermittent technologies given the competition of incumbent generators, using various assumptions regarding the market power. However he does not consider the environmental damage caused by emissions generated by the production of conventional sector. In contrast, in our setting, the electricity price reflects the social cost of producing it, i.e. production and environmental costs.

The second literature, focusing on public policies, has so far ignored the problem of intermittency. In a dynamic framework Fischer and Newell (2008) and Acemoglu et al. (2012), looked at pollution externalities and RD spillovers. Rubin and Babcock (2013), rely on simulation to quantify the impact of price-based mechanisms on the electricity markets. In a qualitative analysis, Menanteau et al. (2003), compare feed-in-tariff and Tradable renewable quota under uncertainty. We take a different approach by developing a highly stylized model and solving it analytically. We also deal with both issues, intermittency and public policy. More precisely we focus on the question of the optimal taxation with intermittent generation. Other papers have looked at pollution externalities. Papers like Ambec and Crampes (2015) and Alzate and Barrera (2012) are the closest to our framework.

Garcia, Alzate and Barrera (2012), develop a highly stylized model of investments with intermittent source and analyze the efficiency of feed-in tariffs and renewable portfolio standards to create incentives for the investment in intermittent technologies generation. They suppose an inelastic demand and a regulated price cap. In contrast, we include consumers' surplus and environmental damage in our analysis, which is more appropriate for analyzing investment and welfare.

Using a generalization of their model of (2012), Ambec and Crampes (2015) examine the impact of public policies aiming to substitute fossil fuel by intermittent renewable sources on the energy mix. They postulate constant environmental marginal damage. In the present paper

³We use the model of Rouillon (2015) to extend the analysis to policy instrument

the assumption of increasing environmental marginal damage is adopted. It seems that this assumption is more appropriate to reflect what is really happening on the electricity market.

The rest of the paper is organized as follows. Section 2 introduces the stylized model. Section 3 describes the first-best energy mix and gives some comparative statics. We show that the optimal investment in intermittent renewable technologies can be explicitly calculated and that it is optimal to keep some conventional capacity in reserve, for topping when renewable sources are lacking. The impact of a tax on pollutants emitted by conventional generators, on electricity production and welfare is then analyzed in Section 4. We also question the ability of the environmental marginal damage taxation in the internalization of the environmental damage in section 5. In section 6, we propose a rate tax capable of implementing a constrained second-best equilibrium. analysis show that the intermittence of renewable sources affects the profitability of conventional sector but promote the investment in intermittent technologies generations. Finally, section 7, we conclude.

2 The model

We consider a model of energy production and supply with intermittent energy. On the demand side, consumers are equipped with traditional meters, they sign fixed-price contracts with retailers on the forward markets. The population size is normalized to 1. Each consumer inverse demand function is $P(D)$. Define $S(D) = \int_0^D P(s)ds$ the consumer's surplus of consuming D kWh of electricity. On the supply side, electricity can be produced by means of two technologies. First, the incumbent firms supply electricity in quantity q , using conventional generators. The cost function $C(q)$ represents their technology. It is assumed that $C'(0) = 0$, $C'(q) > 0$ and $C''(q) > 0$.⁴ The second technology comes from a competitive fringe using intermittent

⁴ As highlighted by Rouillon (2015): "This assumption is appropriate to represent the initial situation where the conventional firms own many generating units, using a large variety of conventional technologies (hydro, nuclear, coal, gas, oil), with different marginal costs of generating electricity, and where the overall capacity of this set of

generators, who seeks to enter the market. The cost of building intermittent units with capacity $\bar{\omega}k$ is $F(k)$. It is assumed that $F(0) = 0$, $F'(k) > 0$ and $F''(k) \geq 0$. The variability is modeled as a random variable ω , reflecting the climatic conditions (sun and/or wind). It is distributed on $\omega \in [\omega_0, \omega_1]$, with cumulative distribution function $G(\omega)$. For all ω , given the installed capacity $\bar{\omega}k$, the intermittent generation will be equal to ωk , at a negligible marginal cost. To simplify and normalize the units, it is assumed that $E[\omega] = 1$. Accordingly, the variance of ω is $V = E[\omega^2] - 1 > 0$ and the intermittent generation has a capacity factor of $1/\bar{\omega}$.⁵

Producing electricity from conventional generators emits air pollutants (SO₂), which causes damages to society. It is assumed that emissions, denoted Z , are proportional to production. Without loss of generality, we normalize to the units, so that $Z = q$. The damage from pollution depends on total pollution Z . The social damage function $d(Z)$ is twice continuously differentiable, increasing and convex i.e. $d''(Z) \geq 0$.⁶

Importantly, we assume that consumer demand does not vary with weather conditions. Furthermore, electricity cannot be stored or transported, the only way to balance supply and demand is to rely on production adjustment or / and price variations.

For the rest of the article, we will use the following linear quadratic specification of the model⁷:

$$P(D) = a - bD, \quad (1)$$

$$C(q) = \frac{1}{2}cq^2, \quad (2)$$

$$F(k) = \left(\gamma + \frac{1}{2}\delta k \right) k, \quad (3)$$

$$d(Z) = z\frac{Z^2}{2}. \quad (4)$$

generating units must be sufficient to match the demand and to prevent black-out".

⁵ $\frac{E[\omega k]}{\bar{\omega}k} = \frac{k}{\bar{\omega}k} = \frac{1}{\bar{\omega}}$.

⁶With this assumption, we simply forbid that the environmental damage of the last unit of pollution decreases as pollution increases.

⁷From our assumptions above, all parameters are positive.

3 Optimal policy

The social problem is to choose the consumption of the consumers D^o , the electric generation $q^o(\omega)$ and emissions $Z^o(\omega)$ of the conventional generators, for all ω and the capacity of the intermittent generators, k^o ,⁸ to maximize

$$\int_{\omega_0}^{\omega_1} [S(D) - C(q(\omega)) - F(k) - d(Z(\omega))] dG(\omega), \quad (5)$$

subject to

$$D = q(\omega) + \omega k, \quad (6)$$

and

$$Z(\omega) = q(\omega), \quad (7)$$

for all ω .

The Lagrangian of this problem writes:

$$L = \int_{\omega_0}^{\omega_1} \left[\begin{array}{l} S(D) - C(q(\omega)) - F(k) - d(Z(\omega)) \\ -\lambda(\omega)(D - q(\omega) - \omega k) \end{array} \right] dG(\omega). \quad (8)$$

Where the Lagrangian multiplier $\lambda(\omega)$ reflects the implicit price of electricity in the state ω ⁹.

Let D^o , $q^o(\omega)$, $Z^o(\omega)$ and k^o be the solution. It satisfies the first order conditions:

$$\lambda(\omega) = C'(q^o(\omega)) + d'(Z^o(\omega)), \quad (9)$$

for all ω .

$$P(D^o) = \int_{\omega_0}^{\omega_1} (C'(q^o(\omega)) + d'(Z^o(\omega))) dG(\omega), \quad (10)$$

⁸ With a slight misuse of language, we will sometimes use k to refer to intermittent capacity. Strictly speaking, it is equal to $\bar{\omega}k$.

⁹Below, it will be assumed that $\lambda(\omega) > 0$, for all ω .

$$F'(k^o) = \int_{\omega_0}^{\omega_1} (C'(q^o(\omega)) + d'(Z^o(\omega)))\omega dG(\omega). \quad (11)$$

The market price reflects the social costs of producing electricity, i.e. production and environmental marginal costs. So that, consumers should raise their consumption as long as their marginal propensity to pay is larger than the expected (implicit) price of electricity minus the environmental marginal damage. The conventional generators should increase their production as long as their marginal cost is smaller than the (implicit) price of electricity minus the marginal damage of the environment. While intermittent generators should increase their capacity as long as their marginal cost of investment is smaller than their expected marginal benefit of investment. In the state of nature ω , the marginal benefit of investing in the intermittent units is the product of the implicit price of electricity plus the environmental marginal damage, times the productivity of the marginal generating unit ω .

Considering the linear-quadratic specification, from (1) to (4), we can show¹⁰:

$$q^o(\omega) = \frac{a - b\omega k^o}{b + s}, \quad (12)$$

$$k^o = \frac{a \frac{s}{b+s} - \gamma}{\delta + b \frac{s}{b+s} (V + 1)}, \quad (13)$$

$$D^o = \frac{a}{b + s} \frac{s \frac{(a-\gamma)s - b\gamma}{a(b+s)} + \delta + b \frac{s}{b+s} (V + 1)}{\delta + b \frac{s}{b+s} (V + 1)}. \quad (14)$$

3.1 Comparative statics

From the results above, we can derive some comparative statics which we see relevant. In particular, how the implicit price of electricity $P(D^o)$ and the intermittent capacity, $\bar{\omega}k^o$, vary with respect to the model's parameters. Our results are given in the table 1 below.

¹⁰ To simplify, we let $s = c + z$.

	a	b	c	γ	δ	V	z
$P(D^0)$	+	-	+	+	+	+	+
ωk^0	+	-	+	-	-	-	+

Table 1. Comparative statics

The results of comparative statics generally go in the expected direction. Nevertheless, it is interesting to analyze two of them.

Firstly, the non-reactivity of consumers and the risk aversion of the conventional operators increase the negative effect of the variability of intermittent sources on the social welfare. This is so because consumers and conventional operators prefer to have, respectively, a constant electricity consumption and production. In fact, more the variability of the climatic conditions increases, fewer intermittent capacities will be installed and inversely larger will be the expected electricity price. Formally, we show that $P(D^o)$ is increasing and the intermittent capacity $\bar{\omega}k^o$ is decreasing in V ¹¹.

Second, the increase of the environmental damage results on a higher expected electricity price. In fact, the higher the environmental damage caused by conventional operators is, the more they will have to pay, which reduces their profits. Inversely, intermittent operators benefit from a higher expected benefit of investment. Thus, their installed capacities are larger when the environmental marginal damage increases. This result is illustrated by the fact that the implicit price of electricity $P(D0)$ and the intermittent capacity $\bar{\omega}k^o$, are increasing in emissions z .

¹¹Since b and c represent, respectively, the risk aversion of consumers and the conventional operators, this result is true only if they are strictly positive, if not, D^o and k^o no longer depend on V .

4 Perfect competition

In this section, assuming perfect competition, we analyze the impact of a tax on pollutants emitted by conventional generators on electricity production and welfare. We consider a market economy with free entry and price-taker producers and retailers.

The system of markets is as follows. There is a full set of spot market and forward markets. On the spot market electricity operators sell to retailers at real-time pricing. Consumers sign forward contracts with retailers at fixed prices.

Finally, assume that the regulator charges a tax to conventional generators on their polluting emitted. Let T denote the tax rate per unit of emissions.

The timing of the decisions is the following. In the first stage, the intermittent generators invest in generating units (k). In the second stage, consumers sign contracts with retailers (quantities \bar{q} at price \bar{p}). In the third stage nature determines the climatic conditions. In the last stage, generating units decide their electricity production on the spot market ($q(\omega)$ and $r(\omega)$ at price $p(\omega)$)¹²

The market equilibrium is now obtained by working backward in the game tree.

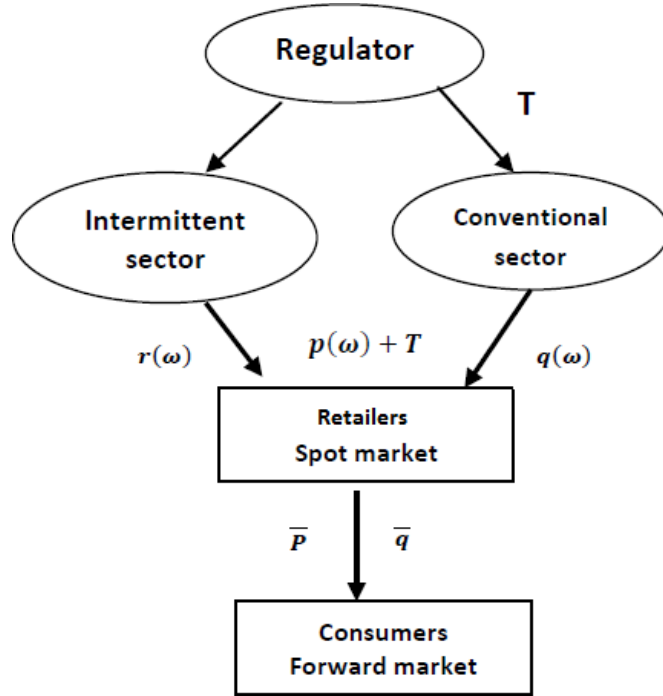
4.1 Spot market

For all ω , let $p(\omega)$ be the equilibrium spot price of electricity in the state ω . Anticipating equilibrium price and quantities of the forward market, the intermittent generators supply ωk ¹³.

The spot market clearing condition is: $D = q(\omega) + \omega k$.

¹²We suppose that the forecasts of operators are constantly updated to minimize the errors when operators submit their bids in the spot market.

¹³They choose $r(\omega) \leq \omega k$ to maximize $\pi = p(\omega)r(\omega)$. With $p(\omega) \geq 0$.



1 2: Figure1: Market design

Conventional generators supply $q^*(\omega)$ such that ¹⁴

$$C'(q(\omega)) = p(\omega) - T. \quad (15)$$

Hence, the conventional operators increase their production as long as their marginal cost, are smaller than the electricity market price minus the tax. The extra costs caused by the introduction of the tax will be charged to the retailers, through the increase of electricity price on the spot market. Retailers in their turn, will charge them to final consumers on the forward market.

4.2 Forward market

Consider the market of contracts. Each consumer demands D such that $P(D) = \bar{p}$. Retailers supply \bar{q} at price \bar{p} . They anticipate they will buy their electricity at the spot price $p(\omega)$, for

¹⁴They choose $q(\omega)$ and $Z(\omega)$ to maximize $\pi = p(\omega)q(\omega) - C(q(\omega)) - TZ(\omega)$, subject to $Z(\omega) = q(\omega)$.

all ω . Thus, in equilibrium, the price of contracts \bar{p} must be equal to the expected price of the electricity on the spot markets $\bar{p} = E[p(\omega)]$, with $p(\omega) = C'(q(\omega)) + T$. Thus competition among retailers drives \bar{p} to be equal to the average of the wholesale price. The forward clearing condition is $D = \bar{q}$.

The equilibrium forward market checks:

$$P(D) = \int_{\omega_0}^{\omega_1} C'(q(\omega))dG(\omega) + T. \quad (16)$$

On the forward market, retailers transfer the costs entailed by the introduction of the tax, to final consumers. Indeed, the latter will bear an increase in their bills by the amount of the tax. So that, consumers should raise their consumption as long as their marginal propensity to pay is larger than the expected price of electricity that includes conventional marginal costs and tax.

4.3 Investment

The intermittent generators anticipate the equilibrium price $p(\omega)$, for all ω and correspondingly choose k to maximize:

$$\pi = \int_{\omega_0}^{\omega_1} (p(\omega) \omega k)dG(\omega) - F(k). \quad (17)$$

Under the assumption of perfect competition, the equilibrium capacity will satisfy:

$$F'(k) = \int_{\omega_0}^{\omega_1} C'(q(\omega))\omega dG(\omega) + T. \quad (18)$$

This means that the intermittent generators should increase their capacity as long as their marginal cost of investment is smaller than their expected marginal profit of investment. This equality shows that intermittent operators benefit from the introduction of the tax due to a higher market price, which induces an increase in their installed capacities.

4.4 Equilibrium outcome

Considering the linear-quadratic specification, from (15) to (16), we can show that the equilibrium outcome (denoted $q^*(\omega)$, $Z^*(\omega)$ for all ω , D^* and k^*) is:

$$Z^*(\omega) = q^*(\omega) = \frac{a - bk\omega - T}{b + c}, \quad (19)$$

$$k^* = \frac{\frac{Tb+ac}{b+c} - \gamma}{\delta + b\frac{c}{b+c}(V+1)} \quad (20)$$

$$D^* = (a - T) \frac{c\frac{b(T-\gamma)+c(a-\gamma)}{(b+c)(a-T)} + \delta + b\frac{c}{b+c}(V+1)}{\delta(b+c) + bc(V+1)} \quad (21)$$

The tax has mainly three effects. First, it increases total costs of conventional generators, following that, their electricity production $q^*(\omega)$, decline¹⁵. Second, these extra costs will be charged to the final consumer through retailers, which increases the electricity prices $P(D^*)$. So that, the electricity consumption decline D^* ¹⁶. Finally, Intermittent operators, benefit when to them, from a higher selling price. Therefore, their installed capacity increase k^* ¹⁷.

4.5 Internalization of the expected environmental marginal damage

In this section we examine the efficiency of the tax in internalization the environmental marginal damage.

Let T^p , be the tax rate equal to the expected marginal damage of emissions, i.e.

$$T^p = \int_{\omega_0}^{\omega_1} d'(Z(\omega)) dG(\omega) \quad (22)$$

¹⁸

¹⁵We can show that: $\frac{\partial}{\partial T}(q^*(\omega)) < 0$.

¹⁶We can show that: $\frac{\partial}{\partial T}(P(D^*)) > 0$ and $\frac{\partial}{\partial T}(D^*) < 0$.

¹⁷We can show that: $\frac{\partial}{\partial T}(k^*) > 0$.

¹⁸Note that in our setting the tax rate should vary with the state of nature which seems unrealistic. In fact usually the rate tax is set every year at a fixed and known rate.

First, let us consider the implementation of the optimal demand level with the taxation at the expected marginal of environment. This is like comparing equations (10) and (16), which represent, respectively, the optimal demand level and the equilibrium demand level under regulated perfect competition. So, if by substituting optimal quantities in equation (16) and T by the expected marginal damage of emissions, we find that the two conditions (10 and 16) coincide, this means that the proposed tax rate here, is optimal.

So, if we inject the optimal quantities i.e. $q^o(\omega)$, $Z^o(\omega)$, D^o and substitute T by the the proposed tax rate, $T^p = \int_{\omega_0}^{\omega_1} d'(Z^o(\omega)) dG(\omega)$, in the equilibrium condition (equation (16)), we get:

$$P(D^o) = \int_{\omega_0}^{\omega_1} (C'(q^o(\omega)) + d'(Z^o(\omega)))dG(\omega), \quad (23)$$

This condition is the same of the optimal state, i.e., equation 10. Thus, the taxation at the expected marginal emissions results in the equality between the equilibrium and optimal demand level. Formally, equations (10) and (16) coincide.

However, using the same reasoning for the investment in intermittent generation, we can show that the assumed tax rate does not provide the optimal level of incentives for the investment in intermittent capacities. To show that, we compare equations (11) and (18), which represent, respectively, the optimal and equilibrium investment in intermittent generations. Thus, if we inject equilibrium quantities i.e. $q^o(\omega)$, $Z^o(\omega)$, D^o and and replace T by T^p , in equation (18), we find that the latter will be equal to :

$$F'(k^o) = \int_{\omega_0}^{\omega_1} (C'(q^o(\omega))\omega + d'(Z^o(\omega)))dG(\omega). \quad (24)$$

As it is shown in the table below, this equation differs from equation (11), that represents the equilibrium level of investment in the intermittent capacities.

This result has economic interpretation. In fact, the objective of the establishment of a tax is to integrate the environmental damage in the costs of conventional operators. This increase

in costs, should result in the reduction of conventional production and consumption and an increase of the intermittent production. However, since consumers are not equipped to receive the price signal, in the short run, this mechanism is interrupted at the level of the demand side.

Optimal policy	Competitive equilibrium
$P(D^o) = \int_{\omega_0}^{\omega_1} (C'(q^o(\omega)) + d'(Z^o(\omega))) dG(\omega)$	$P(D^o) = \int_{\omega_0}^{\omega_1} C'(q^o(\omega)) dG(\omega) + T$
Substituting T by $T^P = \int_{\omega_0}^{\omega_1} d'(Z^o(\omega))dG(\omega)$ implies that optimal and equilibrium competitive conditions are equal.	
$F'(k^o) = \int_{\omega_0}^{\omega_1} (C'(q^o(\omega))\omega + d'(Z^o(\omega))\omega)dG(\omega)$	$F'(k^o) = \int_{\omega_0}^{\omega_1} C'(q^o(\omega))\omega dG(\omega) + T$
If we substitute the optimal quantities and T by $T^P = \int_{\omega_0}^{\omega_1} d'(Z^o(\omega))dG(\omega)$ in the equilibrium competitive conditions, does not yield to the same condition of the optimal state (equation in the left differs from that in the right)	

Table 2: optimal policy vs competitive equilibrium

5 Second-best solution:

While it is clear that the tax rate equalizing the expected marginal damage of emissions, does not yield the first-best resource allocation, there is still a question of what tax rate minimizes the resulting loss associated to imbalance in incentives of investment in intermittent capacities. To answer this question we use previous results to determine a constrained second-best tax rate.

The social problem is to choose the consumption of the consumers D^t , the electric generation of the conventional generators, $q^t(\omega)$, emissions, $Z^t(\omega)$ for all ω , the capacity of the intermittent generators, k^t , and the tax T^t to maximize:

$$\int_{\omega_0}^{\omega_1} [S(D) - C(q(\omega)) - F(k) - d(Z(\omega))] dG(\omega), \quad (25)$$

subject to :

$$D = q(\omega) + \omega k, \quad (26)$$

$$Z(\omega) = q(\omega), \quad (27)$$

for all ω .

$$P(D) - \int_{\omega_0}^{\omega_1} C'(q(\omega)) dG(\omega) = F'(k) - \int_{\omega_0}^{\omega_1} C'(q(\omega)) \omega dG(\omega) \quad (28)$$

The Lagrangian of this problem writes:

$$L = \int_{\omega_0}^{\omega_1} \left[\begin{array}{c} S(D) - C(q(\omega)) - F(k) - d(q(\omega)) \\ -\lambda(\omega)(D - q(\omega) - \omega k) \\ -\beta \left(F'(k) - P(D) - \int_{\omega_0}^{\omega_1} C'(q(\omega)) (\omega - 1) dG(\omega) \right) \end{array} \right] dG(\omega) \quad (29)$$

Where the Lagrangian multipliers $\lambda(\omega)$ and β , reflect respectively the implicit price of electricity in the state ω and the constraint of incentives for each submarket.

Let $D^t, q^t(\omega)$ and k^t and T^t be the solution. It satisfies the following first order conditions:

$$\lambda(\omega) = d'(Z(\omega)) + C'(q(\omega)) - \beta(\omega - 1)C''(q(\omega)), \quad (30)$$

$$P(D) - \beta P'(D) = \int_{\omega_0}^{\omega_1} \lambda(\omega) dG(\omega), \quad (31)$$

$$F'(k) + \beta F''(k) = \int_{\omega_0}^{\omega_1} \lambda(\omega) \omega dG(\omega). \quad (32)$$

Rewrite we obtain:

$$P(D) - \int_{\omega_0}^{\omega_1} \left(C'(q(\omega)) + d'(q(\omega)) \right) dG(\omega) = \Delta, \quad (33)$$

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$$F'(k) - \int_{\omega_0}^{\omega_1} \left(C'(q(\omega)) + d'(q(\omega)) \right) \omega dG(\omega) = \Delta', \quad (34)$$

20

¹⁹ With $\Delta = \beta \int_{\omega_0}^{\omega_1} (\omega - 1) C''(q(\omega)) dG(\omega) + P'(D)$.

²⁰ with $\Delta' = -\beta \int_{\omega_0}^{\omega_1} (\omega - 1) \omega C''(q(\omega)) dG(\omega) + F''(k)$.

$$T^t - \int_{\omega_0}^{\omega_1} d'(Z(\omega))dG(\omega) = \Delta'' \quad (35)$$

²¹, ²²

It is easy to see that the results of the second-best equilibrium are different from those of the optimal state. In fact, If we inject the optimal quantities in equations from (35) to (37), they should be equal to zero²³. However, here, each condition is equal to a term²⁴, which is in general, different from zero. So that, at this stage, all what these results tell us, is that they deviate from the rule of the optimal state, in one sense or another. The question that arises at this stage of analysis is, what is the sense of their deviations from the optimal state? However, we can not answer this question with the general model. In fact, the latter cannot inform us about the direction of variation compared to the optimal state. So, to answer this question, we resolve analytically the model with linear quadratic specification.

Considering the linear-quadratic specification, from (1) to (4), we can show that the equilibrium outcome (denoted $q^t(\omega)$, $Z^t(\omega)$ for all ω , D^t , k^t and T^t) is:²⁵

$$Z^t(\omega) = q^t(\omega) = \frac{a - bk^t\omega}{b + s + bz\frac{\omega-1}{b+\delta+Vc}}, \quad (36)$$

$$D^t = \frac{a + \frac{(a\frac{s}{b+s} - \gamma)(s + \frac{Vbz}{(b+\delta+Vc)})}{\delta - V\frac{bz}{b+\delta} + \frac{\delta+Vc}{b+s} + b\frac{s}{b+s}(V+1)}}{b + s}, \quad (37)$$

²¹with $\Delta'' = b\beta$

²²with $\beta = \frac{\int_{\omega_0}^{\omega_1} (C'(q(\omega)) + d'(q(\omega)))dG(\omega) - P(D)}{\int_{\omega_0}^{\omega_1} (\omega-1)C''(q(\omega))dG(\omega) + P'(D)}$

²³See equations 13, 14 and 25.

²⁴Respectively, Δ , Δ' and Δ''

²⁵ The analytical resolution of the model with the linear-quadratic specification, led to determine the following values of Delta:

$\Delta = -b < 0$ and $\Delta' = V + \delta > 0$

$$k^t = \frac{a \frac{s}{b+s} - \gamma}{\delta - Vbz \frac{\delta + Vc}{(b+\delta+Vc)(b+s)} + b \frac{s}{b+s} (V+1)}, \quad (38)$$

$$\beta = -z \frac{b \frac{kV}{b+s}}{Vc + b + \delta} < 0, \quad (39)$$

$$T^t = \int_{\omega_0}^{\omega_1} d'(Z^t(\omega)) dG(\omega) + b\beta. \quad (40)$$

The interaction between a retail price and tax, both constant for all ω and the intermittency of renewable energy, yield to two phenomena that, on average, promote investment in intermittent capacities.

On the one hand, when there is a lot of wind and/or solar ($\bar{\omega}$), the tax should decrease (because there is little of fossil energy). But it remains constant by assumption. So, when there is a lot of renewable energy that are available, the electricity supply from fossil fuels, is too low compared to the optimum²⁶. As we can see on Figure 2 below, the distance between the point E^* and point B , represents the extra price enjoyed by the intermittent sector, induced by the constant tax rate, when renewable sources are available. Thus, with a slight abuse of language, we can say that it represents the losses of the conventional sector during these periods. So that, when the electricity production of the conventional sector decreases and that of the intermittent sector increases and because of the constant tax rate, the price of electricity is maintained above what it should be at the optimum. While at the same time, it increases the profitability of investment in the intermittent sector during windy and/or sunny periods and reduce that of the conventional sector, which does not profit from the price increase. We can see this on the figure below, in fact in windy and/or sunny days, the electricity price is maintained at the level of $p^*(\omega)$ instead of $p(\bar{\omega})$. So that, the difference between these two prices represents an extra profit for the intermittent sector and a loss of profitability for the conventional sector.

²⁶ $q^t(\omega)$ is decreasing in ω and $q^t(\omega) < q^o(\omega)$

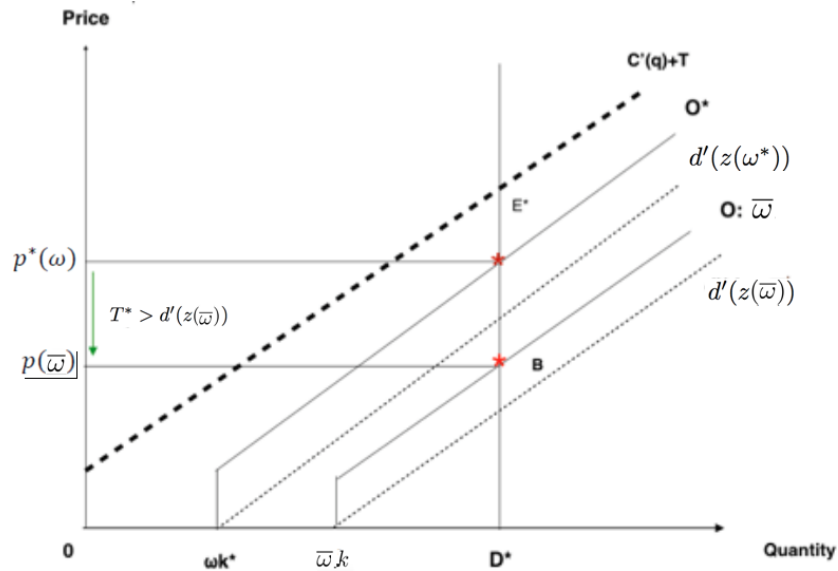


Figure 2: Case with a lot of renewable energy sources

On the other hand, we have the inverse phenomena, when there is little renewable energies available ($\underline{\omega}$). To compensate, the fossil production must be larger and, as a result, the marginal damage is greater. We should therefore reduce fossil production, compared to the optimum. So the constant tax leads to too low electricity prices compared to the optimum. Thus, not only, the profitability of intermittent sector is too low in periods without wind and/or solar, but also the profits of conventional sector. In fact, as we can see on the figure 3, during the periods with little wind and/or sun and because of the assumption of constant tax and demand, the electricity price is kept at the level of $p^*(\omega)$ instead of increasing up $p(\underline{\omega})$. In fact, the distance from point E^* to point A represents the difference between the tax rate that should be during these periods and the tax rate that is really applied on conventional production.

The investment in intermittent capacities, depends on the average of the selling price of renewable energy, which is equal to the average of the electricity selling price, weighted by renewable supply. As selling prices is too high when the intermittent sector sells a lot (very

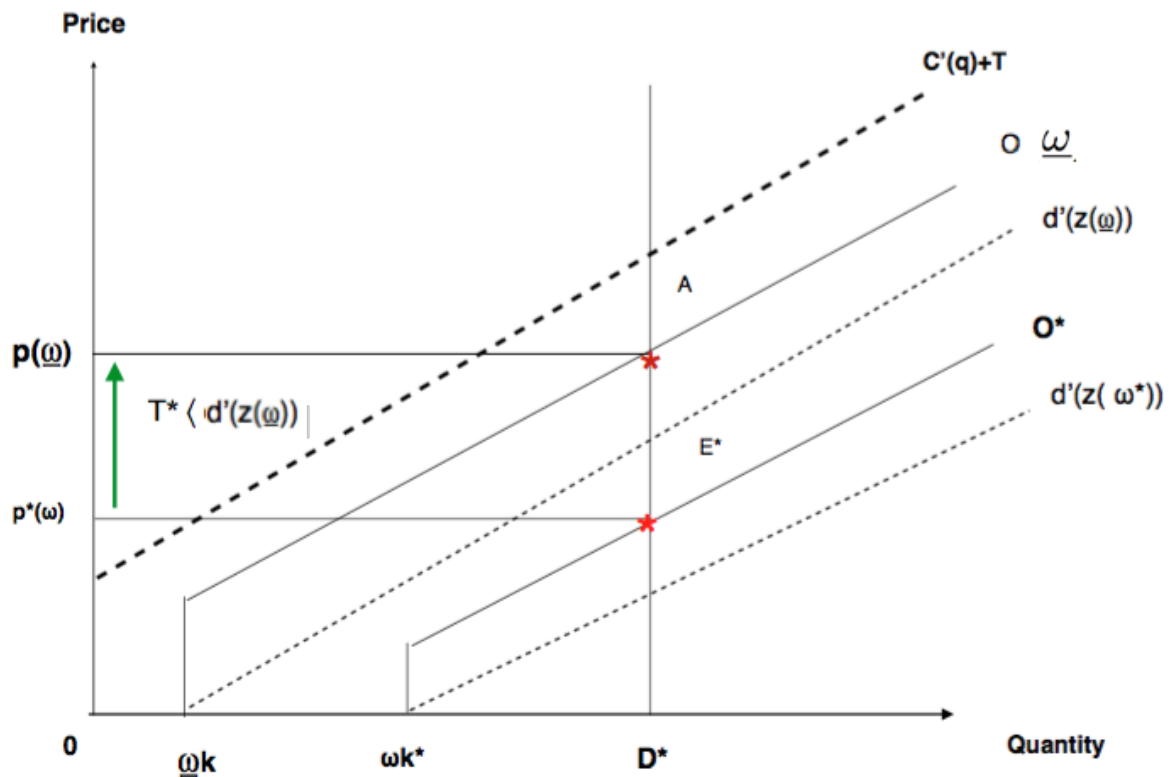


Figure 3: Case with little renewable energy sources

windy, $\bar{\omega}$) and too low when the sector sells little (no wind, $\underline{\omega}$), on average, constant tax, advantage intermittent sector and support investment in renewable capacities.

Finally, as there is an over-investment in the intermittent sector, on average, the environmental marginal damage is lower (because with constant demand, less fossil electricity is used), which justifies applying a lower rate tax than the expected environmental marginal damage that would prevail in the optimal state.²⁷

²⁷ $T^t < \int_{\omega_0}^{\omega_1} d'(Z^o(\omega))dG(\omega).$

6 Conclusion

In this paper, we consider the optimal and the equilibrium development of the intermittent electricity sector, given the competition of the conventional operators. We have shown that it is socially optimal to keep some conventional capacities in reserve for topping when renewable sources are lacking. The assumption of increasing environmental marginal damage, prevents us to decentralize the optimal resources allocation at this tax rate. Under the assumption of perfect competition, the paper also analyses the implementation of a constrained second-best equilibrium, with a constant demand and tax rate. Interestingly, we have shown that the intermittency of renewable sources does not represent a market failure in itself. In fact, we found that, in average, the variation of climatic conditions, not only promote the investment in intermittent generations but also, implies a tax rate below the environmental marginal damage. This result is important from the perspective of policy implication and highlights the importance of the quantity-based instruments for promoting renewable energies. This will be a possible extension of our model in the future.

7 Appendix

7.1 Social optimum

Considering the linear quadratic specification of the model. The optimal allocation $(D^0, q^0(\omega))$ for all ω and k^0) satisfies the following system²⁸:

$$D^0 = q^0(\omega) + \omega k,$$

$$a - bD^0 = s \int_{\omega_0}^{\omega_1} q^0(\omega) dG(\omega),$$

²⁸ To simplify, we let $s = c + z$.

$$\gamma + \delta k^0 = s \int_{\omega_0}^{\omega_1} q^0(\omega) \omega dG(\omega).$$

Using the two first equations to calculate:

$$q^0(\omega) = \frac{a - bk\omega}{b + s},$$

Then substitute into the two last equations and integrate (using $E[\omega] = 1$ and $E[\omega^2] = V + 1$) to write

$$\begin{aligned} a - bD^0 &= s \frac{a - bk}{b + s}, \\ \gamma + \delta k^0 &= s \frac{a - bk(V + 1)}{b + s}, \end{aligned}$$

Finally solve this system to obtain:

$$D^0 = \frac{a + \frac{a \frac{s}{b+s} - \gamma}{\delta + b \frac{s}{b+s} (V+1)} s}{b + s},$$

$$k^0 = \frac{a \frac{s}{b+s} - \gamma}{\delta + b \frac{s}{b+s} (V + 1)}.$$

7.2 Comparative statics

Differentiation of k^0 gives:

$$\frac{\partial k^0}{\partial a} = \frac{s}{\delta(b + s) + bs(V + 1)} > 0,$$

$$\frac{\partial k^0}{\partial b} = -s \frac{a\delta + (a - \gamma)(V + 1)s}{(\delta(b + s) + bs(V + 1))^2} < 0,$$

$$\frac{\partial k^0}{\partial c} = b \frac{a\delta + b\gamma(V+1)}{(\delta(b+s) + bs(V+1))^2} > 0,$$

$$\frac{\partial k^0}{\partial \delta} = -(b+s) \frac{as - \gamma(b+s)}{(\delta(b+s) + bs(V+1))^2} < 0,$$

$$\frac{\partial k^0}{\partial \gamma} = -\frac{b+s}{\delta(b+s) + bs(V+1)} < 0,$$

$$\frac{\partial k^0}{\partial V} = -bs \frac{as - \gamma(b+s)}{(\delta(b+s) + bs(V+1))^2} < 0,$$

$$\frac{\partial k^0}{\partial z} = b \frac{a\delta + b\gamma(V+1)}{(\delta(b+s) + bs(V+1))^2} > 0.$$

The equilibrium price of electricity is

$$P(D^0) = E [P(d^0(\omega))] = a \frac{s}{s + b} \frac{\frac{2}{a}b + \delta + b \frac{s}{s+b} V}{b \frac{s}{s+b} (V+1) + \delta}$$

Differentiation of $P(D^0)$ gives:

$$\frac{\partial P(D^0)}{\partial a} = \frac{\delta + Vb \frac{s}{b+s}}{\frac{\delta b+s}{s} + b(V+1)} > 0,$$

$$\frac{\partial P(D^0)}{\partial c} = b \frac{ab^2V^2s + Vab(2\delta(b+s) + bs) + \delta \frac{(b+s)^2}{s} (a\delta + b\gamma)}{\frac{(b+s)^2}{s} (\delta(b+s) + bs(V+1))^2} > 0,$$

$$\frac{\partial P(D^0)}{\partial \delta} = \frac{bs(as - \gamma(b+s))}{(\delta(b+s) + bs(V+1))^2} > 0,$$

$$\frac{\partial P(D^0)}{\partial \gamma} = b \frac{s}{\delta(b+s) + bs(V+1)} > 0,$$

$$\frac{\partial P(D^0)}{\partial V} = \frac{b^2 s^2 \left(a \frac{s}{(b+s)} - \gamma \right)}{(\delta(b+s) + bs(V+1))^2} > 0.$$

7.3 Perfect competition

$$\frac{\partial k^*}{\partial T} = \frac{b}{\delta(b+c) + bc(V+1)} > 0,$$

$$\frac{\partial q^*(\omega)}{\partial T} = -\frac{1}{b+c} < 0,$$

$$\frac{\partial P(D^*)}{\partial T} = b \frac{\delta(b+c) + Vbc}{(b+c)(\delta(b+c) + bc(V+1))} > 0,$$

$$\frac{\partial D^*}{\partial T} = -\frac{\delta(b+c) + Vbc}{(b+c)(\delta(b+c) + bc(V+1))} < 0.$$

7.4 Second-best tax:

Let D^t , $q^t(\omega)$, k^t and T^* the solution, It satisfies the first order conditions:

$$\lambda(\omega) + (\phi + \beta\omega) C''(q(\omega)) - C'(q(\omega)) - d'(Z(\omega)) = 0,$$

$$P(D) - \phi P'(D) - \int_{\omega_0}^{\omega_1} \lambda(\omega) dG(\omega) = 0,$$

$$\int_{\omega_0}^{\omega_1} \lambda(\omega) \omega dG(\omega) - F'(k) - \beta F''(k) = 0,$$

$$\phi + \beta = 0.$$

Rewrite we get:

$$\lambda(\omega) = d'(Z(\omega)) + C'(q(\omega)) - \beta(\omega - 1)C''(q(\omega)),$$

$$P(D) = \int_{\omega_0}^{\omega_1} \left(d'(Z(\omega)) + C'(q(\omega)) \right) - \beta \left(P'(D) + \int_{\omega_0}^{\omega_1} (\omega - 1)C''(q(\omega)) \right) dG(\omega),$$

$$F'(k) + \beta F''(k) = \int_{\omega_0}^{\omega_1} \lambda(\omega) \omega dG(\omega).$$

we have:

$$\beta = \frac{\int_{\omega_0}^{\omega_1} (d'(Z(\omega)) + C'(q(\omega))) dG(\omega) - P(D)}{P'(D) + \int_{\omega_0}^{\omega_1} (\omega - 1)C''(q(\omega)) dG(\omega)}$$

Substitute it in the first condition we get :

$$P(D^t) = \int_{\omega_0}^{\omega_1} \left(d'(Z(\omega)) + C'(q(\omega)) \right) dG(\omega) - \beta \left(P'(D) + \int_{\omega_0}^{\omega_1} C''(q(\omega)) (\omega - 1) \right) dG(\omega),$$

$$F'(k^t) = \left(d'(Z(\omega)) + C'(q(\omega)) \right) \omega dG(\omega) - \beta \left(F''(k) + C''(q(\omega)) \omega (\omega - 1) dG(\omega) \right),$$

using constraints we have:

$$T^* = \int_{\omega_0}^{\omega_1} d'(Z(\omega)) dG(\omega) - \beta \left(P'(D) + \int_{\omega_0}^{\omega_1} (\omega - 1) C''(q(\omega)) dG(\omega) \right),$$

$$\beta = -z \frac{b \frac{kV}{b+s}}{Vc + b + \delta}.$$

Considering the linear quadratic specification of the model. The optimal allocation ($D^t, q^t(\omega)$, k^t and T^t) satisfies the following system:

$$D = q(\omega) + \omega k,$$

$$P(D) = s \int_{\omega_0}^{\omega_1} q(\omega) dG(\omega) + b \frac{z \int_{\omega_0}^{\omega_1} q(\omega) (\omega - 1) dG(\omega)}{Vc + b + \delta},$$

$$F'(k) = (c + z) \int_{\omega_0}^{\omega_1} q(\omega) \omega dG(\omega) - \frac{z \int_{\omega_0}^{\omega_1} q(\omega) (\omega - 1) dG(\omega)}{Vc + b + \delta} (Vc + \delta).$$

Using the two first condition to get:

$$q^t(\omega) = \frac{a - bk\omega}{b(1 + z) + s \frac{\omega - 1}{b + \delta + Vc}},$$

Then substitute into the two last equations and integrate (using $E(\omega) = 1$ and $E(\omega^2) = V + 1$) to write

$$a - bD = (s) \frac{a - bk}{b + s} - bz \frac{\frac{Vbk}{b + s}}{Vc + b + \delta},$$

$$\gamma + k\delta = (s) \frac{a - bk(V + 1)}{b + s} + (Vc + \delta) \frac{\frac{zVbk}{b + s}}{Vc + b + \delta}.$$

Finally solve the system to obtain:

$$k^t = \frac{a \frac{s}{b + s} - \gamma}{\delta + b(V + 1) \frac{s}{b + s} - \frac{Vbz}{b + \delta} \frac{\delta + Vc}{b + s}},$$

$$D^t = \frac{a + \frac{(a \frac{s}{b + s} - \gamma)(s + \frac{Vbz}{b + \delta + Vc})}{\delta + b(V + 1) \frac{s}{b + s} - \frac{Vbz}{b + \delta} \frac{\delta + Vc}{b + s}}}{b + s}.$$

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