Groupe de Recherche en
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Intermittent electric generation technologies and smart meters: substitutes or complements

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Technologies de production électrique intermittentes et compteurs communicants: substituts ou compléments

Résumé

Nous modélisons un marché électrique simplifié avec des producteurs utilisant des générateurs électriques conventionnels ou intermittents et des consommateurs équipés de compteurs intelligents ou traditionnels. Nous calculons l'investissement dans les technologies intermittentes et les compteurs intelligents dans un optimum social. Nous montrons que la pénétration optimale des compteurs intelligents augmente avec la volatilité du prix électrique au comptant. Par conséquent, les capacités intermittentes et les compteurs intelligents ne sont complémentaires que si la corrélation existant entre l'énergie intermittente et la demande est négative ou si les capacités installées des générateurs intermittents sont suffisamment grandes. Dans le cas contraire, des capacités intermittentes plus importantes contribuent en fait à diminuer la volatilité du prix au comptant électrique, rendant ainsi les compteurs intelligents moins utiles. Nous proposons également une application numérique, calibrée pour représenter la situation du marché électrique français en 2016 et ses objectifs politiques pour l'horizon 2030. Nous montrons en particulier qu'une adoption générale des compteurs intelligents ne serait optimale que si le coût d'installation et de fonctionnement des compteurs intelligents était irréaliste.

Mots-clés: Capacités – électricité – énergies renouvelables – intermittence

Intermittent electric generation technologies and smart meters: substitutes or complements

Abstract

We model a simplified electric market with producers using either conventional or intermittent electric generators and consumers equipped with either smart or traditional meters. We calculate the investment in intermittent technologies and smart meters in a social optimum. We find that the optimal penetration of smart meters is increasing in the volatility of the electric spot price. As a consequence, intermittent capacities and smart-meters are complement, only if the correlation existing between intermittent energy and demand is negative or if the capacity of intermittent generators is large enough. Otherwise, larger intermittent capacities actually help to decrease the volatility of the electric spot price, making smart-meters less useful. We also give a numeral application, calibrated to represent the French electric market in 2016 and policy objective for 2030. We show in particular that a general adoption of smart meters would be optimal only if the cost of installing and operating smart meters was unrealistically low.

Keywords: Capacity choice – electricity – intermittency – renewable energy

JEL: D24, D41, Q41, L11

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1 Introduction

The energy transition aims to prepare for the post-carbon era and to establish a robust and sustainable energy model, facing the challenges of energy supply, depletion of fossil resources and environmental protection. In France, the main objectives of the Energy Transition for Green Growth Act is to increase the share of renewable energy up to 23% of gross final energy consumption in 2020 and to 32% of gross final energy consumption in 2030.[1]

In this paper, we model a simplified electric market with, on the demand-side, consumers on either smart or traditional meters and, on the supply-side, producers using either conventional or intermittent electric generators. Both sides of the market are influenced by climatic and/or social factors (temperature, sun, wind, nebulosity, business cycles, holydays, and so on). All consumers share the same (inverse) demand curve, up to a scaling factor (i.e., the consumer's size). The demand variability is formalized by assuming that the intercept of the (inverse) demand curve is a random variable with known distribution. The consumers can chose either to be equipped with a smart or a traditional meter. Facing real-time pricing, the consumers on smart meters are induced to adapt their demand to the short term position of the electric market. The consumers equipped with traditional meters face flat tariff and their demand is insensitive to the short term position of the electric market. The producers using conventional generators supply a reliable and predictable quantity of electricity, at an increasing marginal cost. Their technology is assumed given . The producers using intermittent generators supply an unreliable and unpredictable quantity of electricity, at a negligible marginal cost. Their production is limited by a capacity constraint, which is endogenously determined by way of investment in new generating units. The marginal cost of building new capacities is assumed increasing, to account for the fact that the implantation sites will be used in a decreasing order of efficiency.

Our framework is designed to determine and analyze in a social optimum, both the investments in the intermittent technologies and the installation of smart meters. The model is kept simple enough in order to allow for the derivation of a closed form solution and for the disambiguation of most comparative statics results. Roughly speaking, the social objective can be represented and summarized along two dimensions, which are to provide electricity at the lowest cost possible, while limiting the volatility of the spot price as much as possible. With this reading in mind, consider first the policy of investing in intermittent generating units. Provided that the renewable technologies are socially efficient, it will directly help to reduce the electric spot price. However, the belief is commonly accepted that, in counterpart, it will increase the volatility of the electric

[1]See: https://www.ecologique-solidaire.gouv.fr/loi-transition-energetique-croissance-verte (visited November 28th, 2017).
See: https://www.legifrance.gouv.fr/affichCode.do?idSectionTA=LEGISCTA000031749063&cidTexte=LEGITEXT000023983208&dateTexte=20160421 (visited November 28th, 2017).

spot price (Wozabal et al., 2014). In fact, our analysis challenges and clarifies this point, by emphasizing the role played by the correlation existing between the electric demand and the intermittent electric supply. We show that the common belief holds true only if the electric demand and the intermittent electric generation are negatively correlated (*i.e.*, on average, the electric demand is smaller during sunny and/or windy hours) *or* if they are positively correlated, but the intermittent generating capacity is already large enough. Otherwise, the development of the intermittent generating technologies allows to reduce both the level and the volatility of the electric spot price.[2] Consider now the policy of instaling smart meters. It will clearly help to smooth the electric demand, by encouraging equipped consumers to react to the electric spot price. However, it has no (direct) effect on the cost of producing electricity. Now, the belief is commonly accepted that the penetration of smart meters will *indirectly* reduce the electric spot price, by inducing a larger intermittent generating capacity in a social optimum. Anew, our analysis shows that this insight is correct only if the electric demand and the intermittent electric generation are negatively correlated *or* if they are positively correlated but the intermittent capacity is already large enough. In other words, cases exist such that the deployment of smart merters induce a smaller optimal intermittent generating capacity, thus increasing the average electric spot price.

This being said, our most important resuls are the following. In a social optimum, the consumers equipped with traditional meters should always face a tariff equal to the *expected* real-time price paid by the consumers equipped with smart meters. The optimal penetration of smart meters is increasing in the volatility of the electric spot price. This is quite intuitive, as smart meter are actually used to smooth it. As a consequence, given that increasing the capacity of intermittent generating units may either increase or decrease the volatility of the electric spot price, depending on the correlation existing between the electric demand and the availability of the renewable energies, the capacity of intermittent technologies and the penetration of smart meters may in theory be either complements or substitutes. In practice, it is commonly accepted that the generalization of smart meters will be needed in response to the penetration of the intermittent generating technologies. We show that this holds true only if the electric demand and the intermittent electric generation are negatively correlated *or* if they are positively correlated but the intermittent capacity is already large enough. In all other cases, the capacity of intermittent technologies and the installation of smart would be substitutes.

In this paper, we also complement our theoretical findings by a numerical application, in order to give some idea of the magnitude of the mechanisms at stake. Though mainly illustrative, the numerical example is meant to reflect the French electric market in 2016 (RTE, 2016). Under our benchmark parametrization, which in particular presumes no correlation between demand

[2] Wozabal et al. (2014) give empirical supports of these results in the case of the German electric spot market.

and intermittent energy, we find that the optimal intermittent capacity will be increasing in the market share of smart meters (i.e., from 51257 MW to 54647 MW). As a result, the average real-time price will be slightly decreasing in the market share of the smart meters (i.e., from 33.65 €/MWh to 32.31 €/MWh). The optimal market share of smart meters is rapidly decreasing in the cost of installing and operating smart meters (i.e., from 100 % for an annualized cost of smart meters of 0.5 €/year, to 46 % for an annualized cost of 30 €/year). This confirms that a general adoption of smart meters would be optimal only if the cost of installing and operating smart meters was unrealistically low (Léautier, 2014).

This paper is part of a rapidly growing body of literature, dealing with the changes of the electric market due to the deployment of the renewable and digital technologies, in the policy context of market deregulation. We will not try to give a exhaustive survey of this literature, as this is far beyond the scope of this paper. Instead, we limit our presentation to a selection of papers which we consider as close to ours, in order to emphasize our contribution.

Firstly, it is worth remarking that the larger part of the literature is empirical and country specific (Crampes and Ambec, 2012). This includes, among others, Benitez et al. (2008), Boccard (2008), Gowrisankaran et al. (2016), Green and Vasilakos (2010, 2011), Kennedy (2005), Lamont (2008), Menanteau et al. (2003), Musgens and Neuhoff (2006, 2007), and Neuhoff et al. (2006, 2007). They provide estimates of the social costs and benefits of the (either optimal or equilibrium) penetration of the intermittent renewable technologies for generating electricity, for different countries and periods. Setting aside secondary differences, the common background can be summarized as follows. Observed electric demands and wind outputs data are combined to build a *residual load duration curve* (Kennedy, 2007), thus accounting for the interaction between the electricity demand and the renewable energy availability. Then, screening curves are used to derive the (either optimal or equilibrium) mix of generating technologies and electric dispatch, in order to satisfy the demand at all times. Some notable departures from this benchmark framework can be found in Benitez et al. (2008), Boccard (2010), Gowrisankaran et al. (2016), Green and Vasilakos (2010, 2011), Musgens and Neuhoff (2006, 2007), and Neuhoff et al. (2006, 2007). Benitez et al. (2008), and Musgens and Neuhoff (2006, 2007) set up inter-temporal models in order to formalize hydropower storage. Gowrisankaran et al. (2016) construct a quite general model, incorporating endogenous demand, with a possibility of curtailment, and a risk of outage of generating units. Green and Vasilakos (2010, 2011) determine supply function equilibria (Klemperer and Meyer, 1989), in order to analyse issues of imperfect competition. Neuhoff et al. (2006, 2007) deal with spatial variation in wind output and transmission constraints within the grid.

This article takes a different direction, by using a stylized microeconomic framework to derive general insights regarding the optimal development of intermittent technologies to generate electricity and smart meters. The corresponding strand of literature is much less developed, including Ambec and Crampes (2012), Bode (2006), Borenstein and Holland (2005), Green and Vasilakos (2010), Joskow and Tirole (2007), Léautier (2014), Rouillon (2015), and Twoney and Neuhoff (2009). For the sake of our presentation, these papers are divided below in two subsets, by distinguishing between those that focus on the supply size of the market and on the competition between conventional and renewable technologies, and those that focus on the demand side of the market and on the organizational features that can be used to make the consumers more reactive to the situation of the spot market.

A first strand of the theoretical literature (Ambec and Crampes, 2012; Bode, 2006; Rouillon, 2014; Twoney and Neuhoff, 2009) deals with the issue of competition on an electric market where the electricity is supplied by conventional generators and renewable generators. Bode (2006) determines the perfect competition equilibrium under several support schemes, financed either through the general public budget or a renewable energy mark-up charged to the final consumers. Implicitly, it is assumed that all consumers pay their electricity at the spot price and the installed renewable capacity is exogenous and not subject to intermittency. The main finding is that the final cost to the consumers may increase or decrease, depending on the support schemes and assumptions. Twoney and Neuhoff (2009) use a similar setting, but add the issue of intermittency explicitly. They determine the market equilibrium under perfect, monopolistic and duopolistic competitions, where the exercise of market power is by the incumbent conventional generators. They show that the average price received by intermittent generators are lower than for conventional generators. Indeed, the conventional generators, having an increasing marginal costs, always set the market price, as marginal generators. Therefore, the latter will be lower (higher) in the periods of high (low) renewable energy. Moreover, Twoney and Neuhoff (2009) find that the difference can be exarcebated in the presence of market power. Ambec and Crampes (2012) and Rouillon (2015) go one step further, by making the energy mix endogenous through the possibility of investing in new generating capacities. Ambec and Crampes (2012) characterize the optimal investment and dispatch between conventional and intermittent generators and discuss its implementability under perfect competition. They consider in turn two polar situations, one where all consumers face prices contingent on the availability of the intermittent source of electricity (first-best optimum) and another where they all face a uniform price (second-best optimum). Rouillon (2015) complements Ambec and Crampes (2012), by assuming that both types of consumers co-exist on the market and by dealing with perfect and monopolistic competitions. Under perfect competition, it is shown that the (second-best) optimal policy is implementable, provided that the conventional and intermittent generators exchange their electric production on a wholesale spot market. By constrast, if a single incumbent firm owns the conventional generators and has market power, the paper shows that the investment in the intermittent technologies will in general be inappropriate.

Another strand of the theoritical literature (Borenstein and Holland, 2005; Green and Vasilakos, 2010; Joskow and Tirole, 2006 and 2007; Léautier, 2014) deals with the design of pricing strategies under conditions of imperfect metering of a variable electric demand. A major problem in electricity markets is that only a fraction of the consumers face and react to real-time pricing (Borenstein and Holland, 2005). The reason is because generalizing real-time pricing requires to equip all consumers with connected *and* communicant meters (Joskow and Tirole, 2006). As long as some consumers remain on traditional meters and face flat rate service, a competitive electricity market will fail to implement the first-best optimum (Borenstein and Holland, 2005). Indeed, in the short run, reallocations of electric consumption between consumers on real-time pricing and flat rate will be socially worth each time they pay different prices. In the long run, the equilibrium spot price will fail to provide adequate incentives to invest in the generating technologies. The literature also investigated the conditions such that an electricity market can implement the second-best optimum, given the existence of price-insensitive retail consumers. Borenstein and Holland (2005) obtain an impossibility, assuming that the retailers can supply linear pricing contracts only. On the contrary, Joskow and Tirole (2007) show that the second-best optimum can be implemented, provided that the retailers offer two-part tariffs contacts, with a fixed fee and flat rate price. However, they argue that the conditions underlying this result are very strong and, in particular, would be violated in the presence of price caps and/or market power on the wholesale market, and in the presence of load profiling and/or load profile heterogeneity on the retail market. Though evoked in Borenstein and Holland (2005) and Joskow and Tirole (2007), the issue of endogenous investment in metering equipment is analyzed in greater details by Léautier (2014). In a socially optimal allocation, Léautier (2014) shows that the marginal value of increasing the proportion of consumers on real-time pricing is proportional to the variance of wholesale prices. This determines the consumers' incentives to adopt smart meters.

To the best of our knowledge, this article is the first attempt to bring together, in a stylized microeconomic framework, the two issues of the optimal development of intermittent capacities and smart meters just surveyed above. The closest papers are Rouillon (2015) and Léautier (2014). The reason why it is interesting to deal with both issues in the same setting is because it allows to emphasize the important role played by the correlation between demand and intermittent supply. As said above, this challenges from the theoretical viewpoint the commonly accepted belief, namely that intermittent capacities and smart meters should necessarily develop in paralel. Although this belief may be true from the empirical viewpoint, this nevertheless drives us to encourage any policy that could increase the correlation between demand and intermittent energy supply. We think that many policies have the potential to influence the correlation between demand and intermittent energies, such as, for example, the introduction of daylight saving time (Havranek et al, 2016), the developement and organization of prosuming consumers (Parag et al, 2016), the choice of the renewable energy mix (Torres et al, 2016), and so on. Such policies would be socially worth and would in fact render the need for smart metering even less urgent.

The outline of this paper is as follows. Section 2 sets out the model. Section 3 deals with the normative analysis. It determines the optimal allocation and gives its comparative statics. Section 4 discusses the development of numerical simulations for the French market, and presents the main empirical results. Section 5 concludes.

2 The model

Consider a simplified electricity system without network constraints. The natural and social factors influencing the electric demand and supply (temperature, sun, wind, nebulosity, business cycles, holydays, and so on) are indexed by a random variable x, with cumulative distribution $F(x)$.[3] The population of consumers is normalized to one. The *aggregate* demand writes $D(p) = (a(x) - p)/b$, where $a(x) > 0$, for all x, and $b > 0$.[4] Define $\overline{a}$ and $V(a)$, respectively, the expected value and the variance of $a(x)$.[5] Each consumer is characterized by his type $t \in [t_0, t_1]$. The consumers' types are assumed to be positive and distributed according to the cumulative distribution $G(t)$ within the population.[6] The demand function of a consumer with type t is $tD(p)$. Since, by assumption, the population is normalized to one and the aggregate demand is equal to $D(p)$, the average type needs to be normalized to one. Formally, $\int_{t_0}^{t_1} t\mathrm{d}G(t) = 1$. The consumers can be equipped with either smart or traditional meters. Let $\mathcal{S} \subseteq [t_0, t_1]$ be the set of all consumers' types equipped with smart meters. Facing real-time pricing, the consumers with types $t \in \mathcal{S}$ can adapt their demand to the short term position of the electric market. We will denote by $p(x)$ the real-time price of electricity in the state x. The consumers with types $t \notin \mathcal{S}$ are equipped with traditional meters. Facing a flat tariff, their demand is insensitive to the short term position of the electric market. We will denote by P the flat price of electricity. The cost of installing and operating the smart and traditional meters are K and κ respectively, with $K > \kappa$.[7] The incumbent plants supply electrical energy in quantity q, using conventional generators (i.e., hydro, nuclear, coal, gas, oil). The cost function $C(q)$ represents their technol-

[3] The use of a unidimensional index to represent all factors influencing the electricity market is made for the sake of notation convenience. It entails no loss of generality.

[4] For the sake of notation simplicity, we make here a slight abuse of notations, since $D(p)$ actually is a function of x.

[5] $\overline{a} = \int_{-\infty}^{+\infty} a(x)\mathrm{d}F(x)$ and $V(a) = \int_{-\infty}^{+\infty} (a(x) - \overline{a})^2 \,\mathrm{d}F(x)$.

[6] We assume that $\mathrm{d}G(t) > 0$ if and only if $t \in [t_0, t_1]$.

[7] If $K \leq \kappa$, then our analysis shows that all consumers should always be equipped with smart meters.

ogy. It is assumed that $C'(0) = 0$, $C'(q) > 0$ and $C''(q) > 0$.[8][9] New plants, using intermittent generators (i.e., solar and wind), seek to enter the market. The cost of building intermittent units with capacity k is $I(k)$. It is assumed that $I(0) = 0$, $I'(k) > 0$ and $I''(k) \geq 0$. Given the installed capacity k, the intermittent generation will be equal to $w(x)k$ in the state x, at a negligible marginal cost. Define $\overline{w}$ and $V(w)$, respectively the expected value and the variance of the capacity factor $w(x)$.[10] Also, define $Cov(a, w)$, the covariance between the demand variability and the intermittent electric generation.[11] We let ρ denote the correlation coefficient, defined by $\rho = Cov(a, w)/\sqrt{V(a)V(w)}$.

To simplify the analysis, only interior solutions will be considered and the following linear-quadratic specification of the model:[12]

$$C(q) = \frac{1}{2}cq^2,$$

$$I(k) = \left(\gamma + \frac{1}{2}\delta k\right)k,$$

will be used in some parts of the paper.

3 Optimal policy

The social problem is to determine the metering equipments of the consumers, the generating capacity of the intermittent units, and the demand and supply of electricity in every states of the world, so as to maximize the *expected* social surplus.

In order to solve it, some more notations need to be introduced. Denote by $S(p)$ the *indirect* aggregate gross surplus when the price of electricity is p.[13] Then the *indirect* gross surplus of a consumer with type t facing the price p is $tS(p)$. Moreover, let $\phi(t)$ be a variable formalizing the meter's equipment of a type t consumer, such that $\phi(t) = 1$ if $t \in \mathcal{S}$ and $\phi(t) = 0$ if $t \notin \mathcal{S}$.[14]

[8] This assumption is used in Twoney and Neuhoff (2009). It is appropriate to represent the initial situation where the incumbent firms own many generating units, using a large variety of conventional technologies (hydro, nuclear, coal, gas, oil), with different marginal costs of generating electricity, and where the overall capacity of this set of generating units is sufficient to match the demand and to prevent black-out. This picture fits quite well the currrent situation of several countries in Europe, where there exists an overcapacity of conventional units remaining in operation till the end of their programmed lifetime.

[9] To simplify, following Ambec and Crampes (2012), we assume implicitly that the cost function $C(q)$ includes the environmental damages related to electricity production. This justifies a scenario where the renewable technologies are socially efficient.

[10] $\overline{w} = \int_{-\infty}^{+\infty} w(x)\mathrm{d}F(x)$ and $V(w) = \int_{-\infty}^{+\infty} (w(x) - \overline{w})^2\,\mathrm{d}F(x)$.

[11] $Cov(a, w) = \int_{-\infty}^{+\infty} (a(x) - \overline{a})(w(x) - \overline{w})\,\mathrm{d}F(x)$.

[12] From our assumptions above, all parameters are positive.

[13] By definition, $S(p) = \int_0^d D^{-1}(z)\ \mathrm{d}z$ where $d = D(p)$. It is immediate to calculate that $S(p) = (a(x)^2 - p^2)/(2b)$. Again, we make here a slight abuse of notations, since $S(p)$ actually is a function of x.

[14] For technical reason, we will admit that $\phi(t)$ can take any value between 0 and 1.

The social problem is to choose $p(x)$, for all x, P, $q(x)$, for all x, k and $\phi(t)$, for all t, to maximize the *expected* social surplus

$$\int_{-\infty}^{+\infty}\left[\int_{t_0}^{t_1}\begin{pmatrix}\phi(t)\left(tS\left(p(x)\right)-\kappa\right)\\+\left(1-\phi(t)\right)\left(tS(P)-K\right)\end{pmatrix}\mathrm{d}G(t)-C\left(q(x)\right)-I(k)\right]\mathrm{d}F(x), \tag{1}$$

subject to the market clearing condition

$$\int_{t_0}^{t_1}\begin{pmatrix}\phi(t)tD\left(p(x)\right)\\+\left(1-\phi(t)\right)tD(P)\end{pmatrix}\mathrm{d}G(t)=q(x)+w(x)k,\text{ for all }x. \tag{2}$$

The lagrangian for this problem writes

$$L=\int_{-\infty}^{+\infty}\begin{bmatrix}\int_{t_0}^{t_1}\begin{pmatrix}\phi(t)\left(tS\left(p(x)\right)-\lambda(x)tD\left(p(x)\right)-\kappa\right)\\+\left(1-\phi(t)\right)\left(tS(P)-\lambda(x)tD(P)-K\right)\end{pmatrix}\mathrm{d}G(t)\\+\lambda(x)q(x)-C\left(q(x)\right)+\lambda(x)w(x)k-I(k)\end{bmatrix}\mathrm{d}F(x) \tag{3}$$

where $\lambda(x)$ is the multiplier associated with the market clearing condition.

The optimal solution will be denoted $p^0(x)$, for all x, P^0, $q^0(x)$, for all x, k^0 and $\phi^0(t)$, for all t. Below, we let $\alpha^0=\int_{t_0}^{t_1}t\phi^0(t)tdG(t)$, which gives the *expected* market share of the consumers equipped with smart meters (see footnote 16). In order to better explain the properties of the optimal solution, we construct and discuss it step by step below. The first-order conditions and comparative statics are derived in the appendix.

3.1 Optimal dispatch

Let us consider as given here, the intermittent units' capacity (i.e., k^0) and the consumers' metering equipment (i.e., α^0 and $\phi^0(t)$ for all t). The regulator's problem then reduces to finding prices $p(x)$, for all x, and P, and conventional units' electric generations $q(x)$, for all x, in order to maximize the *expected* social surplus.

Figure 1 below illustrates the determination of the market equilibrium and the calculus of the resulting *ex post* social surplus in a given state x. The *aggregate* demand of the consumers on smart meters (resp., traditional meters) is $\alpha^0 D\left(p(x)\right)$ (resp., $(1-\alpha^0)D(P)$). The corresponding (inverse) demand curves are shown in Parts (a) and (b) of Figure 1. Part (c) depicts the *aggregate* supply curve by the conventional and intermittent generating units. It is obtained by translating to the right the marginal cost curve $C'\left(q(x)\right)$, by the quantity $w(x)k^0$ generated by the intermittent generating units. By construction, the market equilibrium is obtained when the spot price $p(x)$ and the conventional generation $q(x)$ satisfy $p(x)=C'\left(q(x)\right)$ and $\alpha^0 D\left(p(x)\right)+(1-\alpha^0)D(P)=q(x)+w(x)k^0$. The resulting *ex post* social surplus is the sum of the consumers' gross surplus (i.e., the trapezes in Parts (a) and (b) of Fig.1), less the cost of electric generation (i.e., the triangle in Part (c) Fig.1).

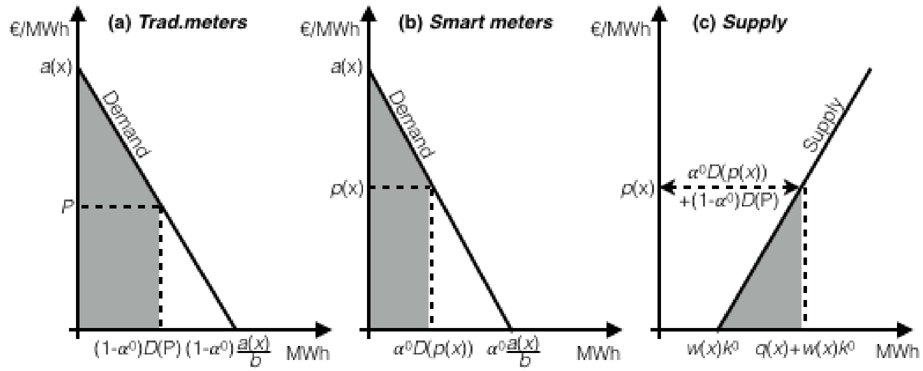


Figure 1: Short term market equilibrium

Rearranging the relevant first-order conditions derived from (3), we can show that the optimal solution satisfies

$$p^0(x) = C'\left(q^0(x)\right), \text{ for all } x, \tag{4}$$

$$P^0 = \int_{-\infty}^{+\infty} C'\left(q^0(x)\right) \mathrm{d}F(x), \tag{5}$$

together with the market clearing condition (2).

In other words, the consumers equipped with the smart meters should always face a price equal to the *ex post* marginal cost of generating electricity. The consumers equipped with the traditional meters should face a price equal to the *expected* marginal cost of generating electricity.

Using (4) and (5), it is immediate that[15]

$$P^0 = \int_{-\infty}^{+\infty} p^0(x)\mathrm{d}F(x),$$

meaning that the flat tariff paid by the consumers equipped with traditional meters should simply reflect the *expected* real-time tariff paid by the consumers equipped with smart meters. Therefore, the fact that the consumers equipped with smart meters adapt their demand to the short term positions of the electric market, whereas the consumers equipped with traditional meters do not, is no justification to discriminate their tariffs on average. This may come as a surprise, because the consumers on traditional meters are actually responsible for a deadweight loss each time the flat tariff differs from the real-time tariff.

[15] Joskow and Tirole (2007) and Léautier (2014) obtain the more general result that $P^0 = \left[\int_{-\infty}^{+\infty} D'(p^0(x))p^0(x)\mathrm{d}F(x)\right] / \left[\int_{-\infty}^{+\infty} D'(p^0(x))\mathrm{d}F(x)\right]$ (when there is no rationing). Here, our expression simplifier because the slope of the demand function is constant (i.e., $D'(p) = -1/b$).

Using the linear-quadratic specification of our model, we can show that

$$p^0(x) = c\left(\frac{\overline{a} - b\overline{w}k^0}{b+c} + \frac{1}{b+\alpha^0 c}\left(a(x) - \overline{a}\right) - \frac{bk^0}{b+\alpha^0 c}\left(w(x) - \overline{w}\right)\right), \quad (6)$$

$$P^0 = c\frac{\overline{a} - b\overline{w}k^0}{b+c}, \quad (7)$$

$$q^0(x) = \frac{\overline{a} - b\overline{w}k^0}{b+c} + \frac{1}{b+\alpha^0 c}\left(a(x) - \overline{a}\right) - \frac{bk^0}{b+\alpha^0 c}\left(w(x) - \overline{w}\right), \quad (8)$$

where[16]

$$\alpha^0 = \int_{t_0}^{t_1} t\phi^0(t)\mathrm{d}G(t)$$

represents the *expected* market share of the consumers equipped with smart meters.

It is immediate to show that the optimal flat tariff P^0 is increasing in $\overline{a}$ and c, and decreasing in b and $\overline{w}$. More interestingly, we note that increasing the intermittent capacity k^0 decreases it, whereas increasing the ratio α^0 of the demand of the consumers on smart meters has no impact.[17]

From this, we can calculate the *ex post* fluctuations of the real-time price around its expected value

$$p^0(x) - P^0 = \frac{c}{b+\alpha^0 c}\left(\left(a(x) - \overline{a}\right) - bk^0\left(w(x) - \overline{w}\right)\right)$$

and its variance[18]

$$V(p^0) = \left(\frac{c}{b+\alpha^0 c}\right)^2 \left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right). \quad (9)$$

The volatility of the real-time price primarily comes from the demand variability (i.e., $V(a)$), the renewable energy intermittency (i.e., $V(w)$ and k^0) and their correlation (i.e., ρ). Clearly, the price volatility is decreasing in ρ. However, the comparative statics with respect to $V(a)$, $V(w)$ and k^0 is ambiguous, depending on the correlation existing between the shocks $a(x)$ and $w(x)$. Let $\underline{k}$ be the intermittent capacity minimizing $V(p^0)$. It is not difficult to show that it is equal to 0 if $\rho \leq 0$ and to $(\rho/b)\sqrt{V(a)/V(w)}$ otherwise. It can then be seen

[16] Formally, α^0 is equal to the *expected* demand of the consumers on smart meters (i.e., $\int_{-\infty}^{+\infty}\left[\int_{t_0}^{t_1}\phi^0(t)tD\left(p^0(x)\right)\mathrm{d}G(t)\right]\mathrm{d}F(x)$) over the *expected* aggregate demand (i.e., $\int_{-\infty}^{+\infty}\left[\int_{t_0}^{t_1}\left(\phi^0(t)tD\left(p^0(x)\right) + \left(1-\phi^0(t)\right)tD(P^0)\right)\mathrm{d}G(t)\right]\mathrm{d}F(x)$). We can simplify it, using $\int_{t_0}^{t_1} t\mathrm{d}G(t) = 1$, and (6) and (7), which implies that $\int_{-\infty}^{+\infty} D\left(p^0(x)\right)\mathrm{d}F(x) = \int_{-\infty}^{+\infty} D\left(P^0\right)\mathrm{d}F(x)$.

[17] The latter confirms result 1 in Léautier (2014).

[18] $V(p^0) = \int_{-\infty}^{+\infty}\left(p^0(x) - P^0\right)^2\mathrm{d}F(x)$.

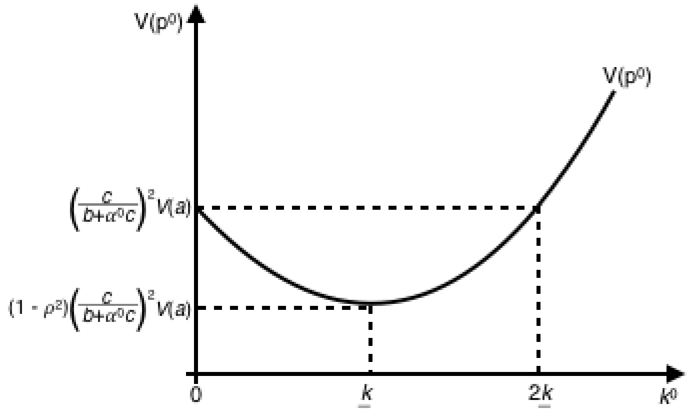


Figure 2: Variance of the real-time price (when $\rho > 0$)

that the price volatility is increasing in $V(w)$ and k^0, if and only if the optimal capacity k^0 is larger than the capacity $\underline{k}$ minimizing $V(p^0)$. Also, we can also prove that $V(p^0)$ is increasing in $V(a)$ if and only if $\rho^2 k^0 < \underline{k}$. The transmission of the shocks (i.e., $a(x)$ and $w(x)$) to the spot prices depends on the elasticities of demand (i.e., α^0 and b) and supply (i.e., c).[19] The volatility of the spot prices is increasing in c and is decreasing in α^0. The comparative statics with respect to b is ambiguous, basically because a more elastic demand has countervailling effects on the spot prices. More precisely, it amplifies the shocks on $a(x)$ and attenuates the shocks on $w(x)$. Overall, we are able to show that the real-time price volatility is increasing in b as long as $k^0 \geq \underline{k}$.[20]

Figure 2 illustrates our discussion above. Assuming that $\rho > 0$, it represents the variance of the real-time price $V(p^0)$, as a function of the optimal capacity of intermittent generating units k^0. Then, $V(p^0)$ is decreasing for $k^0 < \underline{k}$, is increasing for $k^0 > \underline{k}$, and has a minimum equal to $(1-\rho^2)(c/(b+\alpha^0 c))^2 V(a)$ for $k^0 = \underline{k}$. Remark that this minimum is decreasing in ρ and vanishes when $\rho = 1$. Finally, also note that the penetration of intermittent generating units actually helps limiting the variance of the real-time price as long as $k^0 \leq 2\underline{k}$; it becomes detrimental only for $k^0 > 2\underline{k}$.

[19]For all x, the aggregate demand is $\alpha^0 D(p^0(x)) + (1-\alpha^0) D\left(P^0\right) = \left(a(x) - \alpha^0 p^0(x) - (1-\alpha^0)P^0\right)/b$. The derivative of the demand function with respect to $p^0(x)$ is thus equal to α^0/b (in absolute value), showing that the reactivity of the aggregate demand to shocks depends on both α^0 and b.

[20]Note that the comparative statics derived here treats α^0 and k^0 as parameters, although they are endogenously determined in an optimal solution. This is clearly for the sake of simplicity. We will provide a numerical illustration below, in order to fully describe the optimal behavior of all endogenous variables at the same time.

3.2 Intermittent capacity

Let us consider as given here, the real-time price and flat tarif (i.e., $p^0(x)$ for all x and P^0), and the consumers' metering equipment (i.e., α^0 and $\phi^0(t)$ for all t). The social problem then becomes to determine the investment in the intermittent technologies, in order to maximize the *expected* social surplus.

Rearranging the relevant first-order conditions derived from (3), we can show that the optimal solution satisfies

$$I'(k^0) = \int_{-\infty}^{+\infty} C'\left(q^0(x)\right) w(x)\mathrm{d}F(x). \tag{10}$$

Accordingly, the intermittent capacity should be increased as long as the cost of investing in the marginal intermittent unit remains smaller than its *expected* marginal benefit. In the state x, the marginal benefit of intermittent units is the product of the marginal cost of generating electricity from the conventional generators, $C'\left(q^0(x)\right)$, times the production of the marginal generating unit, $\omega(x)$.

Using the linear-quadratic specification of our model, for an interior solution, we can show that[21]

$$k^0 = \frac{\frac{c}{b+c}\overline{aw} - \gamma + \frac{c}{b+\alpha^0 c}\rho\sqrt{V(a)V(\mathrm{w})}}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)}. \tag{11}$$

Accordingly, the optimal capacity of intermittent generating units depends on the demand's level, variability and reactivity (i.e., $\overline{a}$, $V(a)$, b and α^0), the cost of electric generation by conventional units (i.e., c), the cost of building and operating renewable generating units (i.e., γ and δ), the availability and variability of intermittent energy (i.e., $\overline{w}$ and $V(w)$), and the correlation between demand and renewable energy (i.e., ρ). Clearly, the optimal capacity of intermittent units is increasing in $\overline{a}$ and ρ, decreasing in γ and δ, and is increasing in $V(a)$ if and only if ρ is positive. The comparative statics with respect to α^0, b, c, $\overline{w}$ and $V(w)$ is ambiguous. However, we are able to show that the optimal capacity of intermittent generating units is decreasing in b if ρ is positive, increasing in $\overline{w}$ if and only if $k^0 < \overline{a}/(2b\overline{w})$ and increasing in $V(w)$ if and only if $k^0 < \underline{k}/2$.

Figure 3 is useful to understand the comparative statics of k^0 with respect to α^0.[22] It inventories the two reasons for investing in the intermittent generating

[21] An interior solution is obtained as long as $\gamma < \overline{aw}c/(b+c) + \rho\sqrt{V(a)V(\mathrm{w})}/(b+\alpha^0 c)$. Otherwise the optimal capacity in intermittent units is null.

[22] Here again, for the sake of simplicity, we treat α^0 as a parameter, although it is endogenously determined. Below, a numerical illustration is provided to complete the analysis.

technologies, from which the optimal strategy derives. The first rationale is to supply low cost electricity (i.e., merit order argument). It is represented by the horizontal dashed line, plotting the capacity $\overline{k}$ that would be worth investing in the renewable generating units if the renewable energy was continuously available at its average value (i.e., if $w(x) = \overline{w}$ for all x and $V(w) = 0$).[23] The second rationale is to limit the variability of the dispatchable generation (i.e., intermittency argument). It is represented by the broken dashed line, depicting the capacity $\underline{k}$ that would minimize the variance of the real-time price of electricity.[24] In Figure 2, we let ρ^0 be the value of the coefficient of correlation ρ such that these two lines intersect (i.e., $\overline{k} = \underline{k}$).[25] For all α^0, the optimal capacity k^0, given by (11), lies within the shaded area, bounded by the two dashed frontiers in gray, corresponding to the limit cases where $\alpha^0 = 0$ and $\alpha^0 = 1$. Finally, the optimal capacity k^0 is represented by the plain line, for a given α^0 strictly between 0 and 1. As it can be seen, for all α^0, it always lies between the two dashed lines $\underline{k}$ and $\overline{k}$ and intersects them both when $\rho = \rho^0$. Moreover, as the market share of smart meters α^0 increases from 0 to 1, it rotates clockwise around this intersection point. In other words, it gets closer to the horizontal dashed line plotting the capacity $\overline{k}$. This should not come as a surprise, as the development of smart meters helps to limit the variability of the generation of the conventional units. Increasing the market share of smart meters thus justifies to put more weight on the other objective (i.e., merit order argument). This means that k^0 is increasing in α^0 when $\rho < \rho^0$, is decreasing in α^0 when $\rho > \rho^0$, and does not depend on α^0 when $\rho = \rho^0$.

3.3 Metering equipments

Let us finally consider as given, the real-time price and flat tarif (i.e., $p^0(x)$ for all x and P^0), and the capacity in intermittent units (i.e., k^0). The social problem then boils down to determining the consumers' metering equipment, in order to maximize the *expected* social surplus.

Using the relevant first-order conditions derived from (3), we can show that the optimal solution satisfies[26]

$$\phi^0(t) = 1 \quad \text{iff} \quad \begin{array}{c} \int_{-\infty}^{+\infty} p^0(x)t\left(D(P^0) - D\left(p^0(x)\right)\right) \mathrm{d}F(x) \\ -\int_{-\infty}^{+\infty} t\left(S(P^0) - S\left(p^0(x)\right)\right) \mathrm{d}F(x) \end{array} \geq K - \kappa. \tag{12}$$

This inequality emphasizes three terms, listing the different benefits and costs from equipping with a smart meter a consumer with type t. Supplying

[23]Using (11) and substituting $V(w) = 0$, we get $\overline{k} = \left(\frac{c}{b+c}\overline{aw} - \gamma\right) / \left(\frac{c}{b+c}b\overline{w}^2 + \delta\right)$.

[24]Recall that $\underline{k}$ is null if $\rho \leq 0$ and equal to $(\rho/b)\sqrt{V(a)/V(w)}$ otherwise.

[25]It is easily calculated that $\rho^0 = b\overline{k}\sqrt{V(w)/V(a)}$. Note that we implicitly assume in Figure 2 that $\rho^0 < 1$.

[26]Remember that $\phi(t)$ can only take values 0 or 1.

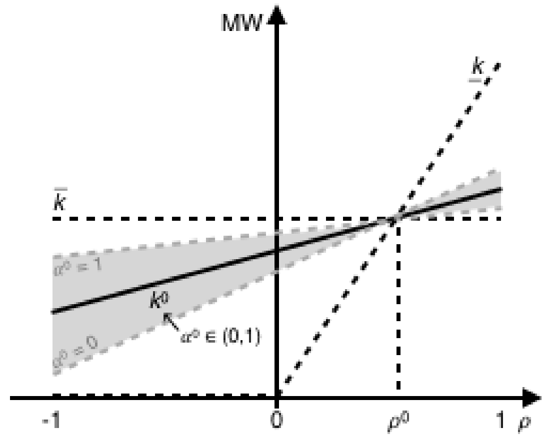


Figure 3: Optimal capacity

a consumer on a smart meter saves costs of generating electricity, because the latter has an incentive to reduce his demand when the price/marginal cost of electricity is high (upper part on the left-hand side of the inequality). However, in counterpart, a consumer on a smart meter incurs a welfare loss, because he bears the risk of price volatility (lower part on the left-hand side of the inequality). Finally, the installation and operation of the smart meter is more costly *per se* (right-hand side of the inequality).

By definition of $D(p)$ and $S(p)$, this condition simplifies to[27]

$$\frac{t}{2b}V(p^0) \geq K - \kappa. \tag{13}$$

Assuming that the equality occurs for an interior solution (i.e., $t_0 < t^0 < t_1$), we can define the optimal marginal consumer's type

$$t^0 = \frac{2b(K-\kappa)}{V(p^0)}, \tag{14}$$

such that every consumer with types $t > t^0$ (resp., $t \leq t^0$) should be equipped with smart (resp. traditional) meters. The corresponding optimal *expected* market share of the consumers equipped with smart meters is

$$\alpha^0 = \int_{t^0}^{t_1} t\mathrm{d}G(t). \tag{15}$$

Clearly, as it is decreasing in t^0, it is immediate from above that the optimal *expected* market share of the consumers equipped with smart meters is decreasing

[27]Moreover, recall that from (4) and (5), we have $P^0 = \int_{-\infty}^{+\infty} p^0(x)\mathrm{d}F(x)$.

in b and $K - \kappa$, and is increasing in $V(p^0)$. These findings are quite intuitive. Smart meters are worthwhile if the consumers are sufficiently reactive (i.e., b is small), if installing and operating them is not too costly (i.e., $K - \kappa$ is small) and if there exists enough price fluctuations (i.e., $V(p^0)$ is large).

At that point, our understanding of the factors influencing the optimal *expected* market share of the consumers equipped with smart meters is partial. Indeed, in an optimal allocation, the variance of the real-time price $V(p^0)$, safisfying (9), is itself a function of α^0 and most parameters of the model. However, accounting for all these indirect effects, we are still able to show that α^0 is increasing (resp., decreasing) in all parameters that increase (resp., decrease) the variance of the real-time price, either directly or indirectly.

Accordingly, the optimal *expected* market share of the consumers equipped with smart meters α^0 is increasing in $V(a)$, $V(w)$ and c, and decreasing in ρ. It is increasing (resp., decreasing) in k^0 if the intermittent capacity k^0 is larger (resp. smaller) than the intermittent capacity $\underline{k}$ minimizing the variance $V\left(p^0\right)$ of the real-time price. Finally, the comparative statics with respect to b is essentially ambiguous. [28]

4 Numerical illustration

To complete our analysis, we now provide a calibration of our model. Although it is mainly for illustrative purpose, the proposed calibration is meant to reflect the French electric market in 2016. We use the annual report of Réseau de Transport de l'Electricité (RTE, 2016), which operates the French power transmission system. In 2016, the electric aggregated consumption was equal to 480.32 TWh. The hourly consumption was equal to 54681.29 MWh, with a standard deviation equal to 11531.66 MWh. The average price of electricity on the spot market was 36.75 €/MWh. The electric generation from intermittent energy sources decomposed to 20.92 TWh for wind energy and 8.26 TWh for solar energy, with installed capacities of 11670 MW and 6772 MW respectively. Overall, the aggregated hourly generation from intermittent energy sources was equal to 3321.8 MWh, with a standard deviation equal to 2055.13 MWh. Accordingly, one can calculate a capacity factor equal to 18.01 %, with standard deviation equal to 11.14 %.[29] We also rely on estimates of the price elasticity of demand for electricity. EPRI (2008) reviews 18 studies giving estimates of the price elasticity of demand for electricity under a variety of conditions. They find that the elasticity of electricity demand is comprised between 0.2 and 0.6, in the short run, and between 0.7 and 1.4, in the long run. Recently, Lijesen

[28] Again, for the sake of simplicity, the comparative statics derived here treats k^0 as a parameter, although it is endogenously determined in an optimal allocation. The numerical illustration in Section 4.

[29] The capacity factor is equal to the hourly output of 2611.63 MWh over the potential output of 14412 MWh. Its standard deviation is equal to the hourly standard deviation equal to 1589.99 MWh over the potential output of 14412 MWh.

(2012), dealing more specifically with *real-time* price elasticities, obtains smaller estimates between 0.01 and 0.1. Finally, in 2016, the share of renewable energies in the the electric consumption attained 21.08 %.[30] Our calibration is made to comply with the French energy laws, prescribing a renewable energy target of 32 % of final energy consumption by 2030,[31] and a smart meter target of 100 % for sites with peak demand smaller than 36 kVA by 2024.[32]

Using this piece of information, we calibrate our model by assuming that the equilibrium of the *regulated* electric market in 2016 is best described by the assumption of perfect competition. Accordingly, the optimal solution calculated previously must both fit the current data and meet the French target for renewable energy. Table 1 below lists our benchmark calibration. The details and calculus can be found in the appendix.

$\overline{a}$	$V(a)$	b	c	γ	δ	ρ	$\overline{w}$	$V(w)$
146.113	531.917	0.002	0.000715	5	0.000011	0	0.180121	0.012418

Table 2 Benchmark calibration

A sensibility analysis will complete this benchmark calibration, by varying the parameters b and ρ.[33] A robustness check with respect to the parameter b is needed due to the lack of consensus about the price elasticity of demand for electricity. Moreover, this factor is critical in our model, since the reactivity of the consumers to real-time prices directly influences the social benefit of smart meters. We will consider values of b between 0.001 and 0.005, corresponding to price elasticities between 0.1 and 0.7 approximately.[34] A sensibility analysis with respect to the parameter ρ is interesting to better highlight the role of intermittent energy sources within the electric system. We will consider values of ρ between -1 and 1.[35] Indeed, the commonly accepted belief is that the penetration of intermittent technologies causes adverse effects to the electric system.

[30] In 2016, the generation from hydroelectricity and biomass were equal to 63.36 TWh and 8.71 TWh respectively.

[31] See: https://www.ecologique-solidaire.gouv.fr/loi-transition-energetique-croissance-verte (visited November 28th, 2017).

[32] See: https://www.legifrance.gouv.fr/affichCode.do?idSectionTA=LEGISCTA000031749063&cidTexte=LEGITEXT000023983208&dateTexte=20160421 (visited November 28th, 2017).

[33] Importantly, it must be noted that the whole set of parameters is automatically updated after varying them, following the same calibration process just described (Cf Tables in appendix).

[34] See the appendix for the calculus of the price elasticity of demand in our model.

[35] Of course, we are perfectly aware that considering the whole range $\rho \in [0, 1]$ is quite irrealistic. However, as noted by Ambec and Crampes (2012, footnote 6, page 322), both cases of positive and negative correlation are possible. For example, Torres et al. (2016) obtain correlation coefficients varying between -0.79 and 0.71, depending on considered scenarios (capacities and energy mix). Finally, we find interesting to think of ρ also as a policy intrument.

Our analysis has made clear that this is so only if we presume a negative correlation between demand and intermittent energy sources or if the intermittent capacity is sufficiently large. Otherwise, the wind and solar energies could in fact supply valuable hedging services. This leads to encourage any public policies capable of increasing the correlation between intermittent energy sources and demand.[36]

Figure 4 represents the optimal capacity of intermittent generating units, as a function of the expected market share of the consumers on smart meters.[37] The main curve plots k^0 for the benchmark specification (i.e., $b = 0.002$ and $\rho = 0$).[38] We verify that it is increasing in the market share of the smart meters (i.e., from 51257 MW to 54647 MW). From our previous analysis, we know that it would actually be increasing (resp., decreasing) for any $\rho < \rho^0$ (resp., $\rho > \rho^0$). Here, we can calculate that $\rho^0 \simeq 0.65$. The horizontal dashed line represents the capacity that would be worth investing provided the renewable energies were perfectly dispatchable (i.e., $\overline{k}$). Here, we can obtain that $\overline{k} = 67038$ MW. From Figure 3, we know that it would also coincide with the optimal capacity of intermittent technologies provided that $\rho = \rho^0$. Figure 4 also highlights the potentially large influence of the correlation between demand and intermittent energy sources on the optimal capacity of intermittent units. This can be seen by comparing the lower and upper boundaries (dashed lines in gray), plotting the capacity that would be optimal if ρ was equal to -1 or 1 respectively. The gap varies from 48720 MW when $\alpha^0 = 0$, to 38255 MW when $\alpha^0 = 1$.[39]

[36]Examples of policies that can influence the correlation between demand and intermittent energies are the introduction of daylight saving time (Havranek et al, 2016), the developement and organization of prosuming consumers (Parag et al, 2016), the choice of the renewable energy mix (Torres et al, 2016), and so on.

[37]See equation 11.

[38]More generally, for any ρ, the optimal capacity would lie in the shaded area, between the lower and upper boundaries (dashed lines in gray) respectively corresponding to ρ equal to -1 and 1.

[39]In the appendix, we observe qualitatively the very same behaviors in Figures 9(a) and 9(b), dealing with the case where $b = 0.001$ and $b = 0.005$. The differences are only to be found in the magnitude of the variations of the optimal capacity of intermittent units.

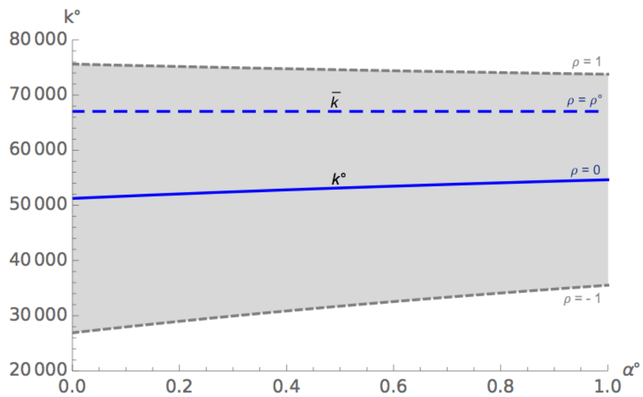


Figure 4: Optimal capacity of intermittent units (Units: %; MW)

Figure 5 depicts the expected real-time price, as a function of the expected market share of the consumers on smart meters.[40] Equation (7) implies that this relation is only indirect, through the variations of the optimal capacity of intermittent generating units. Thus, the underlying mechanisms are exactly the same as above in Figure 4 and corresponding comments. In Figure 5, the main curve plots P^0, assuming that the capacity of intermittent generating units is set optimally for the benchmark specification (i.e., k^0 calculated for $b = 0.002$ and $\rho = 0$).[41] The latter being increasing with α^0, we observe that P^0 is slightly decreasing in the expected market share of the smart meters (i.e., from 33.65 €/MWh to 32.31 €/MWh). From our theoretical analysis, we know that it would actually be decreasing (resp., increasing) for any $\rho < \rho^0$ (resp., $\rho > \rho^0$). The upper dashed line plots the expected real-time price that would prevail with the intermittent capacity set equal to zero. The gap with the main curve thus gives the reduction of the expected real-time price due to the optimal investment in the intermittent technologies. It varies from 4.86 €/MWh if $\alpha^0 = 0$, to 5.18 €/MWh if $\alpha^0 = 1$. Finally, the lower dashed line plots the expected real-time price that would prevail with the intermittent capacity set equal to $\overline{k}$. Recalling that $\overline{k}$ is the capacity that would be worth investing provided the renewable energies were perfectly dispatchable, the gap with the main curve provides a measure of the expected marginal cost of intermittency (i.e., the cost of optimally reducing the intermittent capacity for the sake of limiting the volatility on the spot market). It varies from 1.50 €/MWh when $\alpha^0 = 0$, to 1.18 €/MWh when $\alpha^0 = 1$.

[40] See equation 7.

[41] More generally, for any ρ, the expected real-time price in the optimal outcome would lie in the shaded area, between the lower and upper boundaries in dashed gray lines, respectively corresponding to ρ equal to 1 and -1.

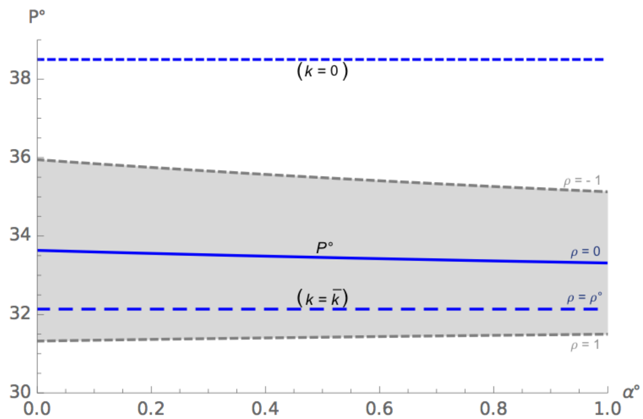


Figure 5: Expected real-time price (Units: %; €/MWh)

Figure 6 represents the variance of the real-time price, as a function of the expected market share of the consumers on smart meters.[42] Equation (9) implies that this relation is both direct and indirect. It is direct because the consumers on smart meters react to real-time price fluctuations in the opposite direction; it is indirect because installing smart meters induces variations of the optimal capacity of intermittent units, which can either increase or decrease the real-time price volatility, depending on the assumptions. In Figure 6, the main curve plots $V(p^0)$, assuming that the capacity of intermittent generating units is set optimally for the benchmark calibration (i.e., k^0 calculated for $b = 0.002$ and $\rho = 0$).[43] We verify that it is decreasing in the expected market share of the consumers on smart meters. In the appendix, we prove that this would hold true for any ρ. This means that the direct effect discussed above (i.e., consumers' reactivity) always dominates the indirect effect (i.e., variations of the optimal intermittent capacities). The lower dashed line plots the variance of the real-time price that would prevail with the intermittent capacity set equal to zero. The gap with the main curve thus measures the increase of the variance of real-time price due to the optimal investment in the intermittent technologies. It varies from 16.70 $(€/\text{MWh})^2$ if $\alpha^0 = 0$, to 10.30 $(€/\text{MWh})^2$ if $\alpha^0 = 1$. Finally, the upper dashed line represents the variance of the real-time price that would prevail with the intermittent capacity set equal to $\overline{k}$. Anew, knowing that $\overline{k}$ gives the capacity that would be worth investing provided the renewable energies were perfectly dispatchable, the gap with the main curve illustrates the optimal trade-off between low cost energy and intermittency, which induces to restrict the use of intermittent technologies compared to $\overline{k}$.

[42]See equation (9).

[43]More generally, for any ρ, the variance of the real-time price in the optimal outcome would lie in the shaded area, between the lower and upper boundaries in dashed gray lines, corresponding to ρ equal to -1 and 1 respectively.

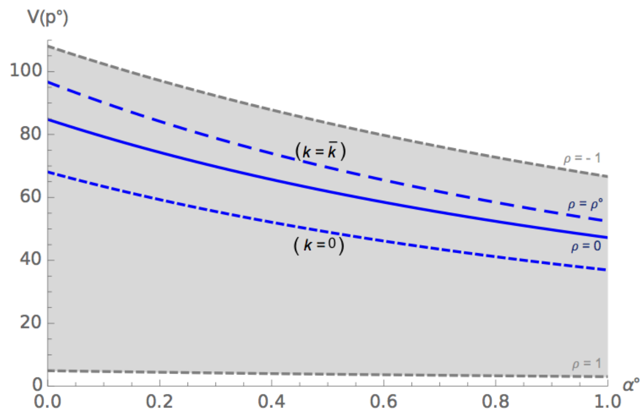


Figure 6: Variance of the real-time price (Units: %; (€/MWh)^2)

In Figures 4 to 6, the expected market of the consumers on smart meters has been treated as a parameter, in order to better highligh its role in the optimal outcome. To complete the analysis, we now deal with its optimal determination.

To do so, some additional piece of information is needed in order to calibrate the distibution of the consumers (i.e., $G(t)$). We use the following data initially collected by Léautier (2014):

(i) the large non residential sites represent 0.1 % of all sites and 42 % of total demand;

(ii) the medium non residential sites represent 1 % of all sites and 15 % of the total demand;

(iii) the small non residential sites represent 13 % of all sites and 10 % of total demand;

(iv) the residential sites represent 86 % of all sites and 32 % of total demand.

Now, assume that the consumers' types are distributed according to the Pareto distribution

$$G(t) = 1 - (t_0/t)^{\mu}, \text{ for all } t \geq t_0,$$

with parameters $t_0 > 0$ and $\mu > 1$.[44] Then, the average type is equal to $\mu t_0/(\mu - 1)$. In our model, it must be normalized to one by assumption. Also, the associated Lorenz curve writes

$$L(f) = 1 - (1 - f)^{1-1/\mu},$$

where f is the percentage of consumers consuming $L(f)$ percent or less of the aggregate electric consumption. Below, we calibrate the Pareto distribution,

[44] Note that $t_1 = \infty$ with the Pareto distribution.

by finding parameters t_0 and μ such that the Lorenz curve approximately fits the data collected by Léautier (2014). We retain the calibration $t_0 = 1/6$ and $\mu = 6/5$. Figure 7 represents the position of the Lorenz curve with respect the three points derived from Léautier (2014).

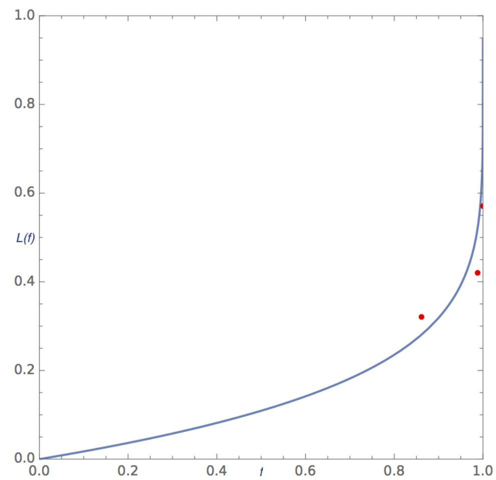


Figure 7: Lorenz curve (Units: %; %)

Figure 8 represents the optimal expected market share of the consumers on smart meters, as a function of the extra cost of installing and operating a smart meter, instead of a traditional meter.[45] The main curve plots α^0 for the benchmark calibration (i.e., $b = 0.002$ and $\rho = 0$), assuming that the capacity of intermittent generating units is set optimally.[46] We see that a general adoption of smart meters (i.e., $\alpha^0 = 1$) is optimal only if $K - \kappa$ is less 0.5 €/year. However, as smart meters become more costly, their optimal deployment decreases to reach 46 % when $K - \kappa$ is equal to 30 €/year. The dashed curve depicts the expected market share of smart meters that would be optimal, in a situation with no capacity of intermittent generating units. Since it lies below the main curve, we conclude that under the benchmark specification, the investment in the intermittent technologies justifies to use smart meters more extensively. However, it must be noted that this result is not general. Indeed, we could display cases, for ρ large enough, such that the main curve would be

[45]In our model, K and κ are the cost of installing and operating the smart and traditional meters, for the whole population, over one period of time. Here, for the sake of interpretation, we normalize the units by first dividing K and κ by 36.6 millions sites and then multiplying them by 8760 hours per year. Accordingly, the x-axis in Figure 7 gives the extra cost of a smart meter, for one site over one year.

[46]More generally, for all ρ, the curve plotting the optimal expected market share of the consumers on smart meters lies between the lower and upper boundaries in dashed gray lines, corresponding to ρ equal to 1 and -1 respectively.

below the dashed curve (e.g, the dashed gray lines, corresponding to ρ equal to -1). In other words, smart meter and intermittent technologies can be either complements or substitutes, depending on the correlation between demand and intermittent energies.

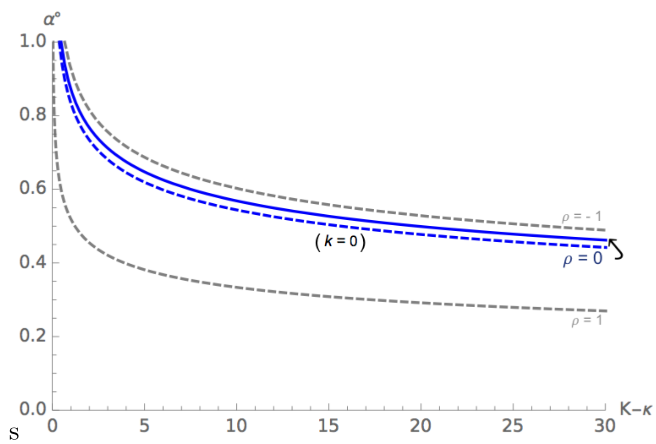


Figure 8: Expected market share of smart meters (Units: €/year; %)

5 Conclusion

In this paper, we bring together, in a stylized microeconomic framework, the issues of the determination of the development of intermittent capabilities and smart meters in a social optimum. Dealing with both issues at the same time allows in particular to highlight the role played by the correlation existing between demand and intermittent supply within the electric system. Our results challenge and clarify two commonly accepted beliefs. The first one is that the penetration of intermittent technologies necessarily causes negative externalities to the electric system. The second one is that the intermittent capacities and smart meters should necessarily be developed in parallel. Our analysis makes clear that this is so only if the electric demand and the intermittent electric generation are negatively correlated *or* if they are positively correlated but the intermittent capacity is already large enough. Otherwise, the development of the intermittent generating technologies may be a win-win policy, by allowing to reduce both the level and the volatility of the electric spot price. Then, the capacity of intermittent technologies and the installation of smart meters would be substitutes rather than complements. This leads to encourage any public policies capable of increasing the correlation between intermittent energy sources and demand. Finally, using data from the French power market, we show that a general adoption of smart meters would be optimal only if the cost of installing and operating smart meters was unrealistically low (0.5 €/year). This finding is important from a policy perspective as it casts doubt on the economic value of the general deployment of smart meters.

6 Appendix

Lagrangian and first-order conditions

The social problem is to choose $p(x)$, for all x, P, $q(x)$, for all x, $\phi(t)$, for all t, and k to maximize the *expected* social surplus

$$\int_{-\infty}^{+\infty}\left[\int_{t_0}^{t_1}\begin{pmatrix}\phi(t)\left(tS\left(p(x)\right)-\kappa\right)\\+\left(1-\phi(t)\right)\left(tS(P)-K\right)\end{pmatrix}\mathrm{d}G(t)-C\left(q(x)\right)-I(k)\right]dF(x),$$

subject to the market clearing condition

$$\int_{t_0}^{t_1}\begin{pmatrix}\phi(t)tD\left(p(x)\right)\\+\left(1-\phi(t)\right)tD(P)\end{pmatrix}\mathrm{d}G(t)=q(x)+w(x)k,$$

for all x.

The lagrangian for this problem writes

$$L=\int_{-\infty}^{+\infty}\left[\begin{array}{c}\int_{t_0}^{t_1}\begin{pmatrix}\phi(t)\left(tS\left(p(x)\right)-\lambda(x)tD\left(p(x)\right)-K\right)\\+\left(1-\phi(t)\right)\left(tS(P)-\lambda(x)tD(P)-\kappa\right)\end{pmatrix}\mathrm{d}G(t)\\+\lambda(x)q(x)-C\left(q(x)\right)+\lambda(x)w(x)k-I(k)\end{array}\right]\mathrm{d}F(x),$$

where $\lambda(x)$ is the multiplier associated with the market clearing condition.

Denoting $\alpha=\int_{t_0}^{t_1}\phi(t)t\mathrm{d}G(t)$,[47] the derivatives of the lagrangian are:[48]

$$\frac{\partial L}{\partial p(x)}=\frac{\alpha}{b}\left(p(x)-\lambda(x)\right)\mathrm{d}F(x)$$

$$\frac{\partial L}{\partial P}=\frac{1-\alpha}{b}\int_{-\infty}^{+\infty}\left(P-\lambda(x)\right)\mathrm{d}F(x)$$

$$\frac{\partial L}{\partial q(x)}=\left(\lambda(x)-C'\left(q(x)\right)\right)\mathrm{d}F(x)$$

$$\frac{\partial L}{\partial \phi(t)}=\mathrm{d}G(t)\int_{-\infty}^{+\infty}\begin{pmatrix}\lambda(x)t\left(D(P)-D\left(p(x)\right)\right)\\-t\left(S(P)-S\left(p(x)\right)\right)\\-(K-\kappa)\end{pmatrix}\mathrm{d}F(x)$$

$$\frac{\partial L}{\partial k}=\mathrm{d}G(t)\int_{-\infty}^{+\infty}\left(\lambda(x)w(x)-I'(k)\right)\mathrm{d}F(x)$$

[47]Remark that $\alpha\in[0,1]$.
[48]The derivatives simplify using $S'(p)=-p/b$ and $D'(p)=-1/b$.

From this, we can show that the optimal solution satisfies the following conditions:

$$p^0(x) = C'\left(q^0(x)\right), \text{ for all } x,$$

$$P^0 = \int_{-\infty}^{+\infty} C'\left(q^0(x)\right) \mathrm{d}F(x),$$

$$I'(k^0) = \int_{-\infty}^{+\infty} C'\left(q^0(x)\right) w(x) \mathrm{d}F(x),$$

$$\phi^0(t) = 1 \quad \text{iff} \quad \tfrac{t}{2b} \int_{-\infty}^{+\infty} \left(p^0(x) - P^0\right)^2 \mathrm{d}F(x) > K - \kappa.$$

Comparative statics of $V(p^0)$ (Equation (9))

We derive here the comparative statics of

$$V(p^0) = \left(\frac{c}{b+\alpha^0 c}\right)^2 \left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right).$$

Let us define below

$$\underline{k} \equiv \max\left\{0, \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right\}.$$

We show below that $\underline{k}$ is the investment in the renewable generating units that minimizes $V(p^0)$. Considering α^0 and k^0 as given, we can calculate that

$$\frac{\partial V(p^0)}{\partial \overline{a}} = 0,$$

$$\frac{\partial V(p^0)}{\partial V(a)} = \left(\frac{c}{b+\alpha^0 c}\right)^2 \frac{b}{\rho}\sqrt{\frac{V(w)}{V(a)}} \left(\frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}} - \rho^2 k^0\right) \gtreqless 0 \Leftrightarrow \rho^2 k^0 \lesseqgtr \underline{k},$$

$$\frac{\partial V(p^0)}{\partial b} = -2\frac{c^2}{\left(b+\alpha^0 c\right)^3} \begin{pmatrix} V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w) \\ + \left(b+\alpha^0 c\right) bk^0 V(w) \left(\frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}} - k^0\right) \end{pmatrix} < 0 \text{ if } k^0 \leq \underline{k},$$

$$\frac{\partial V(p^0)}{\partial c} = \frac{2bc}{\left(b+\alpha^0 c\right)^3} \left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right) > 0,$$

$$\frac{\partial V(p^0)}{\partial \overline{w}} = 0,$$

$$\frac{\partial V(p^0)}{\partial V(w)} = \left(\frac{bc}{b+\alpha^0 c}\right)^2 k^0 \left(k^0 - \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right) \gtreqless 0 \Leftrightarrow k^0 \gtreqless \underline{k},$$

$$\frac{\partial V(p^0)}{\partial \rho} = -2\left(\frac{c}{b+\alpha^0 c}\right)^2 bk^0\sqrt{V(a)V(w)} < 0,$$

$$\frac{\partial V(p^0)}{\partial \alpha^0} = -2\left(\frac{c}{b+\alpha^0 c}\right)^3 \left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right) < 0,$$

$$\frac{\partial V(p^0)}{\partial k^0} = 2\left(\frac{bc}{b+\alpha^0 c}\right)^2 V(w)\left(k^0 - \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right) \gtreqless 0 \Leftrightarrow k^0 \gtreqless \underline{k}.$$

Remark that the last result means that $V(p^0)$ is minimized when $k^0 = \underline{k}$.

Comparative statics of k^0 (Equation (11))

We derive here the comparative statics of

$$k^0 = \frac{\frac{c}{b+c}\overline{a}\overline{w} - \gamma + \frac{c}{b+\alpha^0 c}\rho\sqrt{V(a)V(w)}}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)}.$$

Let us first define

$$\overline{k} = \frac{\frac{c}{b+c}\overline{a}\overline{w} - \gamma}{\frac{c}{b+c}b\overline{w}^2 + \delta}.$$

Note that k^0 coincides with $\overline{k}$ when $V(w) = 0$. Taking α^0 as given, we can calculate that

$$\frac{\partial k^0}{\partial \overline{a}} = \frac{\frac{c}{b+c}\overline{w}}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)} > 0,$$

$$\frac{\partial k^0}{\partial V(a)} = \frac{\frac{c}{b+\alpha^0 c}\frac{1}{2}\rho\sqrt{\frac{V(w)}{V(a)}}}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)} \gtreqless 0 \Leftrightarrow \rho \gtreqless 0,$$

$$\frac{\partial k^0}{\partial b} = -\frac{\begin{array}{c}\frac{c}{(b+c)^2}\overline{a}\overline{w}\left(\frac{\overline{a}\overline{w}-\gamma}{\overline{a}\overline{w}}c\overline{w}^2 + \delta\right)\\ +\frac{c}{(b+c)^2}\overline{a}\overline{w}\frac{c}{(b+\alpha^0 c)^2}V(w)\left((1-\alpha^0)b^2 + \alpha^0\frac{\overline{a}\overline{w}-\gamma}{\overline{a}\overline{w}}(b+c)^2\right)\\ +\frac{c}{(b+\alpha^0 c)^2}\rho\sqrt{V(a)V(w)}\left(\left(b^2+2bc+\alpha^0 c^2\right)\frac{c}{(b+c)^2}\overline{w}^2 + \delta + cV(w)\right)\end{array}}{\left(\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)\right)^2} > 0 \text{ if } \rho \geq 0,$$

$$\frac{\partial k^0}{\partial c} = \frac{\begin{array}{c}\frac{b}{(b+c)^2}\overline{a}\overline{w}\left(\frac{\gamma}{\overline{a}}b\overline{w} + \delta\right)\\ -\frac{b}{(b+c)^2}\overline{a}\overline{w}\frac{b}{(b+\alpha^0 c)^2}V(w)\left((1-\alpha^0)c^2 - \frac{\gamma}{\overline{a}\overline{w}}(b+c)^2\right)\\ +\frac{b}{(b+\alpha^0 c)^2}\rho\sqrt{V(a)V(w)}\left((1-\alpha^0)c^2\frac{b}{(b+c)^2}\overline{w}^2 + \delta\right)\end{array}}{\left(\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)\right)^2},$$

$$\frac{\partial k^0}{\partial \gamma} = -\frac{1}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)} < 0,$$

$$\frac{\partial k^0}{\partial \delta} = -\frac{\frac{c}{b+c}\overline{aw} - \gamma + \frac{c}{b+\alpha^0 c}\rho\sqrt{V(a)V(\mathrm{w})}}{\left(\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)\right)^2} < 0,$$

$$\frac{\partial k^0}{\partial \overline{w}} = \frac{c}{b+c}2b\overline{w}\frac{\frac{\overline{a}}{2b\overline{w}} - k^0}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)} \gtreqless 0 \Leftrightarrow k^0 \lesseqgtr \frac{\overline{a}}{2b\overline{w}},$$

$$\frac{\partial k^0}{\partial V(w)} = \frac{bc}{b+\alpha^0 c}\frac{\frac{1}{2}\frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}} - k^0}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)} \gtreqless 0 \Leftrightarrow k^0 \lesseqgtr \frac{\underline{k}}{2},$$

$$\frac{\partial k^0}{\partial \rho} = \frac{\frac{c}{b+\alpha^0 c}\sqrt{V(a)V(w)}}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)} > 0,$$

$$\frac{\partial k^0}{\partial \alpha^0} = \left(\frac{c}{b+\alpha^0 c}\right)^2 \frac{\left(\frac{c}{b+c}b\overline{w}^2 + \delta\right)bV(w)}{\left(\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)\right)^2}\left(\frac{\frac{c}{b+c}\overline{aw} - \gamma}{\frac{c}{b+c}b\overline{w}^2 + \delta} - \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right) \gtreqless 0 \Leftrightarrow \overline{k} \gtreqless \underline{k}.$$

Note that the comparative statics with respect to c is very ambiguous and cannot be easily characterized.

Justification of Figure 2

We derive here the calculus which justify Figure 2.

We first verify that

$$\overline{k} \gtreqless \underline{k} \Leftrightarrow \rho \lesseqgtr \rho^0 \equiv b\sqrt{\frac{V(w)}{V(a)}}\frac{\frac{c}{b+c}\overline{aw} - \gamma}{\frac{c}{b+c}b\overline{w}^2 + \delta}.$$

Indeed, $\underline{k}$ is equal to $\overline{k}$ when $\rho = \rho^0$ and is increasing in ρ.[49]

We have shown in a previous appendix that

$$\frac{\partial k^0}{\partial \alpha^0} \gtreqless 0 \Leftrightarrow \overline{k} \gtreqless \underline{k}.$$

Given that

$$\overline{k} \gtreqless \underline{k} \Leftrightarrow \rho \lesseqgtr \rho^0,$$

[49] Note that $\overline{k}$ does not vary with ρ.

it is equivalent to

$$\frac{\partial k^0}{\partial \alpha^0} \gtreqless 0 \Leftrightarrow \rho \lesseqgtr \rho^0.$$

Finally, let us show that k^0 always lies between $\underline{k}$ and $\overline{k}$. To see it, first subsitute $\rho = \rho^0$ into

$$k^0 = \frac{\frac{c}{b+c}\overline{aw} - \gamma + \frac{c}{b+\alpha^0 c}\rho\sqrt{V(a)V(w)}}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)}$$

to verify that

$$k^0 = \underline{k} = \overline{k}$$

in this case. If $\rho > \rho^0$, we have just shown that k^0 is decreasing in α^0. Remarking that

$$\lim_{\alpha^0 \to -b/c} k^0 = \frac{\rho}{b}\sqrt{\frac{V(w)}{V(a)}}$$

and

$$\lim_{\alpha^0 \to \infty} k^0 = \frac{\frac{c}{b+c}\overline{aw} - \gamma}{\frac{c}{b+c}b\overline{w}^2 + \delta},$$

we get in particular $\overline{k} < k^0 < \underline{k}$ for all $0 \leq \alpha^0 \leq 1$. Likewise, if $\rho < \rho^0$, we can show that $\underline{k} < k^0 < \overline{k}$.[50]

Comparative statics of α^0 (Equations (9), (14) and (15))

We derive the comparative statics of

$$\alpha^0 = \int_{t^0}^{t_1} t\mathrm{d}G(t),$$

given that

$$t^0 = \frac{2b(K-\kappa)}{V(p^0)}$$

and

$$V(p^0) = \left(\frac{c}{b+\alpha^0 c}\right)^2 \left(V(a) - 2bk^0 Cov(a,w) + \left(bk^0\right)^2 V(w)\right).$$

As $V(p^0)$ is itself a function of α^0, this system defines α^0 implicitly. In order to ease the presentation, we proceed in two steps below.

[50]When $\rho < 0$, to show that $k^0 > \underline{k}$, we use the fact that $k^0 > 0$ (interior solution) and $\underline{k} = 0$.

Let us first consider $V(p^0)$ as a parameter. Total differentiation then directly yields after arrangement

$$\mathrm{d}\alpha^0 = \left(-\frac{\mathrm{d}b}{b} - \frac{\mathrm{d}(K-\kappa)}{K-\kappa} + \frac{\mathrm{d}V(p^0)}{V(p^0)}\right)\left(t^0\right)^2 \mathrm{d}G(t^0), \tag{16}$$

which implies that α^0 is decreasing in b and $K-\kappa$, and is increasing in $V(p^0)$.

Let us now take into account the variations of $V(p^0)$. Using the comparative statics of $V(p^0)$ calculated in a previous appendix, total differentiation yields

$$\begin{aligned}\mathrm{d}V(p^0) =& -2\left(\frac{c}{b+\alpha^0 c}\right)^3\left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right)\ \mathrm{d}\alpha^0 \\ &+ 0\ \mathrm{d}\overline{a} + \left(\frac{c}{b+\alpha^0 c}\right)^2 \frac{b}{\rho}\sqrt{\frac{V(w)}{V(a)}}\left(\frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}} - \rho^2 k^0\right)\mathrm{d}V(a) + ...\end{aligned}$$

Substituting this into (16), we obtain

$$\triangle \mathrm{d}\alpha^0 = -\frac{\mathrm{d}b}{b} - \frac{\mathrm{d}(K-\kappa)}{K-\kappa} + 0\ \mathrm{d}\overline{a} + \left(\frac{c}{b+\alpha^0 c}\right)^2 \frac{b}{\rho}\sqrt{\frac{V(w)}{V(a)}}\left(\frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}} - \rho^2 k^0\right)\mathrm{d}V(a) + ...$$

where we let

$$\triangle \equiv \frac{1}{\left(t^0\right)^2 \mathrm{d}G(t^0)} + 2\left(\frac{c}{b+\alpha^0 c}\right)^3\left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right) > 0.$$

From this, we can finally calculate that

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}\overline{a}} = 0,$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}V(a)} = \frac{1}{\triangle}\left(\frac{c}{b+\alpha^0 c}\right)^2 \frac{b}{\rho}\sqrt{\frac{V(w)}{V(a)}}\left(\frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}} - \rho^2 k^0\right) \gtreqless 0 \Leftrightarrow \rho^2 k^0 \lesseqgtr \underline{k},$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}b} = -\frac{1}{\triangle}\left(\frac{1}{b} + 2\frac{c^2}{\left(b+\alpha^0 c\right)^3}\left(\begin{array}{c} V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w) \\ + \left(b+\alpha^0 c\right) bk^0 V(w)\left(\frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}} - k^0\right)\end{array}\right)\right) < 0 \text{ if } k^0 \leq \underline{k},$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}c} = \frac{1}{\triangle}\frac{2bc}{\left(b+\alpha^0 c\right)^3}\left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right) > 0,$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}(K-\kappa)} = -\frac{1}{\triangle}\frac{1}{K-\kappa} < 0,$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}\overline{w}} = 0,$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}V(w)} = \frac{1}{\triangle}\left(\frac{bc}{b+\alpha^0 c}\right)^2 k^0 \left(k^0 - \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right) \gtreqless 0 \Leftrightarrow k^0 \gtreqless \underline{k},$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}\rho} = -\frac{2}{\triangle}\left(\frac{c}{b+\alpha^0 c}\right)^2 bk^0\sqrt{V(a)V(w)} < 0,$$

$$\frac{\mathrm{d}\alpha^0}{\mathrm{d}k^0} = \frac{2}{\triangle}\left(\frac{bc}{b+\alpha^0 c}\right)^2 V(w)\left(k^0 - \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right) \gtreqless 0 \Leftrightarrow k^0 \gtreqless \underline{k}.$$

Proof that $V(p^0)$ is decreasing in α^0 (Equations (9) and (11))

We prove here that the use of smart meters always reduces real-time price volatility, even when accounting for both its direct (i.e., k^0 assumed given) and indirect (i.e., k^0 varying with α^0) effects on $V(p^0)$ (see Figure 6). Indeed, consider the system

$$V(p^0) = \left(\frac{c}{b+\alpha^0 c}\right)^2 \left(V(a) - 2bk^0 Cov(a,w) + \left(bk^0\right)^2 V(w)\right)$$

and

$$k^0 = \frac{\frac{c}{b+c}\overline{aw} - \gamma + \frac{c}{b+\alpha^0 c}\rho\sqrt{V(a)V(w)}}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)}.$$

We wish to determine the sign of

$$\frac{\mathrm{d}V(p^0)}{\mathrm{d}\alpha^0} = \frac{\partial V(p^0)}{\partial \alpha^0} + \frac{\partial V(p^0)}{\partial k^0}\frac{\partial k^0}{\partial \alpha^0}.$$

Using

$$\frac{\partial V(p^0)}{\partial \alpha^0} = -2\left(\frac{c}{b+\alpha^0 c}\right)^3 \left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right),$$

$$\frac{\partial V(p^0)}{\partial k^0} = 2\left(\frac{bc}{b+\alpha^0 c}\right)^2 V(w)\left(k^0 - \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right)$$

and

$$\frac{\partial k^0}{\partial \alpha^0} = \left(\frac{c}{b+\alpha^0 c}\right)^2 \frac{bV(w)}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)}\left(k^0 - \frac{\rho}{b}\sqrt{\frac{V(a)}{V(w)}}\right),$$

we can write that

$$\frac{\mathrm{d}V(p^0)}{\mathrm{d}\alpha^0} = -2\left(\frac{c}{b+\alpha^0 c}\right)^3 \left(V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w)\right)$$
$$+ 2b\left(\frac{c}{b+\alpha^0 c}\right)^4 \frac{\left(bk^0 V(w) - \rho\sqrt{V(a)V(w)}\right)^2}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)}$$

Noting that

$$2b\left(\frac{c}{b+\alpha^0 c}\right)^4 \frac{\left(bk^0 V(w) - Cov(a,w)\right)^2}{\frac{c}{b+c}b\overline{w}^2 + \delta + \frac{c}{b+\alpha^0 c}bV(w)} < 2\left(\frac{c}{b+\alpha^0 c}\right)^3 \frac{\left(bk^0 V(w) - Cov(a,w)\right)^2}{V(w)},$$

we obtain that

$$\frac{\mathrm{d}V(p^0)}{\mathrm{d}\alpha^0} < -2\left(\frac{c}{b+\alpha^0 c}\right)^3 \begin{pmatrix} V(a) - 2bk^0\rho\sqrt{V(a)V(w)} + \left(bk^0\right)^2 V(w) \\ -\left(bk^0 V(w) - \rho\sqrt{V(a)V(w)}\right)^2 / V(w) \end{pmatrix}.$$

Rearranging the terms under brackets, we can rewrite this as

$$\frac{\mathrm{d}V(p^0)}{\mathrm{d}\alpha^0} < -2\left(\frac{c}{b+\alpha^0 c}\right)^3 V(a)\left(1-\rho^2\right) \leq 0,$$

since $-1 \leq \rho \leq 1$.

Calibration

Here we explain how we calibrate the parameters $\overline{a}$, $V(a)$, b, c, γ, δ, ρ, w and $V(w)$. We use the following data set (see main text for sources):[51]

[51] Note that the notations used here are specific to this appendix. They should not be confused with the notations used in the main text.

	Notation	Value
Initial situation (2016):		
- Average hourly consumption :	D_0	54681.29 MWh
- Variance of hourly consumption:	$V(D_0)$	$(11553.66)^2$ MWh^2
- Average real-time price:	P_0	36.75 €/MWh
- Intermittent power capacity:	k_0	$11670 + 6772$ MW
- Average capacity factor:	w_0	0.180121
- Variance of the capacity factor:	$V(w_0)$	$(0.111437)^2$
- Average biomass production:	q_0^B	991.73 MWh
- Average hydraulic production:	q_0^H	7213.06 MWh
- Share of smart meters:	α_0	0
Objectives (2030):		
- Ratio for renewable energy:	τ_1	0.32
- Share of smart meters:	α_1	1

The parameters $\overline{a}$, $V(a)$, c, $\overline{w}$ and $V(w)$ are chosen such that the initial situation determines a market equilibrium under perfect competition:

$$D_0 = \frac{\overline{a} - P_0}{b},$$

$$V(a) = b^2 V(D_0),$$

$$P_0 = c\frac{\overline{a} - b\overline{w}k_0}{b + c},$$

$$\overline{w} = w_0,$$

$$V(w) = V(w_0).$$

The parameters γ and δ are chosen such that the optimal outcome comply with the policy objectives:[52]

$$\overline{w}k^0 + q_0^B + q_0^H = \tau_1 \frac{\overline{a} - P^0}{b},$$

with k^0 satisfying (11) when $\alpha^0 = \alpha_1$.

As one can see, we have three degrees of freedom (i.e., 6 equations and 9 parameters). In our calibration, the parameter γ is set equal to 5, representing an *annual* amortized cost of a generating unit of 1 MW equal to $5 \times 8760 = 43800$ €/year. Then, we solve the system to determine $\overline{a}$, $V(a)$, c, δ, $\overline{w}$ and $V(w)$, as functions of b and ρ. In the benchmark calibration, we set $b = 0.002$ and $\rho = 0$. We also consider alternative calibrations where $b = 0.001$ and $\rho = 0$, on the one

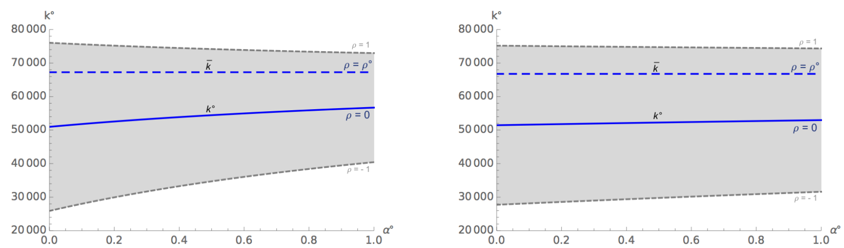
[52]Note that we assume implicitly that the biomass and hydraulic productions will remain constant in the period.

hand, and $b = 0.005$ and $\rho = 0$, on the other hand. The corresponding scenarios are shown the the following table:

	$\overline{a}$	$V(a)$	b	c	γ	δ	ρ	$\overline{w}$	$V(w)$
Benchmark	146.113	531.917	0.002	0.000715	5	0.000011	0	0.180121	0.012418
Low elast.	91.431	132.979	0.001	/	/	0.000014	/	/	/
High elast.	310.156	3324.48	0.005	/	/	0.000009	/	/	/

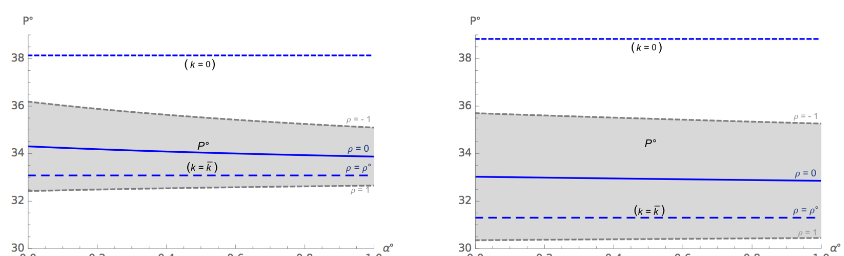
Table 2 Alternative calibrations

Sensibility analysis



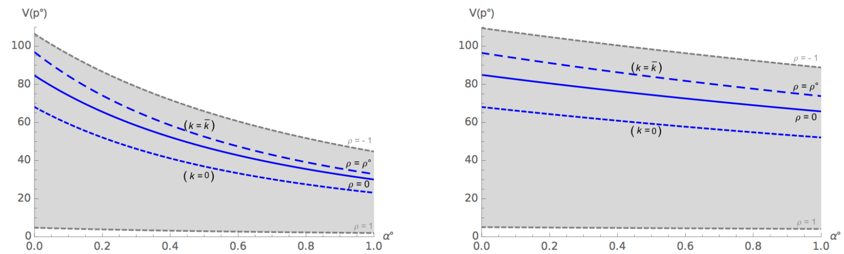
(a) $b = 0.001$**.** **(b)** $b = 0.005$**.**

Figure 9: Optimal capacity of intermittent units (Units: %; MW)



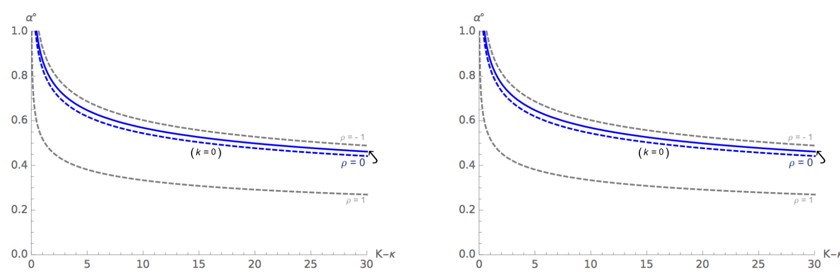
(a) $b = 0.001$**.** **(b)** $b = 0.005$**.**

Figure 10: Expected real-time price (Units: %; €/MWh)



(a) $b = 0.001$. **(b)** $b = 0.005$.

Figure 11: Variance of the real-time price (Units: %; (€/MWh)^2)



(a) $b = 0.001$. **(b)** $b = 0.005$.

Figure 12: Expected market share of smart meters (Units: €/year; %)

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