Groupe de Recherche en
Économie Théorique et Appliquée

The Preference for Monotone Decision Problems

Bernard SINCLAIR-DESGAGNE
HEC Montréal
bsd@hec.ca

Invited contribution

Cahiers du GREThA
n° 2018-15
July

GRETHA UMR CNRS 5113
Université de Bordeaux
Avenue Léon Duguit - 33608 PESSAC - FRANCE
Tel : +33 (0)5.56.84.25.75 - Fax : +33 (0)5.56.84.86.47 - www.gretha.fr

La préférence pour les problèmes de décision monotones

Résumé

Cet article expose certaines conditions sous lesquelles un décideur rationnel accepterait de restreindre a priori ses choix, en échange d'information sur son environnement. Nous montrons que les contraintes imposées conduiront alors le décideur à résoudre un problème de décision monotone. Ceci constitue une première justification de l'omniprésence de ce type de problème dans la vie économique. Vue sous un autre angle, notre analyse explique pourquoi les individus et les organisations s'en remettent souvent à des automatismes et autres routines, et comment le phénomène contribue à façonner l'environnement économique.

Mots-clés: Problèmes de décision monotones; Inattention rationnelle; Attributs de design; Routines

The Preference for Monotone Decision Problems

Abstract

This paper spells out conditions under which a rational decisionmaker will commit ex ante to certain choice restrictions, in order to get extra information about an uncertain state of nature. We show that the envisioned limitations will then bring the decision-maker to solve a monotone decision problem. This provides a first rationale for the observed recurrence of this type of problem in economic life. From another angle, the analysis also explains why individuals and organizations resort to automatic responses and routines in some circumstances, and how this contributes to shape their environment.

Keywords: Monotone decision problems; Rational inattention; Design attributes; Routines

JEL: D01, D02, D89

**Most of this piece has been written when Bernard Sinclair Desgagné was visiting the GREThA in the framework of the Visiting Scholars Programme of the IdEx Bordeaux whose support is gratefully acknowledged.*

Reference to this paper: SINCLAIR-DESGAGNE Bernard (2018) The Preference for Monotone Decision Problems, *Cahiers du GREThA*, n°2018-15.

http://ideas.repec.org/p/grt/wpegrt/2018-15.html.

GRETHA UMR CNRS 5113
Université de Bordeaux
Avenue Léon Duguit - 33608 PESSAC - FRANCE
Tel : +33 (0)5.56.84.25.75 - Fax : +33 (0)5.56.84.86.47 - www.gretha.fr

The Preference for Monotone Decision Problems*

Bernard Sinclair-Desgagné†

HEC Montréal

Abstract: This paper spells out conditions under which a rational decision-maker will commit ex ante to certain choice restrictions, in order to get extra information about an uncertain state of nature. We show that the envisioned limitations will then bring the decision-maker to solve a monotone decision problem. This provides a first rationale for the observed recurrence of this type of problem in economic life. From another angle, the analysis also explains why individuals and organizations resort to automatic responses and routines in some circumstances, and how this contributes to shape their environment.

Keywords: Monotone decision problems; Rational inattention; Design attributes; Routines

JEL classification no.: D01, D02, D89

* This note owes substantially to previous work with Landis Gabel and Antoine Soubeyran, and to many conversations I've had over the years with some friends and colleagues, including Bentley Macleod, Pascale Viala, Lars-Hendrik Röller, Jean-Pierre Ponssard, Pierre Lasserre, Marcel Boyer, Tracy Lewis, John Conlon, Patrick Cohendet, Éric Giraud-Héraud, Pauline Barrieu, Louis Lévy-Garboua, and Murat Yildizoglu. I am, of course, sole responsible for all shortcomings. I also wish to thank the GREThA of the University of Bordeaux for its hospitality (under the IDEX-Bordeaux Visiting Scholars Program) while this piece was being written.

† **Full address:** *Environmental Economics and Global Governance Chair*, Department of International Business, HEC Montréal, 3000 Côte-Sainte-Catherine, Montréal, Canada H3T 2A7; bsd@hec.ca .

1. Introduction

When a decision's payoff is subject to uncertainty, to what extent would a rational decision-maker accept to limit her choices against the possibility of obtaining extra information? At first sight, the answer rests on the value this information brings; but this in turn depends on what the constrained decision-maker will be capable of doing once she gets the additional data.[1] Much of individual or team training involves this precise trade-off: training enables someone to get hold of relevant info, but it also formats a subject to act upon evidence in a certain way. The issue matters as well for organizations: firms must somewhat tie together their business units (especially those respectively in charge of product design and marketing) if they want to effectively gather and process relevant business intelligence (Gold et al. 2001).

Oftentimes, the nature of the obtainable signals - be they numerical, like prices, or subject to measurement - will allow a decision-maker to rank them. Suppose, then, that the decision-maker knows a priori (or believes she knows) how to best react to any given signal. This characterizes a situation with so-called *design attributes*, where "(...) there is a great deal of a priori information about the form of the optimal solution, that is, about how the variables should be related." (Milgrom and Roberts 1992, p. 90) In those circumstances, this paper shows that, in order to get information, the decision-maker will consent on having her choice set reduced to a list of graded moves.

[1] For this reason, the issue considered here does not fit the framework of real options (for some influential presentations, see Dixit and Pindyck 1994, and Tregeorgis 1996). In the latter, a decision-maker weighs up the possibility of making a commitment after receiving information; here, she must commit *before* information comes in.

This result highlights two (objective or subjective) contextual features - rankable signals and design attributes - that will lead a rational decision-maker to *precommit* to view and handle an upcoming situation as a monotone decision problem. Recall that a decision problem is monotone if the decision-maker's observation of a higher signal leads her to choose a higher action. Problems of this sort have been recognized in many settings, such as production planning, financial management, insurance, auctions, contracting, and organizational design. Economists thus keep devoting substantial research effort to their analysis (see, e.g., Athey and Levin 2018, and the references therein). On a different note, this paper offers a first explanation for their prevalence.

One can also see this paper's result as a statement about automatic (or rule-based) versus controlled (or adaptive) decision processes. Dating back at least to Simon (1947)'s and Cyert and March (1963)'s respective accounts of the cognitive limitations of individuals and organizations, there is a sizable and growing literature dealing with bounded rationality and rational inattention (see, e.g., Conlisk 1996, Rubinstein 1998, Dessein et al. 2016, and the references therein), which now draws largely on psychology and the neurosciences (e.g., Kahneman 2003, Angner and Lowenstein 2007, Camerer et al. 2005). This literature notably considers the fact that individuals and organizations often sacrifice on extensive deliberation in exchange for greater awareness of their environment. A generic type of behavior is to develop appropriate reflexes, production habits, standard operating procedures, or routines, the latter meaning that "(...) the parts of an individual's skill which are completely routinized are the parts that he or she does not have to think about - once a routine is switched on in the worker's mind, it goes on to end without further consultation of the higher faculties." (Nelson and Winter 1982, p. 63) We model this conduct as having the choice set reduced to a list of labelled subsets of specific values the decision variables can take; once a given label is picked, all the decision variables are determined. According to our main proposition, a rational decision-maker with enough a priori knowledge will stick to such a scheme in order to grab an informative signal.

The upcoming section will now lay down the mathematical background for this result. Section 3 contains the formal proposition and its demonstration. A concluding Section 4 brings further remarks and observations.

2. Notation and definitions

Consider a decision-making entity (which can be an individual or an organization) with preferences over a set of outcomes $y \in \Upsilon$ represented by a valuation (utility, profit, welfare) function $u : \Upsilon \to \mathbb{R}$. The outcomes stem from a transformation $Y : \mathcal{X} \times \Omega \to \Upsilon$, an instance of it is written $y = Y(x; \omega)$, where $x \in \mathcal{X}$ denotes the decision variables and $\omega \in \Omega$ is an uncertain state of the world. The decision-maker's prior beliefs about the latter are encoded in the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathcal{F}$ is a σ-algebra of subsets (events) of Ω, and $\mathbb{P} : \mathcal{F} \to [0, 1]$ is a probability measure. Assume that, for every x, the composite application $u \circ Y(x; \cdot) : \Omega \to \Upsilon$ is integrable with respect to $\mathbb{P}$; the decision-maker's choice then evolves around the expected valuation $\mathbb{E}_{\mathbb{P}}\left[u\left(Y\left(x; \omega\right)\right)\right]$, where $\mathbb{E}_{\mathbb{P}}$ refers to the expectation operator based on $\mathbb{P}$.

For some measurable space (H, Γ), where H is an arbitrary set (of real numbers, for instance) and Γ is a σ-algebra of subsets of H, let $\Theta : \Omega \rightarrow H$ be a measurable mapping taking values in H. We interpret a realization $\theta = \Theta(\omega)$ as a signal about the state of the world ω. With no loss of generality, we assume that Θ is surjective (or onto), i.e. $\Theta(\Omega) = H$.

The decision-maker could readily seek an element of $\mathcal{X}$ that maximizes $\mathbb{E}_{\mathbb{P}}\left[u\left(Y\left(x;\omega\right)\right)\right]$. Alternatively, she could first commit to a restricted choice set $\overset{\triangle}{\mathrm{X}} \subset \mathcal{X}$ in exchange for the ability to observe $\Theta(\omega)$, then choose in $\overset{\triangle}{\mathrm{X}}$ a maximizer of the conditional (or revised) expected valuation $\mathbb{E}_{\mathbb{P}}[u\left(Y\left(x;\omega\right)\right) \mid \Theta(\omega)]$. Which path would she go for? Two key notions must be defined before we provide a formal answer to this question.

Recall that a *chain* is a totally ordered set.

Definition 1: A *routine* is an injective (or one-to-one) function $r : \overrightarrow{S} \rightarrow \mathcal{X}$, where the domain $\overrightarrow{S}$ of r is a chain.

This definition renders the idea conveyed in the introduction that a routine is like a dial, a knob, a switch or a tuning device. The chain $\overrightarrow{S}$ stands for a graduated ring or strip. Once a spot s on $\overrightarrow{S}$ is hit, specific values of the decision variables are triggered through $r(s)$, and no other spot s' on $\overrightarrow{S}$ activates exactly the same values.

The second notion now evokes some way to represent the objects of a set.

Definition 2: A set A *parametrizes* a set B if there is a bijective (or one-to-one and onto) function $p : A \rightarrow B$.

A *parameter* $a \in A$ thus acts as a label, a code name, or a sticker uniquely ascribed to an object $p(a) = b \in B$. Knowing a, one knows b, and vice-versa.

3. Main result

The issue raised at the start of this paper can now be answered as follows.

THEOREM: Suppose that (i) a chain $\overrightarrow{\mathrm{H}}$ parametrizes the set of signals H, and (ii) for any $\theta \in H$, $\arg\max_{x \in \mathcal{X}} \mathbb{E}_{\mathbb{P}}[u\left(Y\left(x;\omega\right)\right) \mid \Theta(\omega) = \theta] \neq \varnothing$. Then, the decision-maker should find it preferable to rely on a routine if she can thereby gather $\Theta(\omega)$.

PROOF: Observe that, for all $x \in \mathcal{X}$,

$$\begin{aligned} \mathbb{E}_{\mathbb{P}}\left[u\left(Y\left(x;\omega\right)\right)\right] &= \mathbb{E}_{\mathbb{P}}[\mathbb{E}_{\mathbb{P}}\left[u\left(Y\left(x;\omega\right)\right) \mid \Theta(\omega)\right]] \\ &\leq \mathbb{E}_{\mathbb{P}}[\max_{x \in \mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[u\left(Y\left(x;\omega\right)\right) \mid \Theta(\omega)\right]] \ . \end{aligned}$$

Now, consider the subset $\digamma$ of $H \times \mathcal{X}$ defined as $\digamma = \{(\theta, x) \mid x \in \arg\max_{x \in \mathcal{X}} \mathbb{E}_{\mathbb{P}}[u\left(Y\left(x;\omega\right)\right) \mid \Theta(\omega) = \theta]$ for some $\theta \in H\}$. By the Axiom of Choice,[2] there exists a 'uniformization' of $\digamma$, i.e. a 'uniformizing function' $f : H \to \mathcal{X}$ such that, for all $\theta \in H$, the pair $(\theta, f(\theta)) \in \digamma$.[3] Let $\bar{r} = f \circ p : \overrightarrow{\mathrm{H}} \to \mathcal{X}$, where $p : \overrightarrow{\mathrm{H}} \to H$ is the assumed bijection between $\overrightarrow{\mathrm{H}}$ and H. Clearly,

$$\mathbb{E}_{\mathbb{P}}[\max_{x \in \mathcal{X}} \mathbb{E}_{\mathbb{P}}[u\left(Y\left(x;\omega\right)\right) \mid \Theta(\omega)]] = \mathbb{E}_{\mathbb{P}}[\max_{s \in \overrightarrow{\mathrm{H}}} \mathbb{E}_{\mathbb{P}}[u\left(Y(\bar{r}\left(s\right);\omega)\right) \mid \Theta(\omega)]] \ .$$

The function $\bar{r}$ takes values in $\mathcal{X}$ and its domain is a chain. But it may not be one-to-one. We can nevertheless use it to construct a routine.

Take the subset of $\overrightarrow{\mathrm{H}} \times \mathcal{X}$ defined as $\{(s, x) \mid x = \bar{r}(s)\}$. Invoking again the Axiom of

[2] The *Axiom of Choice* is a key (yet debated) axiom of Set Theory. Its usual statement runs like this: *Given a non-empty family $\tilde{A} = \{A_i\}_{i \in I}$ of non-empty sets, there exists a 'choice function' for $\tilde{A}$, i.e. a map $f : I \to \bigcup_{i \in I} A_i$ such that $f(i) \in A_i$ for all $i \in I$* . In the present case, we take H as I and each set $\digamma_\theta = \{(\theta, x) \mid x \in \arg\max_{x \in \mathcal{X}} \mathbb{E}_{\mathbb{P}}[u\left(Y\left(x;\omega\right)\right) \mid \Theta(\omega) = \theta] \ \}$ as A_i .

[3] *Uniformization* is a major topic in Descriptive Set Theory (DST), a branch of mathematical logic which studies sets that are 'well-behaved' (so they avoid, in particular, certain weird consequences of the Axiom of Choice). Such sets are usually complete separable metric spaces (like the set $\mathbb{R}$ of real numbers) called *Polish spaces*. Good introductions to DST can be found in Moschovakis (2009), Sion (1960), and Zapletal (2005).

Choice, there exists a uniformization of this subset, i.e. a partial function $r^{-1} : \mathcal{X} \rightarrow \overrightarrow{\mathrm{H}}$ with domain the range of $\bar{r}$ and image a subset $\overrightarrow{S}$ of $\overrightarrow{\mathrm{H}}$. Its right inverse $r : \overrightarrow{S} \rightarrow \mathcal{X}$ - a function such that $r^{-1} \circ r = id$ - is injective by construction, and $\overrightarrow{S}$ is a chain. This makes r a routine in the sense of Definition 1.

Since $\bar{r}(s) = r(s)$, we have that

$$\max_{s \in \overrightarrow{\mathrm{H}}} \mathbb{E}_{\mathbb{P}}[u\left(Y(\bar{r}\left(s\right); \omega)\right) \mid \Theta(\omega)] = \max_{s \in \overrightarrow{S}} \mathbb{E}_{\mathbb{P}}[u\left(Y(r\left(s\right); \omega)\right) \mid \Theta(\omega)] \ .$$

Taking stock of this relationship, the above inequality becomes

$$\mathbb{E}_{\mathbb{P}}\left[u\left(Y\left(x; \omega\right)\right)\right] \leq \mathbb{E}_{\mathbb{P}}[\max_{s \in \overrightarrow{S}} \mathbb{E}_{\mathbb{P}}\left[u\left(Y(r\left(s\right); \omega)\right) \mid \Theta(\omega)\right]] \ .$$

The latter holds for any $x \in \mathcal{X}$, so

$$\max_{x \in \mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[u\left(Y\left(x; \omega\right)\right)\right] \leq \mathbb{E}_{\mathbb{P}}[\max_{s \in \overrightarrow{S}} \mathbb{E}_{\mathbb{P}}\left[u(Y(r\left(s\right); \omega)) \mid \Theta(\omega)\right]] \ ,$$

which confirms the theorem's assertion. ■

Under proper conditions, a rational decision-maker will then commit to focus her choices on the function $r(\cdot)$'s domain $\overrightarrow{S}$, provided she expects to thereby get a valuable signal. This turns her initial decision problem into facing a number of contingent problems of the form

$$\max_{s \in \overrightarrow{S}} \mathbb{E}_{\mathbb{P}}[u(Y(r\left(s\right); \omega)) \mid \Theta(\omega)] \ . \qquad \text{(MDP)}$$

Since, by assumption, all possible signals $\theta = \Theta(\omega)$ lay on the graded scale $\overrightarrow{\mathrm{H}}$, and what is now the relevant set of decision variables $\overrightarrow{S}$ is aligned on this scale (as the proof shows), problem (MDP) is indeed a *Monotone Decision Problem*.

Behaviorally speaking, solving (MDP) amounts to selecting the appropriate level of a control device which automatically adjusts a collection of primary inputs. The Theorem shows that this somewhat mechanical modus operandi should actually be *preferred* by the decision-making entity, as it enables her to thereby learn something about the state of nature. This agrees with a provocative assertion made some time ago by Alfred North Whitehead (1911):

> (...) it is a profoundly erroneous truism, repeated by all copy-books and by eminent people making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of operations which we can perform without thinking about them.

4. Concluding remarks

When a decision's payoff is subject to uncertainty, we showed that a decision-making entity (an individual or an organization) could set the matter so as to be facing a monotone decision problem. Sufficient conditions for this to occur are that the entity be ex ante capable of (i) holding prior expectations on the additional information she could possibly get, (ii) encoding and ranking the observed extra signals, and (iii) using any received signal optimally. In exchange for the possibility of getting an informative signal, the entity will then commit to use a restricted choice set made of graded marks pointing at specific values of the decision variables.

This result constitutes a first explanation for the repeated occurrence of monotone decision problems in economic life. It suggests that these problems should necessarily (not exclusively, of course) prevail in stable recurrent circumstances, when a priori knowledge and skills are significant. They arise from the desire to then take advantage of this cognitive state (design attributes), by implementing reliable routines that economize on deliberation in order to enhance external awareness.

The construction method invoked in the Theorem's proof was deliberately abstract and general, in order to recoup as many situations as possible. It also was rather unspecific about the implemented routine, apart from the requirement that it should point at contingent value-maximizers. One upshot is that, if several such maximizers exist, then two otherwise identical decision-making entities could adopt different routines, even if they hold similar preferences and beliefs. Additionally, if the first inequality in the above proof is strict, then there would even be room for suboptimal routines. This might support an amount of conservatism towards maintaining (slightly) obsolete production habits. It might also account for the presence of X-inefficiency (Leibenstein 1966) and low-hanging fruits (Gabel and Sinclair-Desgagné 1998) in organizations.

References

Angner, Erik and George Lowenstein (2007), "Behavioral economics,". in *Philosophy of Economics*, vol. 13 (D. Gabbay, P. Thagard, and J. Woods, editors), Elsevier.

Athey, Susan and Jonathan Levin (2018), "The value of information in monotone decision problems," *Research in Economics* 72, p. 101-116.

Camerer, Colin, George Loewenstein, and Drazen Prelec (2005), "Neuroeconomics: How neuroscience can inform economics," *Journal of Economic Literature* 43, p. 9-64.

Conlisk, John (1996), "Why Bounded Rationality?," *Journal of Economic Literature*, 34, p. 669-700.

Cyert, Richard M. and James G. March (1963), *A Behavioral Theory of the Firm* (1st ed.), Prentice-Hall.

Dessein, Wouter, Andrea Galeotti, and Tano Santos (2016), "Rational inattention and organizational focus," *American Economic Review* 106(6), p. 1522-1536.

Dixit, Avinash and Robert Pindyck (1994), *Investment under Uncertainty*, Princeton University Press.

Gabel, H. Landis and Bernard Sinclair-Desgagné (1998), "The firm, its routines, and the environment," in *The International Yearbook of Environmental and Resource Economics 1998/1999: A Survey of Current Issues* (H. Folmer and T. Tietenberg, editors), Edward Elgar.

Gold, Andrew H., Arvind Malhotra, and Albert H. Segars (2001), "Knowledge management: an organizational capabilities perspective," *Journal of Management Information Systems* 18(1), p. 185-214.

Kahneman, Daniel (2003), "Maps of bounded rationality: psychology for behavioral economics," *American Economic Review* 93(5), p. 1449–75.

Milgrom, Paul R. and John D. Roberts (1992), *Economics, Organization and Management*, Prentice-Hall.

Leibenstein, Harvey (1966), "Allocative efficiency vs. X-efficiency," *American Economic Review* 56 (3), p. 392–415.

Nelson, Richard and Sidney Winter (1982), *An Evolutionary Theory of Economic Change*, Belknap Press.

Moschovakis, Yannis N. (2009), *Descriptive Set Theory*, Lecture notes available online.

Rubinstein, Ariel (1998), *Modeling Bounded Rationality*, The MIT Press.

Simon, Simon, H. A. (1947), *Administrative Behavior: a Study of Decision-Making Processes in Administrative Organization* (1st ed.), Macmillan.

Sion, Maurice (1960), "On the uniformization of sets in topolocical spaces," *Transactions of the American Mathematical Society* 96(2), p. 237-245.

Trigeorgis, Lenos (1996), *Real options: Managerial Flexibility and Strategy in Resource Allocation*, The MIT Press.

Whitehead, Alfred North (1911), *An Introduction to Mathematics*, Home University Library.

Zapletal, Jindřich (2005), *Modern Descriptive Set Theory*, Lecture notes available online.

Cahiers du GREThA
Working papers of GREThA

GREThA UMR CNRS 5113

Université de Bordeaux

Avenue Léon Duguit
33608 PESSAC - FRANCE
Tel : +33 (0)5.56.84.25.75
Fax : +33 (0)5.56.84.86.47

http://gretha.u-bordeaux.fr/

Cahiers du GREThA (derniers numéros – last issues)

2018-03: DOYEN Luc, BENE Christophe : A generic metric of resilience from resistance to transformation

2018-04: DOYEN Luc, GAJARDOA Pedro : Viability standards and multi-criteria maximin

2018-05: EMLINGER Charlotte, LAMANI Viola: International Trade, Quality Sorting and Trade Costs: The Case of Cognac

2018-06: BONIN Hubert: Armateurs et négociants et la compétitivité de la cité-port de Bordeaux : les compagnies d'assurances maritimes dans les années 1830-1870

2018-07: V. TARIZZO, E. TROMEUR, O. THEBAUD, R. LITTLE, S. JENNINGS, L. DOYEN: Risk averse policies foster bio-economic sustainability in mixed fisheries

2018-08: ORSATTI Gianluca, STERZI Valerio: Do Patent Assertion Entities Harm Innovation? Evidence from Patent Transfers in Europe

2018-09: ROUGIER Eric, YOL Nicolas: The volatility effect of diaspora's location: A migration portfolio approach

2018-10: NAVARRO Noemí, H. Dan TRAN: Shock Diffusion in Large Regular Networks: The Role of Transitive Cycles

2018-11: CHIBA Fadoua, ROUILLON Sébastien: Intermittent electric generation technologies and smart meters: substitutes or complements

2018-12 : CHHORN Dina: Effect of Microfinance on Poverty and Welfare: New Evidence from 9 provinces in Cambodia

2018-13: BONIN Hubert: Les banquiers et les leçons de l'Histoire : lucidité ou aveuglement ?

2018-14: BALLET Jérôme : Anthropology and Economics: The Argument for a Microeconomic Anthropology

---

La coordination scientifique des Cahiers du GREThA est assurée par Valerio STERZI.
La mise en page et la diffusion sont assurées par Julie VISSAGUET