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A physico-economic model of space debris management

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Abstract

We solve a stylized physico-economic model of orbital environment and space activity, in order to analyse the externality caused by the accumulation of space debris. In line with Gordon (1954) and Schaefer (1957), we focus on the long-term equilibrium of the orbit, induced by a constant rate of satellite launches forever. We show that if, in the long run, the risk of satellite destruction by collision is increasing and convex with the launch rate and becomes arbitrarily large for sufficiently large values of the latter, then the curve representing the long-term expected population of functioning satellites, as a function of the launch rate, has an inverted-U shape. Classically, we then define and compare typical ways of managing the orbital environment (maximum carrying capacity, open-access, social optimum). The maximum carrying capacity is defined as the maximum expected population of satellites that the space sector can sustain in the long run. The physico-economic equilibrium launch rate, that would presumably emerge under conditions of open-access to the orbit, is defined as the launch rate such that the space sector makes no profit. The socially optimal launch rate is the one that maximizes the present value profit of the space sector per launch campaign. Finally, we discus the use standard economic instruments (command-and-control, tax and market) to regulate space activity in order to achieve an optimal outcome. A numerical application based on a realistic calibration is also proposed to illustrate all results.

Keywords: Space economics - Orbital debris - Sustainability

Un modèle physico-économique de gestion des débris spatiaux

Résumé

Nous résolvons un modèle physico-économique simplifié de l'environnement orbital et de l'activité spatiale, afin d'analyser l'externalité causée par l'accumulation de débris spatiaux. Dans la lignée de Gordon (1954) et Schaefer (1957), nous nous concentrons sur l'équilibre à long terme, induit par un taux de lancements de satellites constant pour toujours. Nous montrons que si, à long terme, le risque de destruction par collision d'un satellite est croissant et convexe avec le taux de lancements et devient arbitrairement grand pour des valeurs suffisamment importantes de ce dernier, alors la courbe représentant la population espérée de satellites en fonctionnement à long terme, en fonction du taux de lancements, a une forme en U inversé. Classiquement, nous définissons et comparons ensuite des méthodes typiques de gestion de l'environnement orbital (capacité de charge maximale, libre accès, optimum social). La capacité de charge maximale est définie comme la population maximale espérée de satellites en activité que le secteur spatial peut maintenir à long terme. Le taux de lancements à l'équilibre physico-économique, qui émergerait vraisemblablement dans des conditions de libre accès à l'orbite, est défini comme le taux de lancements tel que le secteur spatial ne réalise aucun profit. Le taux de lancements socialement optimal est celui qui maximise la valeur actualisée du profit du secteur spatial par génération de satellites. Enfin, nous discutons de l'utilisation d'instruments économiques standards (régulation, fiscalité et marché) pour encadrer les activités spatiales afin d'atteindre un résultat optimal. Une application numérique, basée sur un étalonnage réaliste, est également proposée pour illustrer nos résultats.

Mots-clés: Economie de l'espace - Débris spatiaux - Soutenabilité.

JEL: L1, L9, Q2

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1 Introduction

Since the launch of Sputnik in 1957, the number of objects in orbit around the Earth, either satellites or space debris, has increased considerably (ESA, 2019). Unfortunately, the density of space debris in the orbital environment has now reached levels that are beginning to damage the space sector (Weinzierl, 2018) and could even compromise its activity in the future (Kessler and Cour-Palais, 1978; Dolado-Perez et al., 2015). Space debris remains in orbit for long periods of time (decades or centuries, depending on altitudes) and poses a threat for existing satellites (risk of damages by way of collision). Moreover, the process propagates itself, with past collisions contributing to the increase of the stock of space debris.

The issue of space debris was soon recognized as critical by the international community (UN, 1999). In 2007, the United Nations General Assembly has endorsed the Space Debris Mitigation Guidelines (UN, 2010), recommending to limit the debris released by launch vehicles and spacecrafts, to avoid intentional destructions, and to minimize the risk of break-ups, explosions and collisions during and after operational lifetime. These measures are not legally binding under international law and should thus be implemented voluntarily. The degree of compliance in recent years is documented by the European Space Agency's Annual Space Environment Report (ESA, 2019). Importantly, recent simulations (Drmola and Hubik, 2018; Somma et al., 2019) show that, even if these guidelines are followed, space debris will continue to accumulate in the future if the launch rate is not drastically reduced. Accordingly, we will pay much attention below to the limitation of the rate of satellite launches.

In this paper, we construct a stylized physico-economic model of orbital environment and space activity. Following a tradition which has proven to be productive in economics of fisheries (Gordon, 1954; Schaefer, 1957), we consider the long-term equilibrium of the orbit, induced by a rate of satellite launches assumed constant forever. The *expected* lifetime of a satellite may be affected by technical failures and/or collisions with space objects. In the long run, the risk of collisions depends on past rates of launches, as space activity generates additional debris, either directly (i.e., rocket bodies, released objects) or indirectly (i.e., induced collisions), and collisions become more frequent as the density of orbital debris increases. Therefore, our model postulates that the long-term probability of a satellite being destroyed by collision can be formalized as an increasing and convex function of the launch rate.

Within such an analytical framework, the *expected* population of satellites in the long term equilibrium of the orbital environment depends on the launch policy planned today. We give conditions such that the curve representing the relationship between the long term *expected* population of satellites and the constant launch rate has a inverted-U shape. This occurs when the long-term risk of destruction of a satellite by collision is an increasing and convex function of the launch rate, and becomes arbitrary large when the launch rate is sufficiently large. It is important to note that these conditions are quite likely according to existing physical models of the orbital environment (Kessler and Court-Palais, 1978; Somma et al., 2019).

Clearly, this result should recall a well-known finding in the economics of fisheries (Gordon, 1954; Schaefer, 1957), which shows that the catch rate in the long-term equilibrium of the ecosystem is an inverted-U shape function of the fishing effort. This leads us to look for several properties in line with this literature. We calculate the launch rate that maximizes the *expected* population of satellites in the long-term equilibrium, subsequently referred to as the maximum carrying capacity of the orbit. We also derive the physico-economic equilibrium rate of launching, that would presumably emerge under conditions of open-access to the orbit. Typically, it is defined as the launch rate such that the space sector makes no profit and, therefore, has no incentive to change its plan. Finally, we define the socially optimal launch rate, that maximizes the *expected* present-value surplus of the space sector per launch campaign.

We compare these three benchmark situations. From the physical point of view, an overexploitation of the orbit occurs if the rate of launches in the physico-economic equilibrium is larger than the rate of launches maximizing the *expected* population of operational satellites in the long term. We prove that this will happen if the rent from a functionning satellite is high enough, if the overall cost of a launched satellite is small enough and/or if economic agents are not too impatient. It is important to note that both cases *can* happen, with the physico-economic equilibrium rate of launches being either smaller or larger than the rate of launches maximizing the *expected* number of operational satellites. From an economic point of view, an overexploitation of the orbit is characterized when the rate of launches in the physico-economic equilibrium is larger than the socially optimal rate of launches. We show that the physico-economic equilibrium always leads to an overexploitation of the orbit from an economic point of view.

Finally, we show how standard economic instruments can be used to implement a socially optimal outcome, considering in turn an aggregate launch quota, a tax per launch and a market of individual transferable launch quotas. We argue that the first instrument is likely to induce perverse effects if some critical characteristics of satellites cannot be monitored. By contrast, we show that the latter two instruments can implement an optimal rate of launching, if designed correctly. The corresponding (pigouvian) tax level or equilibrium price of an individual launch quota will then induce the space sector to internalize the external cost of a marginal launch.

The economic analysis dealing with orbital congestion and satellites destruction by collisions is recent and quite sparce. To the best of my knowledge, this issue is only considered by Adilov et al. (2014), Adilov et al. (2018), Klima et al. (2016), Macauley (2015) and Muller et al. (2017).[1]

[1] Sandler and Schulze (1981), and Weinzierl (2018) mention this issue. However, they do not analyse it explicitly.

Adilov et al. (2014) propose a model of horizontal differentiation (Salop, 1979), where firms locate satellites around the orbit and consumers have preferences for satellites close to their location. The game has two periods, with launches taking place in period one and damages to satellites occurring in period two. This setting allows to analyse private incentives to launch satellites and to mitigate space debris. The model predicts that, relative to the social optimum, profit-maximizing firms launch too many satellites and choose technologies which create too much debris.

Adilov et al. (2018) construct a simplified dynamic model of the orbital environment, with functionning satellites hit at a rate proportional to the quantity of orbital debris. Firms are assumed to launch satellites as long as the *expected* profit from a marginal launch is positive. Assuming that they form adaptive expectations regarding the future rent and the risk of destruction by collision of a satellite, Adilov et al. (2018) demonstrate several properties of the equilibrium behavior of the space sector. In particular, they show that the equilibrium launch rate first increases with the stock of orbital debris, for low levels of congestion (because firms replace lost satellites), before it decreases above a certain threshold (because the risk of destruction becomes too large). Finally, the model predicts that orbital debris renders orbits economically unprofitable before the occurrence of the Kessler Syndrome.[2]

Klima et al. (2016) use a extensive simulation model of the orbital environment. They consider space debris removal missions by the US, Europe and China. They assume that each country can commit initally to remove zero, one or two debris per year. They simulate the effect of each scenario on the evolution of space debris and the resulting risk of collisions between 2016 and 2165. Finally, they use these projections to calibrate an empirical game in normal form. Their main result is that there exists pure strategy Nash equilibria, where only one player commit to remove one debris per year, and mixed strategy Nash equilibria, where the players randomize between removing zero or one debris per year.

Macauley (2015) develops a simplified model of debris accumulation and formalizes the externality generated by the risk of collision as a proportionate reduction in a spacecraft's lifetime productivity. The main finding is the calculus of the external cost from adding one satellite to the initial fleet. A numerical application, based on realistic calibration of parameters values, shows that it could range from 0.06% up to 2.13% of the production cost of one satellite.

Muller et al. (2017) analyse the choice of a monopolistic firm regarding the number of satellites to be launched initially, knowing that they may be destroyed by collision in the future. The probability of destruction is assumed given and constant over time. In particular, it is invariant with the number of satellites launched initially and damaged subsequently. The company provides and sells some service, as long as *at least one* satellite remains in order. Under

[2]The Kessler syndrome is a scenario such that the density of objects in low Earth orbit becomes so high to render the use of satellites infeasible (Kessler and Court-Palais, 1978).

these assumptions, Muller et al. (2017) derive the private loss in *expected* profit caused by the destruction of satellites and show that it is slightly larger than the cost of reconstructing them.

The present paper extends the previous literature in several directions. Firstly, our model allows a definition of the maximum carrying capacity of the orbit. This concept is new in the economic literature. Secondly, we complement the analysis of the effects of private incentives to use the orbit for profit making under open-access. This issue is also treated in Adilov et al. (2014, 2018) and Muller et al. (2017). However, their investigation is made assuming imperfect competition (horizontal differentiation or monopoly) and/or is limited to two time periods models. Here, we consider perfect competition and the long-term equilibrium of the orbital environment. With the rapid development of a private space economy, known as "New Space" (Weinzierl, 2018), the assumption of perfect competition is becoming increasingly appealing. In particular, as documented by Dos Santos and Le Hir (2016), the satellite market has been marked recently by the arrival of numerous new entrants, who have overcome traditional barriers to entry thanks to the miniaturization of satellites.[3] Besides, our focus on the long-term equilibrium is valuable because it allows to better represent the issue of orbital sustainability. Thirdly, our model makes it possible to formalize an optimal and sustainable management of the orbital envrironment. Such investigation can also be found in Adilov et al. (2014) and Muller et al. (2017). Again, their definition is limited to models with two time periods and thus does not formalize the long-term sustainability properly. Here, we characterize the optimal policy in the long-term stationary state. Finally, we give policy recommandations to mitigate orbital debris, dealing with command-and control, a tax per launch and a market of individually transferable launch quotas. This issue is only considered and partially covered by Adilov et al. (2014), where they derive the tax schedule that implements an optimal outcome as a market equilibrium. Moreover, their formula is not valid in our setting, since it is meant to correct two market failures at the same time, namely, the imperfect competition induced by product differentiation and the externality caused by space debris.

This paper is organized as follows. Section 2 sets out the physical model and states the notion of maximum carrying capacity of the orbit. In Section 3, we construct our economic model to derive the physico-economic equilibrium and socially optimal outcome. Section 4 shows how standard economic instruments (command-and-control, tax, market) can be used to implement an optimal outcome. Section 5 is dedicated to a numerical application based on a simplified specification and a calibration using real data. In Section 6, we conclude and present possible extensions for future researches. Supplementary materials are supplies in a technical appendix.

[3]Dos Santos and Le Hir (2016) analyse small satellites as a disruptive innovation (Christensen, 1997). Relying on more modular and standardized technologies, they are simpler and faster to design and construct. As a result, they display lower performance, but can be offered at a lower price, thus attracting new costumers.

2 The physical model

Consider a given altitude of the low Earth orbit (e.g., between 800 km and 1000 km).[4] Assume that the spatial sector continuously launches q operational satellites per period.[5] The operational lifetime of a satellite orbiting the Earth depends on technical (i.e., fuel exhaustion, components failures) and environmental (i.e., collisions) events. Below, it is formalized as a positive random variable T, distributed according to an exponential probability distribution function $1 - e^{-(\lambda+\mu)T}$, with rate parameter $\lambda + \mu$. The rate of defect by technical failure is denoted by λ. It is considered as exogenously given in this paper. The rate of destruction by collision is denoted by μ. It is endogenously determined, as explained just below.

In our stylized model, following a tradition which has proven to be effective in the economics of fisheries (Gordon, 1954; Schaeffer, 1957), we focus on the equilibrium of the orbital environment, resulting in the long run from a rate of launching q assumed constant forever. In this way, suppose that there exists a relationship $x = \phi(q)$, giving the long-term density of space debris, once the orbit has reached the corresponding stationary state. Assume further that there exists a relationship $\mu = \varphi(x)$, giving the instantaneous risk of destruction of a satellite by collision, as a function of the density of space debris. Overall, this formalization drives us to conclude that the risk of collision in the long run can be written as $\mu = f(q)$, where we let $f(q) \equiv \varphi(\phi(q))$ for the sake of notational convenience.

Many factors are likely to determine the shape of this relationship. Some are under the control of the space sector (e.g., altitudes, technological innovation, mitigation and remediation measures). Others are out of the control of the space sector (e.g., atmospheric density, solar activity). As this is out of the scope of this paper, these factors will be left implicit in the main text, in order to focus attention to the main economic insights. Still, we describe a stylized dynamic model of the orbital environment and debris accumulation in appendix A1, showing how to rationalize our approach and how some of the factors neglected here would intervene.

Below, we will simply postulate the following assumption:

[4] The Earth orbit is divided into three regions, differing by their physical properties and their uses by the space industry. Low Earth orbit lies at an altitude between 200 and 2000 km, medium Earth orbit, between 2000 and 35786 km, and geosynchronous Earth orbit, at exactly 35786 km. Our model considers a given region of low Earth orbit. To simplify, the orbital environment surrounded it is assumed in a stationary state, in order to ensure that the quantity of space objects coming from outside is constant through time.

[5] Satellites differ in many respects (mass, surface, power, etc.). In our model, we consider a standardized prototype, supposedly designed to minimize costs, given the best available technologies and regulations in force. We also assume constant returns to scale, which implies that this template with average characteristics can be produced at different scales. Then, the launch rate can be viewed as a continuous variable, measured along this normalized scale. This allows for the diversity of satellites to be taken into account, while keeping the model tractable.

Assumption 1. $f(0) = 0$, $f'(q) > 0$ and $f''(q) > 0$ for all $q > 0$, and $\lim_{q \to Q} f(q) = \infty$ for some $Q > 0$.[6]

Assumption 1 is illustrated by Figure 1. The intuitions supporting it are the following. Due to natural decay (i.e., atmospheric drag), all debris and satellites eventually fall back to the Earth. Therefore, if there is no launching for a sufficiently long period of time, the orbital environment will ultimately be cleaned up. In such pristine state, the first satellite launched afterwards bears no risk of collision. Then, as the rate of launches increases, the long-term risk of collisions is increasing at an increasing rate. This is quite intuitive and is supported by existing prospective simulations (Somma et al., 2019). Finally, the existence of a threshold level of launching such that the long-term rate of collision becomes infinite, refers to the Kessler syndrome (Kessler and Cour-Palais, 1978). The latter is a scenario such that the density of objects in low Earth orbit and the subsequent risk of collision becomes so high as to render the use of satellites infeasible.

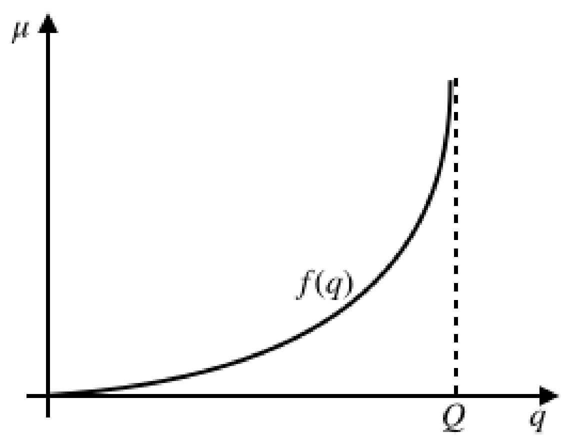

Figure 1: Long-term risk of collision

Under our assumptions, the *expected* number of operational satellites, at any time t_0, converges in the long run to:

$$n(q) \equiv \int_{-\infty}^{t_0} q e^{-(\lambda+\mu)(t_0-t)} \mathrm{d}t, \text{ with } \mu = f(q).$$

Indeed, by assumption, the rate of launches is set equal to q forever. The operational lifetime of a satellite in orbit depends on the rate of defect for

[6]We assume in this paper that Q is finite. Note however that all results remain true if Q is infinite, provided that the last condition is replaced by $\lim_{q \to Q} f(q)/q = \infty$. In a supplementary material, we provide a numerical illustration dealing with this case, where $f(q) = aq^2$, with $a > 0$.

technical reason λ and by collision μ. The probability that a given satellite, launched at time $t < t_0$, remains operational till time t_0, is equal to $e^{-(\lambda+\mu)(t_0-t)}$. The *expected* population of operational satellites at time t_0 is the sum of all generations t of satellites launched previously, q per period, weighted by their (independent) probability $e^{-(\lambda+\mu)(t_0-t)}$ of being still operational at time t_0. It is immediate to integrate the expression above to calculate that:

$$n(q) = \frac{q}{\lambda + f(q)}. \tag{1}$$

Note that this expression can also be seen as the product of the rate of satellite launches, q, times the long-term *expected* lifetime of a satellite, $d(q) \equiv 1/(\lambda + f(q))$.

We can show the following:

Property 1. Under Assumption 1, $n(0) = \lim_{q \to Q} n(q) = 0$ and there exists $\overline{q}$ such that $0 < \overline{q} < Q$ and $n'(q) \gtreqless 0$ iff $q \lesseqgtr \overline{q}$.

Proof of property 1. The first part is immediate. The derivative of $n(q)$ is $n'(q) = (\lambda + f(q) - qf'(q)) / (\lambda + f(q))^2$. The numerator is decreasing (as its derivative is $-qf''(q) < 0$). Note that $n'(0) = 1/\lambda > 0$. Assume that $n'(q) \geq 0$ for all q. Then $n(0) < n(q)$ for all q, which contradicts the fact that $n(0) = \lim_{q \to Q} n(q) = 0$. Therefore, there exists $\overline{q}$ such that $0 < \overline{q} < Q$ and $n'(q) \gtreqless 0$ iff $q \lesseqgtr \overline{q}$. $\square$

Under Assumption 1, the curve representing the *expected* population of operational satellites in the long term typically displays the unimodal shape shown in Figure 1. It is equal to 0 for $q = 0$ and $q = Q$. It is first increasing and then decreasing in q, thus admitting a maximum for some $\overline{q}$ between 0 and Q. The reason why we get this inverted-U shape is intuitively because the *expected* long-term satellite population is the product of the satellites launch rate, q, times their *expected* lifetime, $d(q)$. Indeed, in the long run, an increase of the launch rate has an ambiguous effect on the satellite fleet, as it simultaneously reduces their operational lifetime.

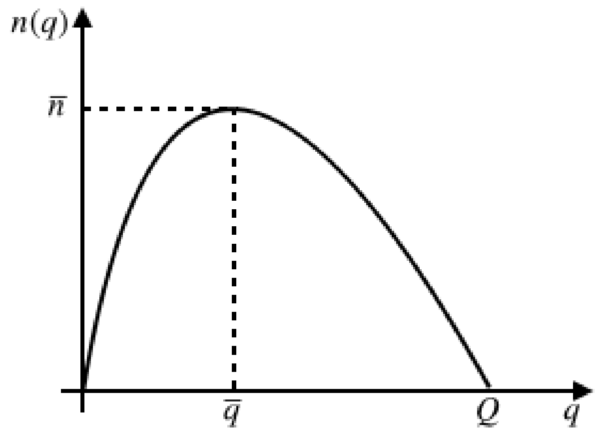

Figure 2: Long-term *expected* number of satellites

Now, let us consider the following definition:

Definition 1. The maximum carrying capacity $\overline{n}$ is the maximum *expected* number of satellites that the space sector can sustain in the long term. Formally, $\overline{n} = \max_q n(q)$. The corresponding *expected* operational lifetime of a satellite is $\overline{d} \equiv d(\overline{q})$.

Using Property 1, the carrying capacity $\overline{n}$ is equal to $n(\overline{q})$. It can be sustained with a launch rate set equal to $\overline{q}$ forever.

3 The economic model

In this section, we propose an economic model in order to investigate two main issues. Firstly, it is important to understand the private incentives of profit-maximizing firms to launch new satellites, as this potentially governs the future evolution of the orbital environment. Secondly, it is useful to propose a normative criterion for characterizing an optimal launching policy from the social point of view.

The economic parameters at stake are the following. The unit cost of designing, building, testing, and launching a standard satellite is denoted by c. A satellite generates a cash-flow per operating period denoted by p, equal to the rents paid by consumers for the services provided, less operating costs.[7] The economic agents are assumed to discount time at the rate of time preference δ.

[7]The rent of an active satellite is assumed a parameter in our model. Using an argument similar to that of the fisheries economy (Gordon, 1954; Schaefer, 1957), this assumption can be justified if the region of the low Earth orbit considered in the model is small enough in comparison to the total population of satellites. An alternative assumption would be that the rent of a satellite decreases with the total population of active satellites in orbit around the Earth.

In order to investigate private incentives to launch new satellites, let us first examine the situation where the orbit is a common-property resource under open-access. By definition, this means that economic agents have the right to launch any number of satellites. Presumably, we assume that they will do so as long as they can expect to achieve a positive benefit per launch.

An economic agent launching an operational payload at any time t_0, bears an immediate cost c and anticipates to receive a flow of revenue p, during the whole operational lifetime of its satellite. Assuming that it discounts time at the rate δ, the *expected* present value $v(q)$ of the revenue generated is given by:

$$v(q) \equiv \int_{t_0}^{\infty} \left(\int_{t_0}^{t} pe^{-\delta(\tau - t_0)} \mathrm{d}\tau \right) (\lambda + \mu) e^{-(\lambda+\mu)(t-t_0)} \mathrm{d}t, \text{ with } \mu = f(q).$$

Indeed, if the satellite remains operational until time $t > t_0$, it will generate a flow of revenue equivalent to $\int_{t_0}^{t} pe^{-\delta(\tau - t_0)} \mathrm{d}\tau$ in present value.[8] The probability that it fails at time t exactly is $(\lambda + \mu) e^{-(\lambda+\mu)(t-t_0)}$. By definition, the *expected* present value of the flow of revenue is the sum, for all possible failure time t, of the product of the last two terms. It is immediate to integrate the expression above to calculate:

$$v(q) = \frac{p}{\delta + \lambda + f(q)}. \tag{2}$$

We can derive the following:

Property 2. Under Assumption 1, $v(0) = p/(\delta + \lambda)$, $\lim_{q \to Q} v(q) = 0$ and $v'(q) < 0$ for all q.

Proof of property 2. The first part is immediate. The derivative of $v(q)$ is $-f'(q)\, p / (\delta + \lambda + f(q))^2 < 0$. □

Property 2 is illustrated by Figure 3, showing the typical shape of the *expected* discounted revenue of a satellite. The intuition behind it is straightforward. In the long run, by Assumption 1, the addition of satellites in the orbit eventually induces a larger density of debris, thus increasing the overall risk of collisions. This reduces the *expected* operational lifetime of any satellite in orbit and, therefore, its *expected* present value. The latter varies from $p/(\delta + \lambda)$ when $q = 0$ (i.e., in the long run, the orbit is clean and there is no risk of collision) to 0 when $q \geq Q$ (i.e., in the long term, the orbit reaches the Kessler syndrome and satellites are immediately destroyed).

[8] Upon integration, we calculate that $\int_{t_0}^{t} pe^{-\delta(\tau - t_0)} \mathrm{d}\tau = p \left(1 - e^{-\delta(t-t_0)}\right) / \delta$.

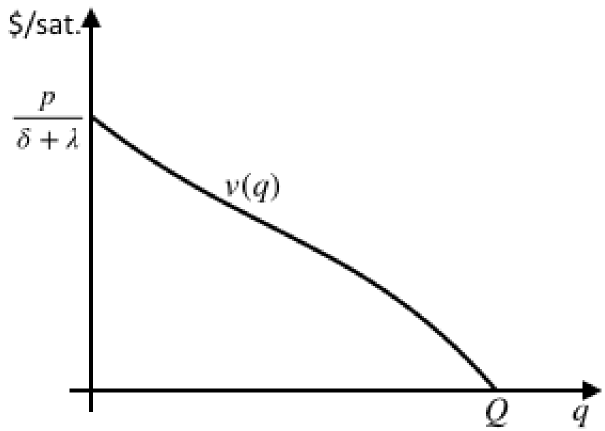

Figure 3: *Expected* discounted revenue of a satellite

Now, recall that if the orbit is under open-access, our assumption is that private agents will launch additional satellites as long as they can derive a positive benefit from them. Formally, this condition is written $v(q) > c$. Conversely, they will have an incentive to cease launching additional satellites if they anticipate a negative benefit from them. Formally, this condition is written $v(q) < c$. Accordingly, the space sector will be satisfied with its strategy if and only if the aggregate launch rate satisfies $v(q) = c$.

We summarize in the following definition our postulate regarding the behavior of private agents under open-access to the orbit:

Definition 2. A physico-economic equilibrium is reached when the *expected* present value of a satellite equals the cost of designing, building, testing, and launching it. Formally, $q = q^*$ where $v(q^*) = c$. The corresponding *expected* number of operational satellites is $n^* \equiv n(q^*)$. The corresponding *expected* operational lifetime of a satellite is $d^* \equiv d(q^*)$.

We call this situation a physico-economic equilibrium because it characterizes a situation where both the state of the orbit is stationary, by definition of $f(q)$, and no economic agent has an incentive to deviate from its plan.

We can show the following proposition:

Proposition 1. If $p/(\delta+\lambda) > c$, there exists a unique physico-economic equilibrium. It is characterized by a launch rate $q = q^*$ such that $0 < q^* < Q$ and $v(q^*) = p/(\delta+\lambda+f(q^*)) = c$. (If $p/(\delta+\lambda) \leq c$, no physico-economic equilibrium exists, in the sense that launching a satellite is never profitable, even when the orbit is clean of space debris.)

Proof of proposition 1. From Property 2, $v(q)$ is strictly decreasing and takes values between $v(0) = p/(\delta+\lambda)$ and $\lim_{q\to Q} v(q) = 0$. If $p/(\delta+\lambda) > c$, then $v(0) = p/(\delta+\lambda) > c > 0 = \lim_{q\to Q} v(q)$. Thus, there exists a unique q^*

such that $0 < q^* < Q$ and $v(q^*) = c$. Clearly, if $p/(\delta+\lambda) \leq c$, then $v(q) < c$ for all $q > 0$. □

Proposition 1 is illustrated by Figure 4, assuming that $p/(\delta+\lambda) > c$. The physico-economic equilibrium is at the intersection between the curve representing the *expected* revenue of a satellite, $v(q)$, and the horizontal line representing the overall cost of a satellite, c. Clearly, given that $v(q)$ is decreasing, only one these situations can exist. Moreover, the physico-economic equilibrium rate of launches q^* lies between 0 and Q.

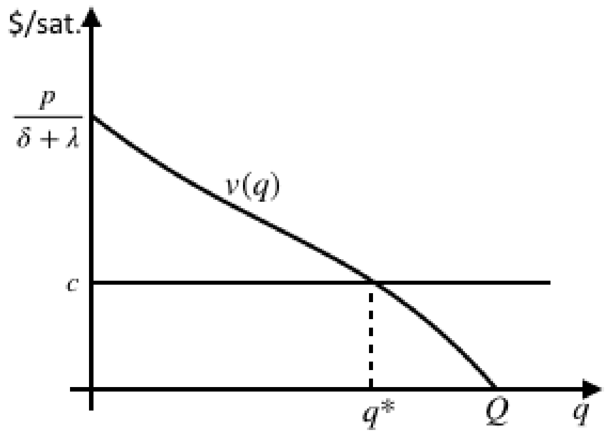

Figure 4: Physico-economic equilibrium

As $f(q)$ is an increasing function, it should be clear that the physico-economic equilibrium rate of launching q^* is increasing in p and decreasing in c, δ and λ. It should also be noted that $q^* < Q$ for all configurations of the parameters. This implies that rational agents have no incentive to initiate a Kessler syndrome, even under conditions of open-access. This confirms in our framework a result previously emphasized by Adilov et al. (2018).

The following property follows quite directly from the definition of a physico-economic equilibrium:

Property 3. In a physico-economic equilibrium, $d^* = 1/(p/c - \delta)$.

Proof of property 3. By Proposition 1, q^* satisfies $v(q^*) = p/(\delta + \lambda + f(q^*)) = c$. Elementary algebra directly yields $d^* = d(q^*) = 1/(\lambda + f(q^*)) = 1/(p/c - \delta)$. □

In other words, under conditions of open-access, the *expected* operational lifetime of a satellite, resulting from the equilibrium behavior of the space sector, only depends on economic parameters c, p and δ. For instance, since the cost of launch increases with altitude, this predicts that satellites placed at higher altitudes should have a longer operational lifetime, all other things being equal.

The UCS Satellite Database corroborates this correlation.[9] Indeed, as reported by satellite operators, the *expected* lifetime of active payloads is on average 6.63 years for the low Earth orbit (out of 511 satellites), 9.38 years for the medium Earth orbit (out of 108 satellites) and 13.47 years for the geosynchronous Earth orbit (out of 481 satellites).

An interesting question is how the physico-economic equilibrium compares with the maximum carrying capacity of the orbit. From the physical point of view, an over-exploitation of the orbit is characterized by a equilibrium launch rate q^* higher than the launch rate $\overline{q}$ maximizing the *expected* population of operational satellites.

Our results are explained in the following property:

Property 4. $q^* \gtreqless \overline{q}$ if and only if $p/c - \delta \gtreqless \overline{q} f'(\overline{q})$.

Proof of property 4. By definition, $\overline{q}$ satisfies $n'(\overline{q}) = 0$ and q^* satisfies $v(q^*) = c$. Here, these conditions are usefully rewritten as $\lambda + f(\overline{q}) = \overline{q} f'(\overline{q})$ and $\lambda + f(q^*) = p/c - \delta$ respectively. Then, clearly, $q^* = \overline{q}$ when $p/c - \delta = \overline{q} f'(\overline{q})$. Moreover, as $f(q)$ is an increasing function, q^* is increasing in $p/c - \delta$. □

In other words, an over-exploitation of the orbit will happen when the rent of a satellite is high, the overall cost of a launched satellite is small and when the economic agents are not too impatient. In such cases, according to Property 1, the long-term satellite population will be smaller than the maximum carrying capacity, because the space sector launches too many satellites, each with a too short lifetime, due to orbit congestion caused by space debris. It is worth noting that inverse cases exist where the equilibrium launch rate q^* is lower than the launch rate $\overline{q}$ that maximizes the long-term *expected* number of operational satellites.

Finally, we propose a normative criterion for characterizing an optimal launch policy from a social point of view. At any instant of time, the space sector launches q satellites. In the long-term equilibrium of the orbital environment, each satellite earns an *expected* present revenue $v(q)$ and costs c. Overall, each generation of q satellites therefore periodically generates a social surplus equal to $s(q) = (v(q) - c)\, q$. From an economic point of view, a social objective should be to maximize the aggregate surplus produced periodically by the space sector.

We summarize in the following definition our normative criterion characterizing an optimal orbital policy:

Definition 3. An optimal launching policy maximizes the social surplus generated periodically by the spatial industry. Formally, $q = q^o$ where $q^o = \arg\max_q s(q)$. The corresponding *expected* number of operational satellites is $n^o \equiv n(q^o)$. The corresponding *expected* operational lifetime of a satellite is $d^o \equiv d(q^o)$.

[9] https://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database

Clearly, a physico-economic equilibrium does not implement an optimal policy. Indeed, by definition, it satisfies $v(q^*) = c$ and thus generates no social surplus. We can show the following proposition:

Proposition 2. If $p/(\delta+\lambda) > c$, there exists a unique optimal policy, characterized by a launch rate $q = q^o$ such that $0 < q^o < Q$ and $s'(q^o) = v(q^o) + v'(q^o)q^o - c = 0$. (If $p/(\delta+\lambda) \leq c$, the optimal orbital policy is $q = 0$.)

Proof of proposition 2. By definition of $s(q)$ and $v(q)$, we have

$$s(q) = \left(\frac{p}{\delta+\lambda+f(q)} - c\right)q. \tag{3}$$

By differentiation, we obtain

$$s'(q) = p\frac{\delta+\lambda+f(q)-f'(q)q}{(\delta+\lambda+f(q))^2} - c$$

and

$$s''(q) = -p\frac{f''(q)q(\delta+\lambda+f(q)) + 2f'(q)(\delta+\lambda+f(q)-f'(q)q)}{(\delta+\lambda+f(q))^3}.$$

For an interior solution, an optimal launching policy must satisfy $s'(q) = 0$. If $p/(\delta+\lambda) > c$, as $s'(0) = p/(\delta+\lambda) - c > 0$ and $s'(q) < 0$ for q sufficiently close to Q,[10] this condition has a solution $q = q^o$ such that $0 < q^o < Q$. Observe that a solution necessarily belongs to the set where $\delta+\lambda+f(q)-f'(q)q \geq 0$ (as otherwise, $s'(q) < 0$). This implies that $s''(q) < 0$ in the relevant region. Then, as $s'(q)$ is strictly decreasing, $s'(q) = 0$ has unique solution determining a maximum of $s(q)$. Clearly, if $p/(\delta+\lambda) \leq c$, then $v(q) < c$ for all $q > 0$, implying that $q = q^o = 0$ is optimal. □

A question of interest is how the physico-economic equilibrium compares with the optimal policy. From an economic point of view, the orbit is over-exploited when the launch rate when the physico-economic equilibrium launch rate q^* is higher than the optimal launch rate q^o.

Our result is stated in the following property:

Property 5. The physico-economic equilibrium always induces an economic over-exploitation of the orbit. Formally, $q^* > q^o$ for all configurations of the parameters.

Proof of property 5. By definition, $v(q^*) = c$ and $v(q^o) + v'(q^o)q^o = c$. Given that $q^o > 0$ and $v'(q^o) < 0$, $v(q^*) = c < c - v'(q^o)q^o = v(q^o)$. Finally, as $v(q)$ is decreasing (see the proof of property 2), this implies that $q^* > q^o$. □

[10] To show this, first note that $f(q) - f'(q)q$ is decreasing (as its derivative is $-qf''(q) < 0$). Thus, $f(q) - f'(q)q \leq 0$ and $s'(q) \leq p(\delta+\lambda)/(\delta+\lambda+f(q))^2 - c$ for all q. Finally, the right-hand side of the latter inequality has a limit equal to $-c$ for $q \to Q$.

Adilov et al. (2014) find a similar result in a different framework, where horizontally differentiated firms have private incentives to launch too many satellites in a two-period model. Their result cannot be directly compared with Property 4, as it brings together both issues of imperfect competition and negative orbital externalities. Moreover, they use a model with two periods of time, whereas we focus on the long-term stationary equilibrium.

Figure 5 is useful to illustrate the determination of q^* and q^o. It decomposes the social surplus $s(q)$ produced periodically by the space sector in its two parts (i.e., $v(q)\,q$ and cq) . The curve $v(q)\,q$ gives the aggregate *expected* revenue in present value from launching q satellites.[11] The line cq represents the corresponding cost. The (vertical) distance between the two curves is equal to $s(q)$. By definition, the physico-economic equilibrium is attained when the space sector generates no surplus. Graphically, this corresponds to the intersection between the two curves, occurring at $q = q^*$. The optimal policy is attained when the distance between the two curves is maximum. Graphically, this is found when the parallel to cq is tangent to $v(q)\,q$, occurring at $q = q^o$.

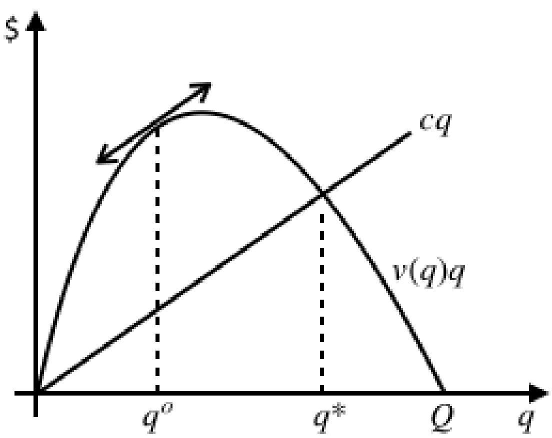

Figure 5: *Expected* revenue and cost of a launching campaign

Figure 6 describes another way to locate q^* and q^o graphically. It is identical to Figure 4, except for the curve representing the *expected* revenue of a *marginal* launch, $v(q)+v'(q)q$ (i.e., the derivative of $v(q)q$). From the proof of Proposition 2, we know that this curve is decreasing as long as it is positive. It lies below the curve representing the *expected* revenue of a satellite, $v(q)$, because it includes the long-term damage generated by a marginal satellite (i.e., $-v'(q)q$). From Figure 4, we know that the physico-equilibrium launch rate q^* is found when the decreasing curve $v(q)$ intersects the horizontal line c. Here, the optimal policy

[11] We know from the proof of Proposition 3 that $v(q)q$ is convex as long as it is increasing. However, it can be either convex or concave otherwise.

q^o is attained when the decreasing curve $v(q) + v'(q)q$ intersects the horizontal line c.

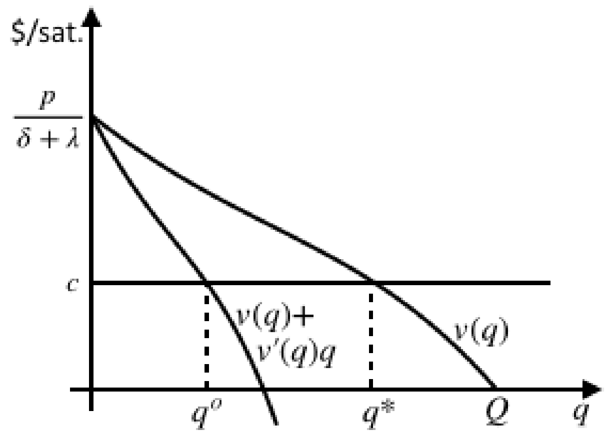

Figure 6: Physico-economic equilibrium vs optimal outcome

Finally, we compare the optimal policy with the maximum carrying capacity of the orbit:

Property 6. In an optimal policy, both situations of under-exploitation and over-exploitation of the orbit from the physical point of view are possible. Formally, there exists $\overline{c}$ such that $0 < \overline{c} < p/(\delta + \lambda)$ and $q^o \gtreqless \overline{q}$ if and only if $c \lesseqgtr \overline{c}$.

Proof of property 6. By Proposition 2, for an interior solution, the optimal rate of launching q^o satisfies

$$s'(q^o) = p\frac{\delta + \lambda + f(q^o) - q^o f'(q^o)}{(\delta + \lambda + f(q^o))^2} - c = 0.$$

Using the implicit function theorem, $\mathrm{d}q^o/\mathrm{d}c = 1/s''(q^o) < 0$ (see the proof of Proposition 2 where we show that $s''(q) < 0$ in a neighborhood of q^o). If $c = 0$, then q^o solves $\delta + \lambda + f(q^o) - q^o f'(q^o) = 0$. As $\overline{q}$ satisfies $\lambda + f(\overline{q}) - \overline{q}f'(\overline{q}) = 0$ and $f(q) - qf'(q)$ is decreasing (as its derivative is $-qf''(q) < 0$), this implies that $q^o > \overline{q}$ if $c = 0$. If $c = p/(\delta + \lambda)$, then it is immediate to show that $q^o = 0 < \overline{q}$. All this proves that there exists $\underline{c}$ such that $0 < \overline{c} < p/(\delta + \lambda)$ and $q^o \gtreqless \overline{q}$ if and only if $c \lesseqgtr \overline{c}$. $\square$

4 Policy implications

In this section, we describe standard economic instruments (command-and-control, tax and market) to regulate space activity, in order to reduce the accumulation of orbital debris and tighten the gap between the physico-economic equilibrium and the optimal outcome.[12] Of course, we are aware that, in reality, implementing such measures would be very complicated, given the international dimension of the problem. Just like other global commons (ozone layer depletion, climate change, international fisheries, etc.), a global agreement is needed before any policy can enter into force.

A first instrument would be to issue an aggregate quota, limiting the quantity of satellites allowed for launch per period. Based on our previous results, it should be set equal to q^o if the objective is to achieve an optimal outcome (see Figures 5 and 6). However, if only the number of satellites is monitored, this policy should be used with caution, as it could ultimately lead to perverse behaviors. Indeed, in our theoretical model, we have considered satellites as standardized units (mass, surface, power, etc.), implicitly designed efficiently to minimize costs. In reality, one way to circumvent the aggregate quota may be for the space sector to design a new generation of more sophisticated satellites, so as to supply more services to consumers and produce higher rents per unit. In addition, throught the implementation of the global quota, this strategy would become profitable in the long term, once the orbit is cleaned up and the life span of satellites is restored. If such a race for sophistication begins, the policy could actually fail, both because the new generation of satellites will be too expensive compared to efficient units and could also contribute more to the accumulation of space debris. In conclusion, if the aggregate quota is the chosen instrument, not only the number of launches needs be monitored, but also all technical characteristics relevant from the perspective of cost and debris accumulation. This might be necessary, even if technology standards generally have detrimental effects in terms of static and dynamic efficiencies (Adilov et al., 2014).

Another instrument would be to impose a tax rate τ paid per satellite launched. Using our results, we can show that it should be set equal to $\tau = -v'(q^o)\,q^o$ if the objective is to attain an optimal outcome. This (pigovian) level coincides with the *expected* loss of revenue caused by a marginal payload to the space sector. It thus induces private actors to internalize their external costs. We can verify that this tax rate actually implements an optimal outcome as follows. From Proposition 1, the physico-economic equilibrium q^* induced by any tax rate τ satisfies $v(q^*) = c + \tau$. From Proposition 2, the optimal allocation q^o satisfies $v(q^o) + v'(q^o)\,q^o = c$. Assuming that $\tau = -v'(q^o)\,q^o$, it follows that $v(q^o) = c + \tau$ and $q^* = q^o$, given that the physico-economic equilibrium is unique by Proposition 1. The underlying intuition can also be discussed using Figure 7. Initially, the physico-economic equilibrium is located at the intersection between $v(q)$ and c. If the unit cost of a satellite is increased by Δ, the physico-economic equilibrium moves at the intersection between $v(q)$ and $c + \Delta$. In Figure 7, the latter coincides with the optimal outcome, since $\Delta = -v'(q^o)q^o$ by construction (i.e., the pigovian tax rate). Importantly, it

[12] Adilov et al. (2014) discuss alternative measures, such as voluntary guidelines and active debris removal. We skip these here, as the model stated above is not designed to address these questions.

should be noted that a tax per satellite should not create perverse incentives to design and launch bigger units, contrary to command-and-control regulations. The reason is because in the physico-economic equilibrium, the space sector makes no extra profit and building more sophisticated (inefficient) satellites could only worsen the situation.

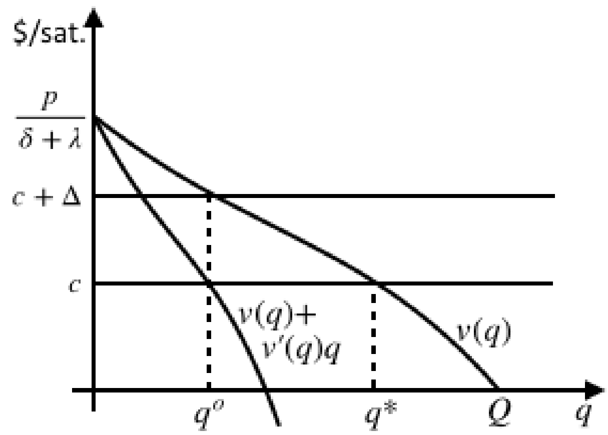

Figure 7: Optimal policy (where $\Delta = -v'(q^o)q^o$)

A last instrument would be to implement a system of individual transferable quotas. Anew, given our previous results, the total quantity of satellites allowed for launch should be set equal to q^o if the objective is to reach an optimal allocation. The aggregate quota is then divided among the firms participating to the quota market. The latters can either launch satellites up to the amount of quota they hold, or they can sell it partially or totally to others. Let us denote by ρ the price of an individual transferable quota. Each firm faces the tradeoff of launching a marginal unit, which is worth $v(q^*) - c$, or selling an individual transferable quota, which is worth ρ. Clearly, in a market equilibrium, it must be the case that $v(q^*) - c = \rho$. As a result, if the total quota of satellites allowed for launch is q^o, we will have $q^* = q^o$ and $\rho = v(q^o) - c$. Moreover, given that q^o satisfies $v(q^o) + v'(q^o)\, q^o = c$ by Proposition 2, the equilibrium price will be $\rho = -v'(q^o)\, q^o$, thus reflecting the external cost of a marginal launch. Using Figure 7, for any total allowable launch rate q, the equilibrium price is equal to the difference between $v(q)$ and c. In particular, it is equal to $\Delta = -v'(q^o)q^o$ when q^o quotas are allocated. Note that the physico-economic equilibrium then coincides with the optimal outcome including in the cost of a satellite the opportunity cost of the foregone quota. As for the case of the pigovian tax, it should be acknowledged here that this mecahnism should not create perverse incentives to design and launch more sophisticated (inefficient) satellites. The reason is because such a strategy would eventually increase the density of space debris and thus depreciate the price of the individual transferable quotas. Moreover, bigger satellites are too costly as compared to optimal units.

5 Numerical application

We propose a specification and a calibration of our model. The aim here is mainly illustrative. Still, the parameters are chosen to fit with actual data, whenever available, in order to give realistic orders of magnitude. A sensitivity analysis is finally performed.

Consider the following specification of the function $f(q)$, giving the rate of defect by collision, once the orbital environment has reached its long-term equilibrium:[13]

$$f(q) = \frac{aq}{Q-q}, \text{ for } 0 \leq q < Q,$$

where $a > 0$. Accordingly, we can calculate (see Appendix A2):[14]

$$\overline{q} = \frac{1}{1+\sqrt{a/\lambda}} Q, \tag{4}$$

$$q^* = \frac{p-(\delta+\lambda)c}{p+(a-\delta-\lambda)c} Q \tag{5}$$

and

$$q^o = \frac{-\frac{c}{p}(\delta+\lambda)+\sqrt{\left(1+\frac{c}{p}(a-\delta-\lambda)\right)\frac{\delta+\lambda}{a}}}{1+\frac{c}{p}(a-\delta-\lambda)+\sqrt{\left(1+\frac{c}{p}(a-\delta-\lambda)\right)\frac{\delta+\lambda}{a}}} Q. \tag{6}$$

Our calibration relies on the following data. We deal with the region of low Earth orbit between 800 km and 1000 km, which is already known to have reached a "runaway" level (Kessler and Anz-Meador, 2001; Kessler et al., 2016).[15] The DISCOS database reports a total of 593 payloads placed in this region since 1957 (i.e., about 10 launches per year).[16] Based on long-term simulations, Farinella and Cordelli (1991) predict an annual probability of collision of an intact satellite of about 0.15, at the end of a time span of 500 years, under their business-as-usual scenario.[17] Somma et al. (2019) argue that a typical

[13] Alternative specifications with two or three parameters are $aq^b/(Q-q)^b$, $-a\ln(1-q/Q)$ and aq^b. Parameter Q is infinite in the latter case.

[14] Note that all results are linearly proportional to Q.

[15] A "runaway" environment is such that no equilibrium is possible, as long as the population of intact objects remains constant (Kessler and Anz-Meador, 2001; Kessler et al., 2016).

[16] https://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database

[17] In Farinella and Cordelli (1991), the annual rate of collision for a satellite is the product of a constant coefficient, equal to $3 \cdot 10^{-10}$, times the number of orbiting fragments larger than 1 cm, which will reach $5 \cdot 10^8$ objects within 500 years according to their simulation.

satellite placed in low Earth orbit has an active lifetime of 8 years. In 2016, the total revenue from satellite services was \$127 700 millions (SIA, 2017). With a total population of 1459 operational satellites (all altitudes between 200 km and 35786 km included), the average revenue per satellite was \$87.53 millions. According to Dos Santos and Le Hir (2016), the cost of designing, building and testing a satellite is between \$100 and \$400 millions. Adding the launch price, we will set the overall cost of a typical satellite equal to \$300 millions. Finally, we will assume that the space sector discount time at a rate equal to 5 % a year.

Based on these information, we propose to use the following benchmark calibration: $Q = 20$ (satellite/year), $a = 0.15$ (%/year)[18], $\lambda = 1/8$ (%/year), $p = 87.53$ (\$millions/satellite/year), $c = 300$ (\$millions/satellite) and $\delta = 0.05$ (%/year).[19] Accordingly, we obtain the following results:

$$\overline{q} = 9.54 \text{ (satellites/year)}, \ \overline{n} = 36.44 \text{ (satellites) and } \overline{d} = 3.82 \text{ (years)}$$

$$q^* = 8.75 \text{ (satellites/year)}, \ n^* = 36.21 \text{ (satellites) and } d^* = 4.14 \text{ (years)}$$

$$q^o = 4.45 \text{ (satellites/year)}, \ n^o = 26.49 \text{ (satellites) and } d^o = 5.96 \text{ (years)}$$

We comment these outcomes using Figures 8 to 10.

Figure 8 represents the *expected* population of satellites $n(q)$ as a function of the launching rate q. The top of this curve at point $\overline{A}$ gives the maximum carrying capacity of the orbit. It is equal to $\overline{n} = 36.44$ satellites, sustainable if the space sector launches $\overline{q} = 9.54$ satellites per year forever. Point A_0, which coordinates are the average number of launches since 1957 (10 satellites/year) and the population of active payloads orbiting between 800 km and 1000 km in 2018 (57 operational satellites according to UCS database), is displayed for the sake of comparison. The reason why point A_0 lies well above the curve representing $n(q)$ is because the current risk of collision is still negligible, as the orbit has not reached its stationary state yet.[20] In other words, the large fleet of active satellites that we currently benefit is probably unsustainable. In the long run, according to this simulation, only a maximum of $\overline{n} = 36.44$ satellites can be sustained and will necessitate to refill the orbit with for $\overline{q} = 9.54$ satellites/year. Points A^* and A^o respectively plots the outcomes arising in a physico-economic equilibrium and socially opimal policy. Note that no physical over-exploitation of the orbit occurs in the physico-economic equilibrium, since point A^* lies at the left of point $\overline{A}$.

[18] According to Farinella and Cordelli (1991), parameter a is chosen to satisfy $f(10) = 0.15$, given $Q = 20$.

[19] Below, we provide a sensitivity analysis to extend this benchmark configuration.

[20] All simulation models predict a large increase of space debris in the future under their business-as-usual scenario (Drmola and Hibik, 2018; Eichler and Reynolds, 1997; Farinella and Cordelli, 1991; Somma et al., 2018).

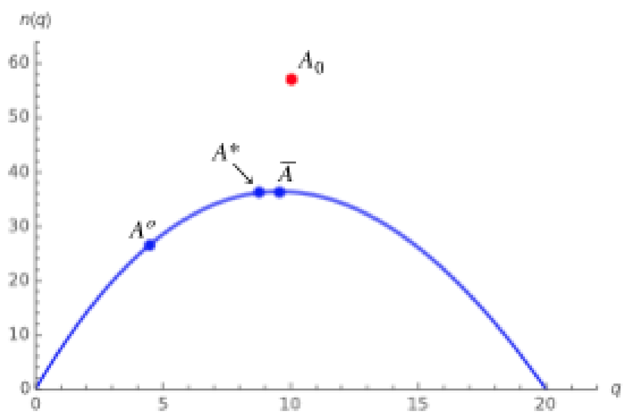

Figure 8: Maximum carrying capacity (units: sat./year; sat.)

Figure 9 illustrates the physico-economic equilibrium and the optimal outcome. The inverted U-shaped curve displays the *expected* present revenue $v(q)\,q$ of each generation of q satellites launched yearly. The line represents the total cost cq. The intersection of these two curves at point A^* determines the physico-economic equilibrium outcome. From above, it is attained with $q^* = 8.75$ launches per year. The dashed line parallel to cq is used to locate the optimal policy situation. By construction, the distance between $v(q)\,q$ and cq is maximum at point A^o when it is tangent to $v(q)\,q$. From above, it is attained for $q^o = 4.45$ launches per year.

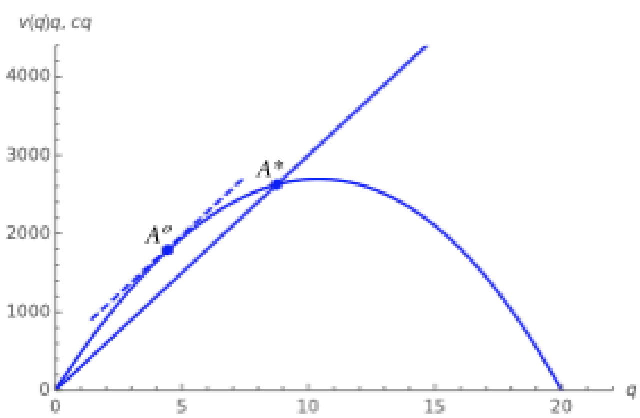

Figure 9: Physico-economic equilibrium *vs* Social optimum (units: sat./year; $millions)

Figure 10 plots the *expected* lifetime of a satellite $d(q)$ as a function of the launch rate q. It is decreasing from 8 years for $q = 0$, inducing a clean orbital environment in the long run, to 0 year for $q \to 20$, inducing the occurrence of the Kessler syndrome in the long run by assumption in our benchmark calibration. The satellite's *expected* lifetimes associated with the three outcomes of interest of our model are displayed. Points $\overline{A}$, A^* and A^o respectively stand for the maximum carrying capacity, the physico-economic equilibrium and and social optimum. The long-term *expected* operational lifetimes are $\overline{d} = 3,82$ years, $d^* = 4.14$ years and $d^o = 5.96$ years, corresponding to launch rates $\overline{q} = 9.54$ satellites/year, $q^* = 8.75$ satellites/year and $q^o = 4.45$ satellites/year respectively.

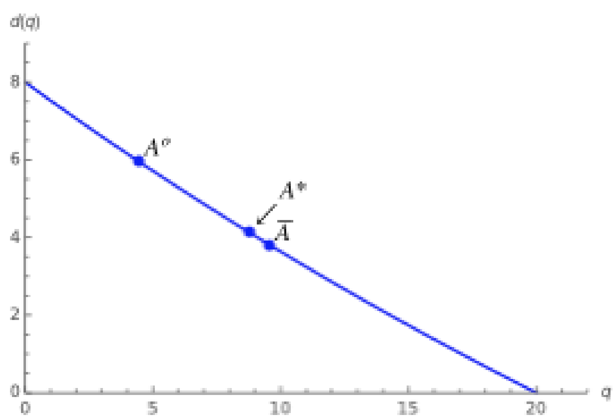

Figure 10: *Expected* lifetime of a satellite (units: sat./year; years)

From the policy perspective, we can also calculate the alternative economic instruments evoked in Section 4. Under the command-and-control approach, the aggregate quota of satellites allowed must be set equal to $q^o = 4.45$ launches per year. The launch tax must be set equal to $\tau = v'(q^o)\, q^o = \$101.69$ millions/satellite. Finally, under a system of individual transferable quotas, the aggregate number of launches allowed must be set equal to $q^o = 4.45$ launches per period. In a market equilibrium, the price of an individual transferable quota is then equal $\rho = v'(q^o)\, q^o = \$101.69$ millions/satellite. To grap a better idea of this result, these amount can be compared with the overall cost of a satellite, which lies between \$100 and \$400 millions (Dos Santos and Le Hir, 2016).

Finally, we provide a sensitivity analysis, in order to check the robustness of our results to changes in the benchmark calibration. We randomize the six primary data of our calibration (i.e., Q, $f(10)$, λ, p, c and δ), by letting them vary by more or less 33,33% around their baseline values.[21] In order to get a

[21] Precisely, we draw $Q \in [13.33, 26.67]$, $f(10) \in [0.1, 0.2]$, $\lambda \in [0.0833, 0.1667]$, $p \in [58.3505, 116.7009]$, $c \in [200, 400]$ and $\delta \in [0.0333, 0.0667]$, according to uniform distributions. Parameter a is calculated by solving $f(10) = 10a/(Q - 10)$.

representative picture of all possible configurations, we implement $5^6 = 15625$ random calibrations. We calculate the corresponding outcomes, i.e., launch rates, *expected* populations of active satellites, *expected* operational lifetimes and economic instruments. The corresponding data are represented in the form of box-and-whisker plots in parts *a* to *d* of Figure 11, respectively.[22]

Let us first consider the interior of the boxes, which includes 50% of all simulations by construction.[23] Thus, most of the time, the maximum carrying capacity $\overline{n}$ lies between 24.37 and 49.31 active satellites. It is attained with a launch rate $\overline{q}$ between 7.07 and 11.92 satellites each year. The launch rate in the physico-economic equilibrium q^* exhibits a larger dispersion, as it varies between 4.28 and 11.97 satellites per year. The resulting long-term *expected* population n^* varies between 18.33 and 43.87 payloads. Finally, the launch rate in a social optimum q^o generally lies between 2.07 and 6.20 payloads per year. This allows to sustain an *expected* population of active satellites n^o varying between 12.57 to 36.72 units in the long run. Note that the latter is not that far from the two others outcomes, despite a launch rate dramatically lower. The reason of this is to be found in the longer *expected* lifetime d^o that the optimal policy induces in the long term, between 5.37 and 6.84 years. This is to be compared with an *expected* lifetime $\overline{d}$ between 3.30 and 4.33 years in the maximum carrying capacity situation, and d^* between 3.26 and 5.29 years in the physico-economic equilibrium. Finally, the damage of a marginal launch at the social optimum in general varies between 55.34 and 153.97 $millions/satellite.

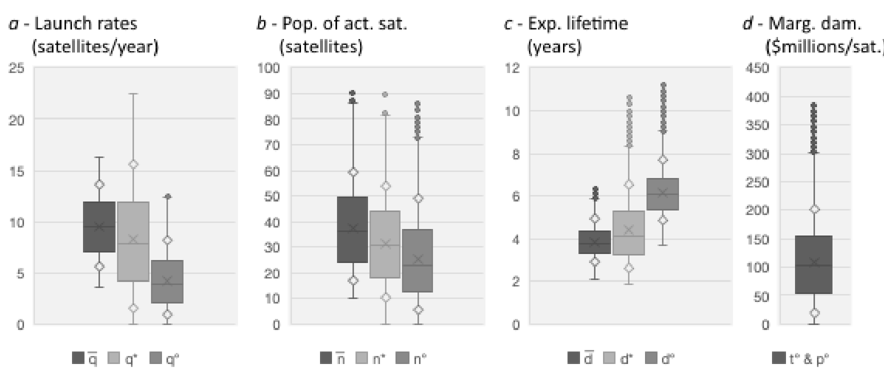

Figure 11: Sensitivity analysis (over 15625 simulations)

Let us now take a look at extreme values. For the sake of concision, we focus on the launch rates and the economic instruments only. Filtering the 15625

[22]The boxes plots the first and third quartiles. The band inside the box is the median. The cross represents the mean. The diamonds represent the first and ninth deciles. The ends of the whiskers are the lowest datum still within 1.5 interquartile range of the lower quartile, and the highest datum still within 1.5 interquartile range of the upper quartile. "Outliers" are plotted as individual points.

[23]Also outcomes inside the segments joining the diamonds represent 80% of all data.

simulations to sort out all cases such that the physico-economic equilibrium launch rate $\overline{q}$ is larger than 21.26 satellites per year leaves us with 16 outcomes (i.e., 0.1% of all cases). Interestingly, they coincide with configurations such that the optimal launch rates q^o is larger than 11.67 satellites per year. Reciprocally, filtering the 15625 simulations to sort out all simulations such that the socially optimal launch rates q^o is larger than 12.23 satellites per year leaves us with 16 outcomes (i.e., 0.1% of all cases). Likewise, they coincide with configurations such that the physico-economic equilibrium launch rates $\overline{q}$ is larger than 20.45 satellites per year. This means that launch rates are simultaneously very high in the physico-economic equilibrium and in the social optimum. Looking for a general pattern in the parameters inducing these cases, we observe that all parametrizations satisfy $a \leq 0.09$, $Q \geq 24.83$, $c \leq 267.02$ and $p \geq 90.05$ (and no clear-cut regularity for the other parameters). Hence, they occur only under the conjonction of favourable physical and economical conditions, as compared to our benchmark calibration, that is, if the orbital environment is not too constraining (i.e., a is small enough and Q is large enough) *and* if the space activity is economically attractive (i.e., c is small and p is sufficiently large). Now, filtering the 15625 simulations to sort out all cases such that the marginal damage in the social optimal plan is larger than 339.84 $millions/satellite leaves us with 16 outcomes (i.e., 0.1% of all cases). Again, looking for a general pattern in the parameters generating these cases, we find that $\lambda \leq 0.09$, $c \leq 230.5$ and $p \geq 107.95$ (and no clear-cut regularity for the other parameters). Accordingly, a large pigovian tax or equilibrium price of an individual launch quota occur only if the rate of failure of a satellite is small enough, if the cost of a satellite is small enough *and* if the annual rent of a satellite is large enough.

6 Conclusion

In this paper, we have developed a simplified physico-economic model of orbital environment and space activity. Focusing on the long-term orbital equilibrium, we have assumed that the risk of collision is an increasing and convex function of the launch rate, becoming arbitrary large when the launch rate is large enough. Then the *expected* population of satellites in the long term is shown to be a inverted-U shape function of the launch rate. This setting allows us to define and calculate the maximum carrying capacity of the orbital environment, the physico-economic equilibrium launch rate (that would emerge under conditions of open-access to the orbit) and the socially optimal launch rate (that maximizes the surplus of the space sector per period). All three outcomes has been compared, in order to highlight conditions of either a physical or an economic overexploitation of the orbit. Finally, we have described economic instruments (command-and-control, tax, market) to regulate the space activity and explained how to use them optimally.

This work paves the way for much future research. From a theoretical point of view, an interesting extension will be to build a physical model of the orbital environment, with three state variables, one for the debris population, one for the end-of-life satellite population and one for the active satellite population.[24] The objective will be to use optimal control theory to characterize an optimal orbit exploitation trajectory, and differential game theory to describe a non-cooperative orbit exploitation trajectory. The main contribution of this extension will be to focus on transition dynamics, instead of focusing on stationary states. Another will be to consider complementary strategies other than the launch rate for managing the orbit congestion, such as automatic de-orbiting capabilities and active debris removal. Ideally, the model should remain simple enough to be resolved analytically. Under this condition, it will be possible to derive general results about the optimal design of satellites and the conditions for cost-effectiveness of space debris removal techniques.

In the longer term, applied research will be needed to design more realistic models, taking better into account the diversity of space debris and the relationships between the different altitudes of the Earth's orbit. On this global scale, a physico-economic equilibrium will be defined as a state such that each region of the Earth's orbit is exploited until all economic rents have dissipated. However, the externalities generated by space objects will depend on their destination altitude, due to the Earth's attraction and atmospheric drag, determining their post-mission residence time. In addition, altitude will play an asymmetric role, since satellites placed in higher orbits represent a future threat to those placed lower, while the opposite is not true. These mechanisms deserve to be studied and evaluated if appropriate economic incentives are to be developed to ensure optimal exploitation of the Earth's orbit.

Appendix

A1. A stylized physical model of the orbital environment

We describe here a simplified model of the orbital environment of a region of low Earth orbit. We use the following notations:

$$q\left(t\right) = \text{launch rate at time } t;$$

$$r\left(t\right) = \text{debris removal at time } t;$$

$$x\left(t\right) = \text{stock of orbital debris at time } t;$$

$$y\left(t\right) = \text{stock of operational satellites at time } t.$$

We propose to formalize the evolution of the orbital environment by way of the following dynamical system:

$$\dot{x}\left(t\right) = \alpha q\left(t\right) - r\left(t\right) + s(t) - \beta x\left(t\right) + \left(\lambda + \varphi\left(x\left(t\right)\right)\right) y\left(t\right),\ x\left(0\right) = x_0,$$

[24] An intermediate attempt is already made in Appendix A1 for illustrative purpose.

$$\dot{y}(t) = q(t) - (\lambda + \varphi(x(t)))\, y(t), \; y(0) = y_0,$$

with $\varphi(.)$ an increasing and convex function, giving the risk of fatal collision between satellites or space debris.

The first differential equation describes the evolution of the stock of orbital debris, $x(t)$. The first two terms, $\alpha q(t)$ and $r(t)$, depend on the activity of the space sector. On the one hand, $\alpha q(t)$ debris are released in the orbit, as a byproduct of the launching activities (rocket bodies, space objects), with $\alpha \geq 0$. Parameter α gives the quantity of debris per launch. On the other hand, $r(t)$ debris are removed from the orbit, thanks to the active debris removal activities.[25] The third term, $s(t)$, reflects orbital debris coming from nearby altitudes, either scattered after collisions or decayed by the atmospheric drag. Inversely, the third term, $\beta x(t)$, represents the decay of orbital debris due to the atmospheric drag, with $\beta > 0$. Its inverse, $1/\beta$, can be interpreted as their average lifetime in orbit. The last term, $(\lambda + \varphi(x(t)))\, y(t)$, represents the addition of the recently "dead" satellites to the stocks of orbital debris.

The second differential equation describes the evolution of the stock of operational satellites, $y(t)$. The first component, $q(t)$, is the result of the lauching activity of the space sector. The last term, $(\lambda + \varphi(x(t)))\, y(t)$, gives the number of satellites that defects, either for technical (i.e., fuel, failures) or environmental (collisions) reasons. The rate of decay by technical failure is denoted by λ. The rate of decay by collision is denoted by $\varphi(x(t))$.

Now, let us consider constant rates of launching and debris removal forever, i.e., $q(t) = q$ and $r(t) = r$ for all t. Assume further that the nearby orbits are in a stationary state, implying that $s(t) = s$ for all t. We wish to calculate the stationary solution of the dynamical system. Thus, we search $x(t) = x^*$ and $y(t) = y^*$ such that $\dot{x}(t) = \dot{y}(t) = 0$ for all t. Summing the two differential equations yields

$$\dot{x}(t) + \dot{y}(t) = (1+\alpha)\, q - r + s - \beta x(t) = 0,$$

which implies that

$$x(t) = x^* = \frac{(1+\alpha)\, q - r + s}{\beta}.$$

Then, substituting this into the second differential equation yields

$$\dot{y}(t) = q - \left(\lambda + \varphi\left(\frac{(1+\alpha)\, q - r + s}{\beta}\right)\right) y(t) = 0,$$

[25] Active debris removal activities have attracted attention recently and might become profitable in the future (Somma et al., 2017). Although not mentionned in the main text of this article, active debris removal can be seen as implicit in our framework, as long as the effort of debris removal remains constant over time (either in absolute value or relative value, i.e. as a ratio of the stock of debris).

which gives that

$$y(t) = y^* = \frac{q}{\lambda + \varphi\left(\frac{(1+\alpha)q - r + s}{\beta}\right)}.$$

It is worth noting that these closed-form expressions echoe and rationalize the model used in the main text. In Section 2, we have postulated the existence of a function $x = \phi(q)$, giving the quantity of debris in the long term as a function the launch rate q. Our calculus here suggest that a possible specification could be $\phi(q) \equiv x^* = ((1+\alpha)q - r + s)/\beta$. Also, we have defined a function $\mu = f(q)$, giving the risk of collision in the long term as a function of the launch rate q. Our calculus here show that a possible specification could be $f(q) \equiv \varphi(x^*) = \varphi(((1+\alpha)q - r + s)/\beta)$. Finally, we have calculated in section 2 the *expected* population of satellites in the long term as $n(q) = q/(\lambda + f(q))$. The fact that this coincides with the expression of y^* obtained here confirms again that the dynamical system considered above is a way to rationalize our model in the main text.

Moreover, let us bring attention to the fact that the closed-form stationary solution obtained here allows to highlight some assumptions left implicit in the main text. Indeed, it appears that the function $f(q)$ depends on:

1/ the behavior of the space sector, through the quantity of debris released after a launch, α, and the quantity of space debris removed periodically, r;

2/ the physical characteristics of the orbit, here formalized by the rate of decay of space debris, β, and the function giving the rate of collision, φ;

3/ the state of the nearby orbits, represented here by the debris coming from other altitudes, s.

We can also supply conditions such that the stationary solution is locally stable. Linearization of the dynamical system around its stationary solution gives

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = A \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} \alpha q(t) - r(t) + s(t) \\ q(t) \end{bmatrix},$$

where

$$A = \begin{bmatrix} -\beta + \varphi'(x^*) y^* & \lambda + \varphi(x^*) \\ -\varphi'(x^*) y^* & -\lambda - \varphi(x^*) \end{bmatrix}.$$

Sufficient conditions for local stability are

$$\text{trace}(A) = -\beta + \varphi'(x^*) y^* - \lambda - \varphi(x^*) < 0$$

and

$$\det(A) = \beta(\lambda + \varphi(x^*)) > 0.$$

The first condition is satisfied if

$$\frac{\frac{q}{\beta}\varphi'\left(\frac{(1+\alpha)q-r+s}{\beta}\right)}{\left(\lambda+\varphi\left(\frac{(1+\alpha)q-r+s}{\beta}\right)\right)^2} < 1.$$

The second condition is always satisfied.

A2. Calculus when $f(q) = aq/(Q-q)$ (with $a > 0$ and $0 \leq q < Q$).

Carrying capacity: Using (1), the *expected* population of satellites in a long-term orbital equilibrium is

$$n(q) = \frac{(Q-q)\,q}{\lambda(Q-q)+aq},$$

from which we can show that

$$n(0) = \lim_{q\to Q} n(q) = 0$$

and

$$n'(q) = \frac{\lambda(Q-q)^2 - aq^2}{(\lambda(Q-q)+aq)^2}.$$

As the denominator is strictly decreasing (its derivative is $-2(\lambda(Q-q)+aq) < 0$), $n(q)$ admits a unique maximum for $q = \overline{q}$ satisfying $n'(\overline{q}) = 0$. It is immediate to calculate

$$\overline{q} = \frac{1}{1+\sqrt{a/\lambda}}Q.$$

Physico-economic equilibrium: From (2), the *expected* present value of the cash-flow generated by a satellite is

$$v(q) = \frac{p(Q-q)}{(\delta+\lambda)(Q-q)+aq},$$

from which we can calculate

$$v(0) = \frac{p}{\delta+\lambda},$$

$$\lim_{q\to Q} v(q) = 0,$$

and

$$v'(q) = -\frac{apQ}{\left((\delta+\lambda)(Q-q)+aq\right)^2} < 0.$$

Accordingly, if $p/(\delta+\lambda) > c$, there exists a unique physico-economic equilibrium $q = q^*$ satisfying $v(q^*) = c$. From this, we can obtain

$$q^* = \frac{p-(\delta+\lambda)c}{p+(a-\delta-\lambda)c}Q.$$

Optimal policy: Using (3), the social surplus from each generation of q statellites is

$$s(q) = \left(\frac{p(Q-q)}{(\delta+\lambda)(Q-q)+aq} - c\right)q,$$

from which we can get

$$s'(q) = p\frac{(\delta+\lambda)(Q-q)^2 - aq^2}{\left((\delta+\lambda)(Q-q)+aq\right)^2} - c.$$

An optimal launching policy must satify $s'(q) = 0$. Some algebra shows that this condition is equivalent to

$$(1+A)x^2 + 2ABx - (1-AB)B = 0,$$

where we let $A = ac/p$, $B = (\delta+\lambda)/a$ and $x = q/(Q-q)$. Assuming that $p/(\delta+\lambda) > c$, this is readily solved using standard methods:[26]

$$\triangle = 4(1+A(1-B))B > 0,$$

$$x^- = \frac{-AB - \sqrt{(1+A(1-B))B}}{1+A}$$

and

$$x^+ = \frac{-AB + \sqrt{(1+A(1-B))B}}{1+A}.$$

As we seek a positive root, we disregard x^-, which is clearly negative, and keep x^+, which can be shown to be positive given that $p/(\delta+\lambda) > c$. Indeed, after subsitution of $A = ac/p$ and $B = (\delta+\lambda)/a$, we get

$$x^+ = \frac{-\frac{c}{p}(\delta+\lambda) + \sqrt{\left(1+\frac{c}{p}(a-\delta-\lambda)\right)\frac{\delta+\lambda}{a}}}{1+a\frac{c}{p}}.$$

Then, some elementary algebra shows that the inequality $x^+ > 0$ is equivalent to

[26]To verify that $\triangle > 0$ if $p/(\delta+\lambda) > c$, rewrite it as $\triangle = 4(A+1-AB)B$. Given that $A > 0$ and $B > 0$, it is sufficient that $1-AB = 1-(\delta+\lambda)c/p > 0$, which is true if $p/(\delta+\lambda) > c$.

$$-a\left(\delta+\lambda\right)\left(\frac{c}{p}\right)^2+\left(a-\delta-\lambda\right)\frac{c}{p}+1>0.$$

Note that the left-hand side is an equation of second degree in c/p. Moreover, it is equal to 1 if $c/p=0$ and to 0 if $c/p=1/\left(\delta+\lambda\right)$. Also, the coefficient of $\left(c/p\right)^2$ is negative. Clearly, given that $c/p<0$ is not feasible, the inequality will be met if and only if $0\leq c/p<1/\left(\delta+\lambda\right)$. Finally, substituting $x=q/\left(\overline{q}-q\right)$ into the expression of x^{+}, we can obtain

$$q^{o}=\frac{-\frac{c}{p}\left(\delta+\lambda\right)+\sqrt{\left(1+\frac{c}{p}\left(a-\delta-\lambda\right)\right)\frac{\delta+\lambda}{a}}}{1+\frac{c}{p}\left(a-\delta-\lambda\right)+\sqrt{\left(1+\frac{c}{p}\left(a-\delta-\lambda\right)\right)\frac{\delta+\lambda}{a}}}Q.$$

Policy instruments: The pigovian tax τ and the equilibrium price ρ of an individual transferable quota is equal to the forgone revenue of inframarginal satellites due to a marginal launch

$$-v'\left(q\right)q=\frac{apQ}{\left(\left(\delta+\lambda\right)\left(Q-q\right)+aq\right)^2}q,$$

evaluated for $q=q^{o}$.

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